

Contents

1	Weak convergence of stochastic processes — 1
	Introduction — 1
2	Weak convergence in metric spaces — 4
2.1	Cylindrical measures — 4
2.2	Kolmogorov consistency theorem — 6
2.3	The finite dimensional family for Brownian motion — 9
2.4	Properties of Brownian motion — 11
2.5	Kolmogorov continuity theorem — 14
2.6	Exit time for Brownian motion and Skorokhod theorem — 16
2.6.1	Skorokhod theorem — 19
2.7	Embedding of sums of i.i.d. random variable in Brownian motion — 21
2.8	Donsker's theorem — 23
2.9	Empirical distribution function — 25
2.10	Weak convergence of probability measure on Polish space — 29
2.10.1	Prokhorov theorem — 34
2.11	Tightness and compactness in weak convergence — 36
3	Weak convergence on $C[0, 1]$ and $D[0, \infty)$ — 41
3.1	Structure of compact sets in $C[0, 1]$ — 41
3.1.1	Arzela-Ascoli theorem — 41
3.2	Invariance principle of sums of i.i.d. random variables — 45
3.3	Invariance principle for sums of stationary sequences — 48
3.4	Weak convergence on the Skorokhod space — 50
3.4.1	The space $D[0, 1]$ — 50
3.4.2	Skorokhod topology — 52
3.5	Metric of $D[0, 1]$ to make it complete — 53
3.6	Separability of the Skorokhod space — 56
3.7	Tightness in the Skorokhod space — 58
3.8	The space $D[0, \infty)$ — 60
3.8.1	Separability and completeness — 64
3.8.2	Compactness — 64
3.8.3	Tightness — 66
3.8.4	Aldous's tightness criterion — 67
4	Central limit theorem for semi-martingales and applications — 70
4.1	Local characteristics of semi-martingale — 70
4.2	Lenglart inequality — 72

4.3	Central limit theorem for semi-martingale — 76
4.4	Application to survival analysis — 81
4.5	Asymptotic distribution of $\hat{\beta}(t)$ and Kaplan-Meier estimate — 85
5	Central limit theorems for dependent random variables — 89
6	Empirical process — 110
6.1	Spaces of bounded functions — 114
6.2	Maximal inequalities and covering numbers — 119
6.3	Sub-Gaussian inequalities — 125
6.4	Symmetrization — 126
6.4.1	Glivenko-Cantelli theorems — 129
6.4.2	Donsker theorems — 131
6.5	Lindberg-type theorem and its applications — 133
Bibliography — 142	