## Contents

1	Max	well's Equations		
	1.1	Introduction		
	1.2	Gradient, Curl and Divergence		
	1.3	Integral Theorems for the Case of $\mathbb{R}^3$		
	1.4	Maxwell's Equations in Integral Form		
		1.4.1 The Law of Induction		
		1.4.2 Gauss' Law		
		1.4.3 The Law of Biot and Savart		
		1.4.4 The Lorentz Force		
		1.4.5 The Continuity Equation		
	1.5	Maxwell's Equations in Local Form		
		1.5.1 Induction Law and Gauss' Law		
		1.5.2 Local Form of the Law of Biot and Savart		
		1.5.3 Local Equations in All Systems of Units		
		1.5.4 The Question of Physical Units		
		1.5.5 Equations of Electromagnetism in SI System		
		1.5.6 The Gaussian System of Units29Scalar Potentials and Vector Potentials35		
	1.6 Scalar Potentials and Vector Potentials			
		1.6.1 A Few Formulae from Vector Analysis		
		1.6.2 Construction of a Vector Field		
	`	from Its Source and Its Curl		
		1.6.3 Scalar Potentials and Vector Potentials		
	1.7	Phenomenology of the Maxwell Equations		
		1.7.1 The Fundamental Equations and Their Interpretation 47		
		1.7.2 Relation Between Displacement Field and Electric Field 50		
		1.7.3 Relation Between Induction and Magnetic Fields 52		
	1.8	Static Electric States		
		1.8.1 Poisson and Laplace Equations		

ix

•



		1.8.2 Surface Charges, Dipoles and Dipole Layers		
		1.8.3 Typical Boundary Value Problems		
		1.8.4 Multipole Expansion of Potentials		
	1.9	Stationary Currents and Static Magnetic States		
		1.9.1 Poisson Equation and Vector Potential		
		1.9.2 Magnetic Dipole Density and Magnetic Moment 84		
		1.9.3 Fields of Magnetic and Electric Dipoles		
		1.9.4 Energy and Energy Density		
		1.9.5 Currents and Conductivity		
2	Syn	ometries and Covariance of the Maxwell Equations		
	2.1	Introduction		
	2.2	The Maxwell Equations in a Fixed Frame of Reference 97		
		2.2.1 Rotations and Discrete Spacetime Transformations 98		
		2.2.2 Maxwell's Equations and Exterior Forms		
	2.3	Lorentz Covariance of Maxwell's Equations		
		2.3.1 Poincaré and Lorentz Groups		
		2.3.2 Relativistic Kinematics and Dynamics		
		2.3.3 Lorentz Force and Field Strength		
		2.3.4 Covariance of Maxwell's Equations		
		2.3.5 Gauge Invariance and Potentials		
	2.4	Fields of a Uniformly Moving Point Charge		
	2.5	Lorentz Invariant Exterior Forms and the Maxwell Equations 14		
		2.5.1 Field Strength Tensor and Lorentz Force		
		2.5.2 Differential Equations for the Two-Forms $\omega_F$ and $\omega_F$ . 14.		
		2.5.3 Potentials and Gauge Transformations		
		2.5.4 Behaviour Under the Discrete Transformations 149		
		2.5.5 * Covariant Derivative and Structure Equation 150		
3	Ma	xwell Theory as a Classical Field Theory		
÷	3.1			
	3.2	Lagrangian Function and Symmetries in Finite Systems 152		
	5.2	3.2.1 Noether's Theorem with Strict Invariance		
		3.2.2 Generalized Theorem of Noether		
	3.3	Lagrangian Density and Equations of Motion for a Field Theory 15'		
	3.4	Lagrangian Density and Equations of Motion for a Field Theory 11.		
	3.5	<b>0 0 1</b>		
	5.5	Symmetries and Noether Invariants1633.5.1Invariance Under One-Parameter Groups169		
		3.5.2 Gauge Transformations and Lagrangian Density 17		
		3.5.2 Gauge Hanstonnations and Eaglangian Density		
		3.5.4 Interpretation of the Conservation Laws		
	3.6	Wave Equation and Green Functions		
	5.0	3.6.1 Solutions in Noncovariant Form		
		3.6.2 Solutions of the Wave Equation in Covariant Form		
		- 5.0.2 Solutions of the mare Equation in Covariant Form (1, 1, 1, 10)		

	3.7	Radiation of an Accelerated Charge
4	Sim	ple Applications of Maxwell Theory
	4.1	Introduction
	4.2	Plane Waves in a Vacuum
		and in Homogeneous Insulating Media
		4.2.1 Dispersion Relation and Harmonic Solutions 199
		4.2.2 Completely Polarized Electromagnetic Waves 205
		4.2.3 Description of Polarization
	4.3	Simple Radiating Sources
		4.3.1 Typical Dimensions of Radiating Sources
		4.3.2 Description by Means of Multipole Radiation 216
		4.3.3 The Hertzian Dipole
	4.4	Refraction of Harmonic Waves
		4.4.1 Index of Refraction and Angular Relations
		4.4.2 Dynamics of Refraction and Reflection
	4.5	Geometric Optics, Lenses and Negative Index of Refraction 232
		4.5.1 Optical Signals in Coordinate and in Momentum Space . 232
		4.5.2 Geometric (Ray) Optics and Thin Lenses
		4.5.3 Media with Negative Index of Refraction
		4.5.4 Metamaterials with Negative Index of Refraction 247
	4.6	The Approximation of Paraxial Beams
	1.0	4.6.1 Helmholtz Equation in Paraxial Approximation
		4.6.2 The Gaussian Solution
		4.6.3 Analysis of the Gaussian Solution
		4.6.4 Further Properties of the Gaussian Beam
		1
5		al Gauge Theories
	5.1	Introduction
	5.2	Klein–Gordon Equation and Massive Photons
	5.3	The Building Blocks of Maxwell Theory
	5.4	Non-Abelian Gauge Theories
		5.4.1 The Structure Group and Its Lie Algebra
		5.4.2 Globally Invariant Lagrange Densities
		5.4.3 The Gauge Group
		5.4.4 Potential and Covariant Derivative
		5.4.5 Field Strength Tensor and Curvature
		5.4.6 Gauge-Invariant Lagrange Densities
		5.4.7 Physical Interpretation
		5.4.8 *More on the Gauge Group
	5.5	The U(2) Theory of Electroweak Interactions
		5.5.1 A U(2) Gauge Theory with Massless Gauge Fields 295
		5.5.2 Spontaneous Symmetry Breaking
		5.5.3 Application to the U(2) Theory

	5.6	Epilogue and Perspectives		17		
6	Class	sical Field Theory of Gravitation		)9		
	6.1	Introduction		)9		
	6.2	Phenomenology of Gravitational Intera		0		
		6.2.1 Parameters and Orders of Magn	nitude	0		
		6.2.2 Equivalence Principle and Univ	versality	2		
		6.2.3 Red Shift and Other Effects of	Gravitation 31	6		
		6.2.4 Some Conjectures and Further	Program	2		
	6.3	Matter and Nongravitational Fields .		22		
	6.4	Spacetimes as Smooth Manifolds				
		6.4.1 Manifolds, Curves, and Vector	Fields	25		
		6.4.2 One-Forms, Tensors, and Tenso	or Fields	32		
		6.4.3 Coordinate Expressions and Te	nsor Calculus 33	35		
	6.5 Parallel Transport and Connection					
		6.5.1 Metric, Scalar Product, and Ind		13		
		6.5.2 Connection and Covariant Deri	vative	15		
		6.5.3 Torsion and Curvature Tensor F	Fields	19		
		6.5.4 The Levi-Civita Connection .		51		
		6.5.5 Properties of the Levi-Civita Co		;3		
		6.5.6 Geodesics on Semi-Riemannian	n Spacetimes 35	56		
		6.5.7 More Properties of the Curvatu	re Tensor	50		
	6.6	The Einstein Equations		53		
		6.6.1 Energy-Momentum Tensor Field	ld in Curved Spacetime . 36	53		
		6.6.2 Ricci Tensor, Scalar Curvature,	, and Einstein Tensor 36	54		
		6.6.3 The Basic Equations		6		
	6.7	Gravitational Field of a Spherically Sy	mmetric Mass Distribution 37	1		
		6.7.1 The Schwarzschild Metric		12		
		6.7.2 Two Observable Effects		14		
		6.7.3 The Schwarzschild Radius is an	n Event Horizon 38	32		
	6.8	Some Concluding Remarks		35		
Bibl	liogra	ohy		37		
Som				20		
3011	ie msi	orical Remarks		,7		
Exe	rcises	· · · · · · · · · · · · · · · · · · ·		<b>)</b> 5		
Sele	cted S	olutions of the Exercises	40	)3		
Inde	ex.		42	29		
Aho	out the	Author	43	33		
			· · · · · · · · · · · · · · · ·			