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Facility Location Planning for Distribution Networks and Infrastructure Locations

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For my father & son.

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CHAPTER I

INTRODUCTION

This chapter includes parts of the paper:

Götzinger M., Brandt T., Neumann D. (2012). Green Facility Location – A Case Study, *18th Americas Conference on Information Systems (AMCIS 2012)*

1. Introduction and Motivation

Facility location is one of the most important strategic questions to many businesses and public institutions offering products or services. Location is essential for achieving an optimal cost position, fulfilling guaranteed service times to customers or ensuring accessibility. Mathematical models, generally referred to as facility location problems (FLP), support decision makers in the planning process of establishing or relocating existing facilities. FLPs have permanently received attention by researchers and are widely discussed since the 1960s. Applications comprise a wide range of economic and daily live, reaching from the location of warehouses through telecommunication switching centers to fire stations (Current, Daskin, & Schilling, 2002). The focus of this thesis is on two applications for facility location models: First, distribution network planning, i.e. determining locations of warehouse facilities, and second, infrastructure planning, i.e. determining charge point locations for electric vehicles. Consequently, the remainder of this chapter is structured along those two dimensions.

1.1. Distribution Network Planning

Finding the optimal warehouse location is a problem that occurs in all stages of the supply chain planning process. It is an important aspect in planning a new supply chain from scratch. Companies are entering new markets and need to establish a distribution structure with warehouses in order to ensure a smooth distribution. It is just as important for reevaluating existing supply chains: Demand is not stable over a period of years, which requires adaptation of the existing warehouse structure. The same is true for a company acquiring a competitor: due to the acquisition parallel distribution structures may exist and a consolidation is required. Above examples illustrate that the challenge for supply chain planners lies in finding optimal warehouse locations in their planning process. Optimal in a distribution planning context mainly refers to a cost-minimal solution by taking into consideration accessibility of customers. Quantitative, data-based facility location models support supply chain planners in modeling the reality, while accounting for various logistical aspects. Recent company press reports on new warehouse openings indicate that businesses require comprehensive location models in order to capture those aspects simultaneously. The following news releases and press reports are representative for those business needs and reflect actual requirements related to location models:

- Volkswagen (VW) revealed in June, 2013 its new distribution centers in Roane County, TN. The facility functions as a redistribution center for service warehouses in the U.S., Canada, Mexico and Germany. VW officials stated that the facility will help to reduce delivery times of components (Krafcik, 2013).
- Grainger opened in July, 2013 a new distribution center in Minooka, IL. The facility is one of 15 distribution centers operating within the U.S., Canada and Mexico, and serves as a central distribution center. The news report states that *"the distribution center helps enable Grainger to deliver products same-day or next-day to its customers nationwide"* (W. W. Grainger Inc., 2013).
- Procter & Gamble (P&G) is about to establish a new warehouse in Franklin County, PA, which is expected to open in July, 2014. The premises is leased from a real estate investment company and operated by a contractor. A P&G spokesman stated that the company is active in consumer goods and not in running warehouses. The site was chosen due to its proximity to highways and populations centers (Fitch, 2013).
- W.P. Carey Inc., a real estate investment trust, announced in July, 2013 that it has acquired a logistics center from H&M Hennes & Mauritz AB in Poznan, Poland. The press report says that *"the center is subject to a long-term, triple-net lease that is fully guaranteed by H&M. Located in Poznan, [...], the modern center is critical to the supply chain of H&M in Europe. It is H&M's European distribution center for*

> *Eastern Europe, as well as its primary e-commerce and online-retail logistics hub for Europe"* (W. P. Carey Inc., 2013).

Above examples, derived from a business context, illustrate clearly that location planning requires integrated models. The reports of H&M, Grainger and VW indicate that distribution structures comprise multiple stages, e.g. central and regional warehouse facilities. The hierarchical setup allows companies to realize economies of scale, as transportation costs in practice are generally of non-linear nature. Freight rates are typically a function of weight and distance. Furthermore, VW and Grainger representatives emphasize the need for delivery time reduction and meeting promised service times. The examples of H&M and P&G point out that many companies are currently not operating their own warehouses, but rather rely on service providers and real estate companies. Thereby, companies avoid investing their own capital in fixed assets and only incur operational/running costs. Consequently, companies operate more flexible and it is easier to change locations, thus enabling a dynamic planning of their distribution structures.

It can be summarized that facility location models need to consider above characteristics in an integrated manner, rather than just the isolated single aspects. Those characteristics, derived by companies' actual requirements with regard to distribution structures, are (1) a hierarchical setup, (2) economies of scale in transportation cost, and (3) service time requirements. Dynamic aspects (4) additionally play an important role, as this allows companies to plan adaptations to their structures in the course of time. Especially in cases where companies lease their warehouses from real estate companies and operate those with contractors, it is easier for them to alter warehouse locations, as assets are not owned.

1.2. Infrastructure Planning

Infrastructure is defined as the basic equipment and facilities of an economy, belonging to its economic capital stock. Examples include transport networks, such as roads and railways, but also utility services and disposal systems, such as energy, water and communication (Klodt, 2013). Gas stations, which provide traditional, fuel or gas powered means of transportation – such as automobiles or trucks – with energy, are part of an economy's infrastructure. The rise of electric vehicles leads to a change and reorientation of this repowering infrastructure. The primary power source, electricity, does not need to be provided in form of stored fuel or gas, but can virtually be provided at any location with an existing electricity grid. Many countries and especially cities have set themselves high targets in respect to electric vehicle employment rates (HM Government, 2009; Mayor of London, 2009; The White House Office of the Press Secretary, 2008). The federal government of Germany, for example, announced in its *National Electromobility Development Plan* to bring one million electric vehicles on the road by 2020 (Cabinet of Germany (Die Bundesregierung), 2009). The city of Amsterdam launched its initiative *Amsterdam Electric* in 2009, which has the objective to reduce CO_2 emissions to zero for the entire transportation system by 2040. The goal is to have 200,000 electric vehicles on the road by then (Government of Amsterdam (Gemeente Amsterdam), 2013). Along with those bullish plans of electric vehicle employment rates, an adequate infrastructure for refueling those is required. Even more than that, an existing, well developed infrastructure is an enabler for reaching the high targets of governments and city councils. One of the main disablers is closely linked with the charging infrastructure, namely an effect described as "*range anxiety*". The term describes the fear of electric vehicle users of running out of battery in a place, where no charging infrastructure exists (Eberle & Von Helmolt, 2010). Consequently, infrastructure planning is high on the agenda of many cities and regions. A good example is Transport for London (TfL), the local government body responsible for the transport system in Greater London, England. TfL issued a guideline *"Guidance for implementation of electric vehicle charging infrastructure"* in 2010 (Transport for London, 2010). The document is aimed at providing London's 33 borough officers a guideline, which assists them in the electric vehicle charge point infrastructure planning and implementation process. At the beginning of the planning process, the guide suggests *"in order to select the most suitable sites for on-street and off-street public charging points across a borough a desktop evaluation should be undertaken that takes account of the following factors: Demand (existing/potential) […], visibility/accessibility […], road space […], footway space […], potential to create Green Hubs […]"* (Transport for London, 2010, p. 25). Thus, at the beginning of the process a desktop evaluation is recommended. The factor demand, especially potential demand, is the one out of those five, which is not straight forward to capture or measure. In order to define optimal locations for charge points, a systematic methodology needs to be established, which is easily transferable to other boroughs or cities.

The guide lists potential locations for charge points, amongst which are *"town centers, high streets, tourist attractions"*, *"leisure centers and sports facilities"*, *"retail outlets"*, *"parks and other green spaces"*, and *"education facilities"* (Transport for London, 2010, p. 24). It furthermore states *"the demand for the charging point sites will be dictated by the users' journey purpose. […] the most suitable locations are those which have the above-listed attractors nearby […]"* (Transport for London, 2010, p. 25). Charging demand is thus closely linked to the destinations of user's errands.

Summarizing the requirements above, it can be concluded that a methodology, which is supporting city planners in optimally locating charge points, should consider the integration of user's trip destinations, as they determine the charging demand. Accordingly, anticipated future charging demand will be based on the spatial distribution of above mentioned facilities and infrastructure.

2. Research Outline and Key Concepts

As introduced, this thesis deals with two aspects of facility location planning. Aspect one considers the further advancement of existing models in the application of distribution network planning. The main contribution is in merging the existing isolated approaches into a single integrated model. This allows a more realistic modeling of real life applications and planning situations. Aspect two transfers an existing facility location concept to a new, innovative application, which is the planning of a charge point infrastructure for electric vehicles. This section continues with a general introduction to the topic of facility location problems. It subsequently presents selected concepts of distribution network planning, which are important for the research of this thesis, and finally addresses the aspects of electric vehicle charging infrastructure planning.

2.1. Introduction to Facility Location Problems

Facility location problems (FLP) date back to the 17th century. Fermat (1601-1665) is generally credited to be the first one, who stated the problem in written form: *"given three points in the plane, find a fourth point such that the sum of its distances to the three given points is a minimum"* (Kuhn, 1967 as cited in Drezner, Klamroth, Schöbel, & Wesolowsky, 2002, p. 3). The solution of the problem is usually attributed to Evangelista Torricelli (1608-

1647; Drezner et al., 2002). Almost three centuries later the German economist Alfred Weber discusses the problem in an industrial context: a central facility needs to be located among a set of demand points – each with a weight associated depicting the quantity shipped – in such a way that the weighted sum of distances from the demand points to the central facility is minimized. In the appendix of his book *"Theory of the location of industries"*, Georg Pick gives the first mathematical formulation of the problem (Weber, 1909).

The problems described by Fermat and Weber are of continuous nature and locate facilities at the median of the given points in the plane. This reflects that the location – the *"fourth point to be found"* or the *"central facility"*, respectively – can be placed anywhere in the plane. In the context of business applications, such as warehouse locations or airline hubs, so called discrete facility location problems dominate: The facilities to be established are selected among a given set of candidate sites, which are part of a network or graph. Current, Daskin, & Schilling (2002) mention different reasons, why there is such a strong interest and long history of research on location analysis and modeling: Location decisions are relevant to individuals and organizations in many aspects of daily life. They are of long-term, strategic nature and they can impose economic externalities, such as pollution or noise. It is furthermore very hard to solve location models optimally, especially when they grow in size. However, the most important reason is that there is no general location model, which can be applied to all potential or existing applications. Consequently, models are application specific. The objective function, the constraints and variables look different, depending on the context and application they are used in (Current et al., 2002).

As an introduction to the topic, three basic facility location problems are introduced, which form the foundation for the models developed in the course of this thesis.

The first one is the p-median problem, the second one is the uncapacitated facility location problem (UFLP), and the third one is the maximum coverage location model. All three problems are discrete problems, located on a graph or network. Nodes represent the demand locations, e.g. customers, and facility sites. Edges are the existing connections between nodes. The formulations presented are based on Drezner & Hamacher (2002).

The p-median problem formulated by Hakimi (1964) is based on a set of demand nodes and a set of potential facility sites. Distances between both are given. The model then locates exactly p facilities, while minimizing the demand weighted distance between located sites and demand nodes.

Using following notation, the mathematical problem can be formulated:

I = the set of demand nodes, indexed by i

J = the set of potential facility sites, indexed by j

h_i = demand at node i

d_{ij} = distance between node i and potential site j

p = number of facilities to be located

The decision variables of the problem are:

$$y_{ij} = \begin{cases} 1 \text{ if demand node } i \text{ is assigned to a facility at node } j \\ 0 \text{ if not} \end{cases}$$

$$x_j = \begin{cases} 1 \text{ if facility } j \text{ is selected} \\ 0 \text{ if not} \end{cases}$$

The problem then can be formulated as follows:

Minimize $$\sum_{i \in I} \sum_{j \in J} h_i d_{ij} y_{ij} \tag{1.1}$$

subject to $$\sum_{j \in J} x_j = p \tag{1.2}$$

$$\sum_{j \in J} y_{ij} = 1 \qquad \forall i \in I \tag{1.3}$$

$$y_{ij} - x_j \leq 0 \qquad \forall i \in I, j \in J \tag{1.4}$$

$$x_j \in \{0,1\} \qquad \forall j \in J \tag{1.5}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{1.6}$$

The objective function (1.1) minimizes the demand-weighted total distance. Constraint (1.2) ensures that exact p facilities are being opened. Constraint set (1.3) ensures that each demand node is assigned to exactly one facility. Constraint set (1.4) guarantees assignment of demand nodes only to open facilities. Constraint sets (1.5) and (1.6) restrict the decision variables of opening facilities and assigning demand to facilities to be binary. The number of facilities to be selected p is given by the decision maker and not a decision variable of the model. Consequently fixed cost associated with the construction and operation of facilities needs not to be incorporated into the model formulation. Moreover this cost does not influence the selection of candidate sites for fixed values of *p*.

This can be seen as a decisive disadvantage of the p-median problem, as the planners need to be aware of how many facilities to be established. If this is unknown, the second basic model, UFLP, can be considered. Again, demand nodes and potential facility site are known sets. In addition to the p-median problem, fixed cost of the facility sites are given and demand is associated with a distance-unit and demand-unit based transportation cost. Hence, the number of locations is a trade-off between transportation and facility fixed cost. The above notation needs to be extended with:

c_{ij} = cost per unit demand per unit distance between node i and potential site j

f_j = fixed cost for potential site j

The mathematical formulation of UFLP then is:

$$\text{Minimize} \quad c_{ij}\sum_{i\in I}\sum_{j\in J}h_i d_{ij} y_{ij} + \sum_{j\in J} f_j x_j \tag{2.1}$$

$$\text{subject to} \quad \sum_{j\in J} y_{ij} = 1 \qquad \forall i \in I \tag{2.2}$$

$$y_{ij} - x_j \leq 0 \qquad \forall i \in I, j \in J \tag{2.3}$$

$$x_j \in \{0,1\} \qquad \forall j \in J \tag{2.4}$$

$$y_{ij} \in \{0,1\} \qquad \forall i \in I, j \in J \tag{2.5}$$

The objective function (2.1) now minimizes the total transportation and facility fixed cost. Constraint (2.2) guarantees that each demand node is assigned to exactly one facility. Constraint (2.3) again ensures that demand nodes are only assigned to open facilities. Constraints (2.4) and (2.5) are standard integrality and non-negativity constraints.

The above models, p-median and UFLP, are based on minimizing total distance or cost, respectively, for all given demand nodes. In contrast to that, the third model, the maximum coverage location model, is based on maximizing total demand covered with a given number of facilities to be established.

Using the following additional notation,

D_c = *coverage distance,*

$N_i = \{j \mid d_{i,j} \leq D_c\}$ = *set of all potential facility locations that cover demand of box i,* and

$$z_i = \begin{cases} 1 \text{ if demand node } i \text{ is covered} \\ 0 \text{ if not} \end{cases}$$

the maximum coverage location problem is defined as follows:

Maximize $$\sum_{i \in I} h_i z_i \tag{3.1}$$

subject to $$\sum_{j \in N_i} x_j - z_i \geq 0 \qquad \forall i \in I \tag{3.2}$$

$$\sum_{j \in J} x_j = p \tag{3.3}$$

$$x_j \in \{0,1\} \qquad \forall j \in J \tag{3.4}$$

$$z_i \in \{0,1\} \qquad \forall i \in I \tag{3.5}$$

Objective function (3.1) maximizes the total demand covered. Constraint (3.2) guarantees that only demand nodes are counted as covered, if a location is established that covers demand of that node. Constraint (3.3) ensures that only the predefined number of facilities *p* is established. Constraints (3.4) and (3.5) are binary constraints.

The presented models form the basis for plenty of other facility location models that have been formulated. Extensions to those models include, but are not limited to, multi-commodity settings, hierarchical setups, capacitated versions, approaches dealing with uncertainty, multi-objective optimization and dynamic setups. Comprehensive reviews and surveys on models with those extensions can be found in Melo, Nickel, & Saldanha-da-Gama (2009, general review); Revelle & Eiselt (2005, general review); Revelle, Eiselt, & Daskin (2008, general review including covering models); Farahani, Asgari, Heidari, Hosseininia, & Goh (2012, covering models); Sahin & Süral (2007, hierarchical); Snyder (2006, uncertainty); Owen & Daskin (1998, dynamics and uncertainty); Arabani & Farahani (2011, dynamics); and Farahani, SteadieSeifi, & Asgari (2010, multi-objective models).

2.2. Distribution Network Planning

As the introduction pointed out, businesses impose various requirements towards their distribution structures. In the following, seminal research articles presenting approaches of integrating either hierarchy, economies of scale in transportation cost, or dynamics into mixed integer linear facility location programs are briefly outlined. Regarding the service time aspect, no special research article is presented, as this is implemented by combining a covering model with a facility location model. The distance from warehouse to customer is used as a proxy for service time.

Going back to the remaining three aspects, a hierarchical setup in a facility location problem was presented by Kaufman, Eede, & Hansen (1977) in form of a plant and warehouse location problem. This problem is explicitly formulated as a mixed integer linear program. The authors furthermore clearly indicate that concave (nonlinear) cost functions need to be approximated linearly. The problem is formulated with assignment based decision variables, which means that each link of plant-warehouse-customer has its own decision variable x_{ijk}, where i denotes the plant, j the warehouse and k the customer location. In contrast to that, also individual decision variables for each link can be established, e.g. plant-warehouse x_{ij} and warehouse-customer x_{jk}. Additional constraints then ensure flow conservation, i.e. a warehouse is not able to send more than it received. Kaufman et al. (1977) minimize total cost in their model, including transportation and warehouse fixed cost. A branch-and-bound algorithm is developed to solve the problem optimally and computational results are reported.

O'Kelly & Bryan (1998) introduce a method of including nonlinear cost functions into a hub location problem. The work takes up an approach of Balakrishnan & Graves (1985), which they use in a network flow problem for less-than-truckload shipments. Economies of scale are achieved by bundling shipments on mutual arcs. The hub location problem of O'Kelly & Bryan (1998) takes airline hub operations as an application, with economies of scale arising from bundling passengers on hub-hub connections. The nonlinear cost functions are divided into segments, which can be approximated by linear functions. Consequently, the nonlinear nature of cost is reflected in the problem formulation, but the program remains linear. As a result, traditional decision variables, denominating the amount shipped from location A to location B, are split up into a set of new decision variables. The new number of decision variables is equivalent with the number of segments in the new piecewise linear cost function.

Ballou (1968) was among the first to point out that facility location planning must be dynamic, rather than static, in certain situations. He states that *"the effect of the future time dimensions cannot be neglected in location analysis"* (Ballou, 1968, p. 271). It is essential to obtain a location plan *when* and *where* to relocate warehouses within the planning period, given that accurate prediction of demand for longer periods can be made. The model proposed is a profit maximizing model, requiring a recursive solution procedure. This is implemented by dynamic programming, solving the static problems for each time period in a first step. Starting with the last time period, this is followed by iterations, which recursively optimize the previous periods.

Hinojosa, Puerto, & Fernández (2000) combine hierarchical and dynamic aspects in a single mixed integer programming facility location model. The location decisions for two layers, plants and warehouses, are made simultaneously, along with opening and closing decisions in the examined time horizon. Two subsets for each location layer are introduced, split into facilities, which are open from the beginning of the planning period, and facilities, which mark candidate sites. Each facility is allowed to change its status only once in the overall planning period: Initial facilities can stay open or being closed, and candidate sites can only be established, but not closed again. The authors develop an approach to solve the model, consisting of Lagrangian relaxation, dual ascent method and a heuristic construction phase.

Above concepts have shown different approaches, how requirements imposed by businesses with regard to distribution structure planning can be tackled. Nonetheless, above research only considers single aspects of those requirements. Distribution costs are a significant portion of a company's total cost and thus it is essential considering those requirements simultaneously in an integrated planning model. Two chapters of this thesis deal with facility location analysis and the application to distribution network planning. Chapter II considers the joint integration of three aspects, namely hierarchical setup and simultaneous location decision of two layers, economies of scale in transportation cost by a piecewise linear cost function and implementation of service time requirements. Chapter III extends the location model by dynamic aspects, allowing planning for a series of consecutive time periods. The model additionally indicates location and relocation decisions of warehouses at different layers in the course of time.

2.3. Infrastructure Planning

As explained in the introduction, charging demand for electric vehicles is closely linked to the destinations of users' errands. Those destinations are generally referred to as *points of interest*, such as restaurants, banks, or retail outlets, to name a few. The access to points of interest data, namely category and spatial location in a city, allows creating a proxy for electric vehicle charging demand in the planning area. Differentiation for categories is essential, as the potential duration of stay at a restaurant or gym is generally longer than withdrawing money from an ATM or buying a newspaper from a newsstand. As the recharging of an electric vehicle consumes more time than it does for traditional gas-fired cars, first mentioned examples are consequently more relevant for recharging a car than last-mentioned. The spatial location is important, in order to obtain a comprehensive, spatially differentiated overview of charging demand within a city or region. The combination of the points of interest information, as a proxy for anticipated future charging demand, with a classical facility location model permits to optimally plan the charging infrastructure for electric vehicles in a defined planning area. The result, for instance, will be a set of city blocks, in which it is optimal to establish a charging infrastructure. In the consecutive step, the infrastructure planner then can consider the other factors mentioned in the introduction, such as visibility/accessibility, road space, or footway space within this area. Consequently, with a data-based methodology, which provides anticipated charging demand, the search for optimal locations is much more focused and user-centric. There is no need for city planners to assess the entire city with regard to the other factors, but only focus on them as soon as a city block is specified.

As a consequence, the systematic methodology to be developed is to support infrastructure planners with the step of finding optimal locations. A big advantage of such an approach is that it systemizes the planning process, is reproducible, transparent and easy to implement. Such a process guarantees quick results and allows infrastructure planners to focus on defining the micro location of charge points. Micro location in this context refers to the actual site of the charge point within the selected city block, that is, for example, on-street next to a parking space in front of a grocery store.

Transferring existing concepts to new applications is a major challenge in academic research. The challenge, in turn, is not necessarily on the modeling aspect, as existing models can often be implemented, but rather on the choice of input factors to the model. This is exactly the case for creating a methodology and model for planning a charge point infrastructure, as described above. Best suitable for the problem is a maximum coverage location model, which maximizes demand covered, based on a given number of charge points to be established. The problem can also be reformulated, i.e. optimizing for the minimum number of charge points required to serve a given demand. It is obvious, that the challenge in this case is not the creation of a suitable facility location model, but rather, the challenge lies in deriving appropriate input factors defining demand. The outline above describes how this issue can be approached and is further addressed in Chapter VI.

3. Thesis structure

The two sections above derived requirements from real-world applications in a business context related to facility location problems. In accordance to those, each of the following three chapters presents a research paper, focusing on these aspects and filling the research gap. Fig. 1 shows the structure of the thesis embedded in two dimensions of facility location models, namely *Methodology/Integrated Models* and *Innovative Applications*. The arrows considering those aspects are schematic, only indicating the emphasis of the chapters, as all three contain elements of both. *Methodology/Integrated Models* in this context refers to integrating different planning aspects into a single facility location model, with the focus on methodological aspects of model formulation, whereas *Innovative Applications* focuses on transferring familiar concepts to new, especially innovative, applications, such as the planning of a charging infrastructure for electric vehicles. The challenge in this case is less on the model formulation, but rather on deriving adequate input. The gray boxes in Fig. 1 additionally provide a condensed view of key concepts and accomplishments of each chapter.

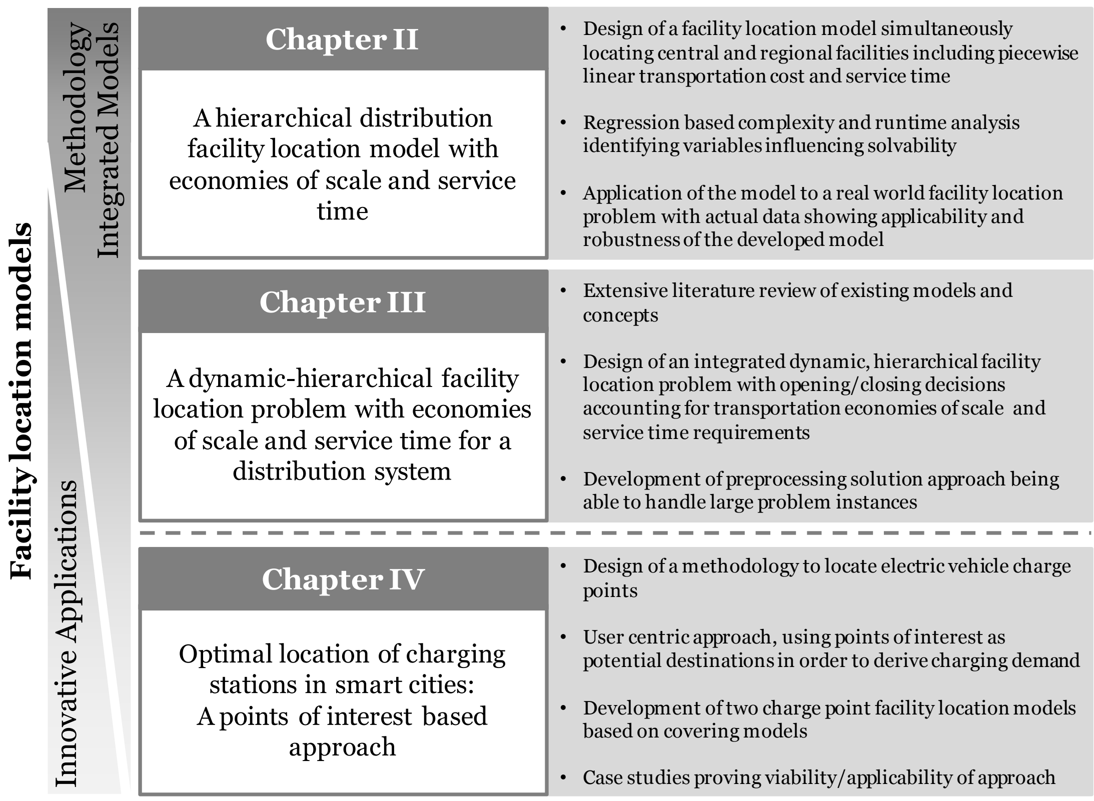

Fig. 1. Thesis Overview.

The second chapter develops, derived by requirements of the actual business environment, a facility location model which simultaneously locates central and regional warehouse facilities. The model is formulated as a mixed integer linear program accounting for a piecewise linear transportation cost function and service time requirements. The applicability to real-world location problems is emphasized by a case study. Chapter three extends the previous approach by considering not only spatial, but also temporal aspects of facility location. The model makes decisions about warehouse openings and closings in course of the planning horizon. It is still of linear nature, with the advantage of using exact linear programming solution procedures to solve the model. For larger program instance, a preprocessing approach is developed, which is able to solve those. Chapter four, on the other hand, innovatively applies known facility location models, namely covering models, to a new application of location planning: the development of an electric vehicle charging infrastructure. A point of interest based approach is developed, which allows obtaining anticipated charging demand, which is the major input for the location model. Based on the demand, the model provides an optimal infrastructure plan.

3.1. Summary: A hierarchical distribution facility location model with economies of scale and service time

The second chapter presents the *SiLCaRD* model, which conduces to planning distribution network setups. The *SiLCaRD* model is a facility location model, which integrates different aspects into a single location model.

Aspect one is the hierarchical nature of distribution systems. This is realized with four layers in the model developed in the research. Layer one, which is of optional nature, comprises of a virtual plant/global warehouse, which denotes the goods' origin. Layer two and three are the distribution facilities, which are central and regional warehouses. The fourth layer is made up by customer locations. Simultaneous location decisions for central and regional warehouses are taken.

Aspect two is considering the existence of nonlinear freight rates in practice. Those are generally a function of weight and distance. The integration of a nonlinear function would lead to a nonlinear program, which in turn requires different solution techniques. Consequently, the approach of piecewise linearization was applied to the cost function. This approach divides the nonlinear function into segments that can be approximated linearly. As a result, the program is still of linear nature, and known solution techniques can be applied.

Aspect three is the inclusion of a service time requirement, which is either requested externally by customers, or offered by companies, e.g. promising next day delivery. The challenge regarding service time, generally measured by accessibility in form of a drive time radius, is that the inclusion of drive time would lead to a bi-objective objective function. This requires developing a methodology, which is able to assess and compare the two different units, time and cost. Instead, a proxy for drive time, which is a maximum allowed distance between customer and regional warehouse, is employed. A correlation analysis, with a coefficient of correlation of 99%, shows that this is a sound approach.

An extensive computational study with randomly generated data is carried out, requiring a net runtime of more than 18 days for solving all instances. The analysis reveals limitations up to which sizes problems can be solved optimally. Problems with up to 160 facilities (central, regional, and customer) and up to four linear segments are solved optimally. The largest problems, consisting of 200 facility sites and four linear segments, are still solved with a single-digit optimality gap of 3%. The optimality gap denotes the relative difference of best integer solution found after a given time and the then valid lower bound. The analysis provides valuable insight for supply chain planners with regard to practical applicability to real-world problems.

The computational study is enhanced by a regression-based complexity and runtime analysis, which is based on variations of the model with regard to specific model parameters. Those are number of facility/customer sites, number of linear segments, inclusion/exclusion of a virtual plant/global warehouse, and inclusion/exclusion of a service time requirement. The regression results show an adjusted R^2 of 0.834 and significant t-statistics for all four variables. Most naturally the number of facility/customer sites influences the runtime negatively (99.9% confidence level), as well as the number of linear segments (95% confidence level). Both lead to larger problem instances, the higher the number of facilities or segments. The inclusion of a virtual plant/global warehouse or a service time requirement (both at 99.9% confidence level), on the other hand, contributes positively to runtime. Coefficient results can be used by distribution structure planners to get indications on adequate problem sizes and expected runtime.

In the last step, the model is validated for practical purposes by applying it to a real world facility location problem, using actual demand and cost data. All considered case study problems are solved well below the predefined timeframe of twelve hours, with the largest problem consisting of three linear segments, 45 central candidate sites, 191 regional candidate sites and 661 customer sites. Managerial implications complete the research article.

3.2. Summary: A dynamic-hierarchical facility location problem with economies of scale and service time for a distribution system

Chapter three contains a decision support model for dynamic facility location planning. The model developed in this paper extends the one presented in chapter two by a temporal component and is formulated as a mixed integer linear program.

In contrast to a static model, the dynamic version allows supply chain planners to get indications not only *where* to open a facility, but also *when* to open, and respectively close it. The formulation implies an initial setup of warehouses. Consequently two sets for each warehouse layer are created: The set of facilities which are open from begin on and the set of candidate facility sites. Location decision may only be changed once for each set, i.e. open warehouse can only be closed, but not reopened, and candidate sites can only be opened, but not closed again. This approach has mainly practical reasons and is widely accepted in academia. Warehouse movements require a huge organizational effort upfront and a certain time after the movement until they are fully operational and back at implied service levels. Potential investments coming along with a movement are not considered, as today many companies choose to neither own nor operate their warehouses, as laid out in the introduction. First mentioned effects are not easily measured monetarily, but are the reason why a company wants to avoid changing facilities too often. In the worst case facilities *'jump'* back and forth between two locations. Furthermore, planning horizons in most applications are ten years or less, which additionally supports this approach of allowing status to change only once.

The hierarchical setup, as in the *SiLCaRD* model, includes a virtual plant/central warehouse, which is optional, central and regional warehouse facilities, for which the location decision is derived simultaneously, and customer locations. Economies of scale in transportation cost are included in form of a piecewise linear approximation of the nonlinear cost function. Service times are modeled with a proxy of a maximum allowed distance from customers to regional warehouses. A regression analysis is performed, evaluating the relationship of drive time and distance. The regression returns an R^2 of 0.985 and a significance level of 0.1%, i.e. a confidence level of 99.9%. Distance is thus guaranteed to be a fair proxy for drive time.

The mathematical formulation of the problem leads to a separate decision variable for each transportation link (warehouse-warehouse or warehouse-customer), linear segment and time period. Thus, they have four indices including time period, segment, origin and destination.

A computational analysis with random data and average values of solved problem instances shows limitations with respect to solvability as the instances grow in size. Several larger problem instances, with 60 up to 100 facility sites, six time periods and three linear segments, show a single digit optimality gap within the allowed solve time of three hours. Optimality gap again refers to the best mixed integer solution found after three hours in relation to the then valid lower bound. Some instances even report an *"out of memory"* status of the system.

This is the reason, why an approach is developed, which is able to handle and solve those instances. This approach is similar to dynamic programming, as it divides the large dynamic problems into smaller static problems. Those are solved optimally and the resulting facility locations serve as an input to the dynamic problem. As a consequence, the original dynamic problem is drastically reduced in size and can subsequently be solved. Results obtained are superior to the ones which previously showed an optimality gap and also *"out of memory"* instance can be computed. With the application of this solution procedure, objective values are improved by up to 62%. This approach is referred to as a *preprocessing solution approach* in the chapter.

The computational analysis is rounded up with a case study, now using real-world company data. The case study shows, that the dynamic hierarchical facility location model including economies of scale and service time is a valuable tool for planning distribution structures. Three different scenarios of different model sizes are presented and all are solved well below the given time frame of twelve hours. As the problem is of strategic nature, real-time results are not required, and twelve hours served as a natural practical threshold, starting computation in the evening, obtaining results the next morning. More importantly, the case study also compares static with dynamic planning results and the effects of economies of scale as opposed to purely linear transportation cost. From the results can be concluded, that the dynamic version with economies of scale should be preferred over a static or linear cost version in planning situations, where temporal aspects and nonlinear cost are existent.

3.3. Summary: Optimal location of charging stations in smart cities: A points of interest based approach

The fourth chapter is an extended version of the research article that has been accepted for the *International Conference on Information Systems 2013*. This research article addresses the location of electric vehicle charge points by means of facility location models.

Electric vehicles are reaching the mass market, inter alia because of advancing battery technologies and governments and administrations putting up optimistic electric vehicle development plans supported by incentives. To facilitate these plans, an adequate charging infrastructure is required. This is where the chapter steps in, as it provides a decision support system allowing city planners to plan a charging infrastructure. The core issue determining optimal locations for charge points is anticipated future charging demand. A novel approach is developed to derive that demand. The hypothesis is that charging behavior is influenced by destinations of electric vehicle users and their respective category. For example, these destinations, which are called points of interest (POI), can be restaurants, universities, or hair dressers.

The existing, well developed charging infrastructure of Amsterdam is taken as a reference. A database with usage behavior of established charging stations is the basis for further analysis. It contains information on the average length of charging sessions and users per day. A measure defining *charge point importance* is established, which is based on average daily usage and users patronizing a station. Evaluating the charge point importance and the surrounding points of interest allows inferring on the influence of POIs on charging behavior. A multiple linear regression analysis is used in order to derive statistical significance. The adjusted R-squared value is at 0.1, which is acceptable in social studies. The main categories, such as food, stores or health, show significant results, with confidence levels at 95%.

The charge point importance of individual charging stations is then used to calculate a *POI category rank*. The POI category rank differentiates categories by their influence on charging behavior and is a proxy for charging demand. In a consecutive step, all POIs in the planning area, e.g. a whole city or borough, get assigned their respective POI category rank. The planning area then is divided into a grid of boxes with identical edge's length. Summing up the POI category ranks of all POIs within a box or given radius allows assigning a charging demand proxy to each individual box, which is denominated as *box factor*.

This spatial charging demand then serves as input for two facility location models, namely the *maximum coverage facility location model (MCFL)* and the *minimum charge point location model (MCPL)*. Both are covering location models, based on the basic example in the introduction. A coverage radius defines the area, which is covered when locating a charge point. The MCFL maximizes demand covered with a given number of charge points to locate. The MCPL, on the other hand, minimizes the number of charge points for a given demand coverage ratio. Case studies for Amsterdam and Brussels are conducted, showing planning results for a charge point infrastructure.

A computational study in combination with a parameter sensitivity analysis evaluates the performance of both models and the influence of parameter settings on results. Parameters, which can be used by city planners to adjust the models to their specific planning needs, are the following: calculation of POI category ranks, definition of the planning area (total grid size and edge length of grid boxes), box factor calculation, and charge point coverage radius. Both models additionally have specific ones, which is the number of CPs to be located p for the MCFL and proportion of demand to be covered c for the MCPL. POI category ranks in the paper were derived by the reference city Amsterdam and actual charging behavior at existing charge points. Thus they are not subject to the sensitivity analysis. However, city planners can modify the derived POI category ranks, depending on their or a city's specific preferences, giving e.g. banks a higher weight. Edge length of grid boxes influences the granularity of the planning results with respect to exact location. The smaller the edge length, the more detailed is the result with regard to location. Box factor calculation can be influenced by a radius defining the sphere of POIs, which contribute to the box factor. The radius can exceed the box, thus accounting also for outside POIs. This is especially important in order to account for agglomeration of POIs that fall into adjacent boxes. The bigger the radius is chosen, the smoother and less delimited is the demand. Contrariwise, a small radius allows for local demand peaks. Coverage radius, on the other hand, enables defining the density of the charging infrastructure. A high coverage radius leads to a bigger area in which the demand is covered by an established charging station compared to a small coverage radius. The specific parameters have an effect on how much of the planning area is covered, by indicating the number of charge points p in the MCFL and the total demand covered c in the MCPL.

Finally, an iterative algorithm is developed, which is locating charge points iteratively by selecting high demand areas in a decreasing manner. In the facility location model the locations are selected simultaneously, optimizing for the objective function. The advantage of the algorithm is that it provides quick results. Although those are inferior to the location model within a single digit percentage gap, the algorithm proves to be an excellent mean for adjusting above model parameters quickly. The percentage gap of total demand covered for p=100 between the MCFL and the iterative algorithm is 2%.

4. Conclusion and Outlook

This dissertation presented three research chapters addressing different aspects in facility location modeling and planning. While chapter two and three extended and integrated existing approaches into single planning models, chapter four transferred the concept of linear facility location programs to an innovative application. Specifically, today's distribution network structures require integrated planning approaches, considering various aspects simultaneously, rather than separately. Innovative application of known concepts, on the other hand, enables economic sectors to plan their structures, such as an electric vehicle infrastructure, more efficiently.

In summary, in chapter two a distribution infrastructure planning model named *SiLCaRD* was developed, which incorporates the simultaneous planning of different warehouse levels, while at the same time considering transportation economies of scale and service time requirements. An extensive computational study with random data and a case study using actual company data certified the practical applicability of the model. A regression-based complexity/runtime analysis provides supply chain planners a tool to infer on solvable problem sizes.

Chapter three then extended the *SiLCaRD* model by additionally accounting for temporal aspects. Not only is the choice made for the location, but also how to change facility structures over time. Due to limitations regarding solvability in the conducted computational analysis, a preprocessing approach was developed. This approach enables planners solving large problem instances. A comparison of results between the dynamic models, consecutively solved static models and purely linear transportation cost models was presented. The comparison supports the fact that the dynamic model addresses the reality of today's structures in a more accurate way, and that results differ significantly from the other models.

Chapter four addressed a current topic, which is electric vehicle infrastructure planning. A novel methodology deriving future anticipated charging demand was developed. This in turn served as an input factor for two covering facility location models, which allow planners to systematically plan a charging infrastructure for a city or urban district. Implications on how planning parameters influence results are presented, as well as an iterative algorithm leading to quick results for adjusting those parameters.

This dissertation addressed two different topics in the field of facility location planning. Integrated models for distribution network planning and a methodology for electric vehicle infrastructure planning were developed. Still, the research in those topics opened up various opportunities to continue with research in future works:

Distribution Network Planning

- The *SiLCaRD* model and its dynamic extension integrated several aspects into a single planning model. Nonetheless, additional aspects may be integrated in future research. Those aspects include, but are not limited to, multi-product settings, warehouses with capacity limits, or additional layers, such as external suppliers.
- The dynamic version of the model may also serve as a starting point for integrating uncertainty aspects, e.g. for future demand. Instead of using the set T for defining the time periods, it may be used to define different future scenarios. Consequently, the objective function and formulation needs to be adapted and probabilities may be assigned to each scenario. Results then would indicate an optimal future warehouse structure, based on the probabilities of demand scenarios.
- Both models used a distance proxy for service time requirements. It could be investigated how to integrate service time and cost into a bi-objective location model. A scheme defining how to weigh and assess the trade-off between service time and cost then needs to be developed.
- The computational analysis has shown that there are limitations with regard to solvability for larger problem instances. While adequate real-world problems have been solved in the research and a preprocessing approach for the dynamic version has been developed, efficient heuristics or algorithms to either obtain quicker solutions or solve larger models optimally can still facilitate the usability.

Infrastructure Planning

- The methodology presented to obtain anticipated future charging demand makes use of point of interest data in the planning area. Although these POIs are trip destinations for electric vehicle users, additional data of structural and environmental nature may enhance results. Examples for such additional data include traffic flows, vehicle trip data, usage patterns, or population data.
- The parameter sensitivity analysis pointed out in which way the settings influence results and allow city planners to adjust the methodology to specific environmental needs. The development of a guideline on how to use the setting of parameters would give further insight.
- The methodology of using points of interest data to infer on future charging demand can be transferred to other aspects facilitating smart city planning, such as:
    - Attractiveness of city districts with regard to residence or quality of life, by deriving an infrastructure supply index, which can additionally be differentiated by categories. A potential output can support house hunters, but also city planner in order to set a focus for future city planning. Planning results could read like *“district A’s health supply index is at 60% and its grocery index at 82%”*.
    - Targeted advertising, by deriving a potential customer-fit index, e.g. city block A has a high share of gyms, and thus, the potential for advertising related goods, such as sportswear or health products, is higher than in other blocks.
    - Business location planning, by deriving potential future demand of an area with regard to services or products and additionally establishing a competitor index.

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CHAPTER II

A HIERARCHICAL DISTRIBUTION FACILITY LOCATION MODEL WITH ECONOMIES OF SCALE AND SERVICE TIME

Abstract

In this paper we develop a linear multi-stage mixed integer facility location model, which determines the location of central and regional warehouses simultaneously, accounting for transportation economies of scale by a piecewise linear cost function and for service time requirements: SiLCaRD (simultaneous location of central and regional distribution facilities). The model's rigor is tested with an extensive computational analysis. We furthermore examine the effects of selected variables on complexity/runtime of the model by a multiple regression analysis. A case study with real world freight rate and demand data is conducted, showing the models applicability to real world problems. Managerial implications are provided.

Keywords: *Hierarchical facility location problem, distribution networks, economies of scale, service time, decision support systems*

1. Introduction

The problem of facility location is present to human beings in every aspect of daily life. It comprises finding optimal locations for retail outlets providing goods and services to consumers, for doctors and hospitals providing medical services, or for telecommunication and electricity switching stations providing infrastructural connections to the end user, just to name a few.

Facility location problems are not new to the operations research, decision support systems and transportation research communities. In particular single-level problems have been studied to a great extent. As shown above, facility location problems cover a wide field of applications in the business environment and the public sector. Besides those single-level problems, hierarchical problems are, from a practical point of view, more relevant to most real-world applications: The above examples have in common that they are part of a hierarchical system. Retail outlets are supplied by a regional warehouse which in turn gets its supplies from a central warehouse or a manufacturing plant. Doctors and hospitals offer different services and treatments, patients may be referred from one doctor to another (e.g. specialist, hospital, special treatment center, etc.). Infrastructural nodes are also part of bigger networks with a supplying and/or receiving function.

In this paper we want to focus on a production – distribution network setting. The research was motivated by a real world application. In the manufacturing/consumer goods sector companies are facing the challenge of establishing efficient and cost-effective distribution structures. Those often comprise multi-tier networks, reaching from manufacturing plants over warehouses to customers: Manufacturing plants are usually on the highest and customers on the lowest level. The intermediate layer comprises warehouses, which again can be in a hierarchical setting (e.g. central vs. regional warehouses). In the consumer goods industry, many retailers do not produce products themselves, but purchase those from suppliers, which in this case make up the plant layer. Multiple layers often originate from differing requirements: consolidation of goods from various sources, minimization of overall stock level in the system, predefined service times to customers, etc. Fig. 1 illustratively depicts such a production-distribution network.

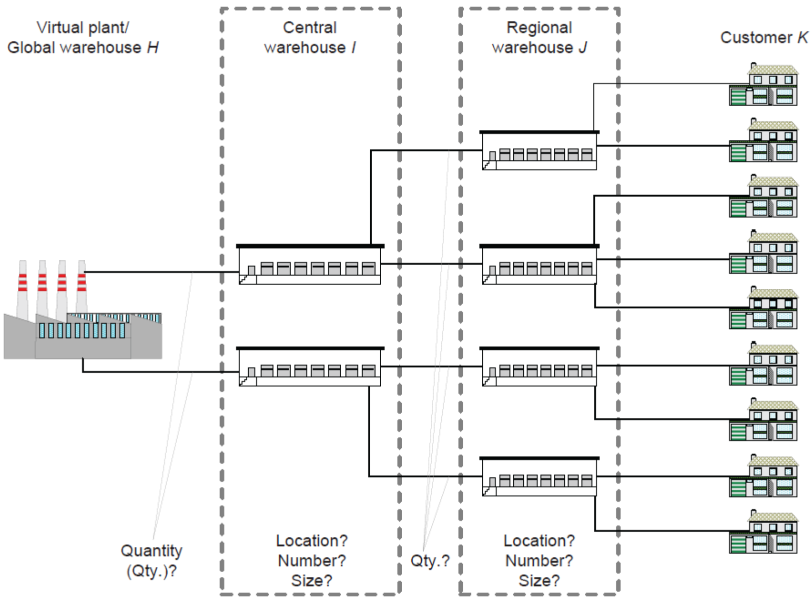

Fig. 1. Schematic overview of a production - distribution network.

In the following we want to give three references of companies from different disciplines, operating (production-) distribution networks:

- Geberit AG, active in sanitary technology, operated a decentralized logistics network until 2005: Plants delivered their products directly into the markets. In 2007 it radically changed its logistics structure to a centralized one: Plants deliver goods to a central warehouse in Germany, which in turn delivers goods to customers and regional satellite warehouses in order to ensure critical service times (LOCOM Consulting GmbH & Geberit AG, 2008).

- H&M, a global fashion retailer, operates in Hamburg a central transit terminal through which a high proportion of sold items pass. They are forwarded to country distribution centers, where goods are either delivered to stores directly or put in a warehouse, where stores can call for products when required (Kihlén, 2005).

- Rossmann , a drugstore chain active in six European countries, operates one central warehouse and five regional warehouses in order to serve the demand of its retail stores (Dirk Rossmann GmbH, 2012).

Generally companies that operate complex logistics systems, such as a production distribution network, periodically face questions that deal with the design of their network.

Those usually arise from a certain event or a review of existing structures. The following examples provide a representative cross section of such events/reviews and the corresponding questions to be answered:

- New market entry: New countries/markets are included in the company's sphere of presence and new logistic structures have to be established. Question: Where should warehouses be located in order to serve this market?
- Demand growth: Demand has outgrown the capacities of existing structures. Question: Should existing warehouses be extended or closed in order to establish new structures, i.e. warehouses, in different (better) locations?
- Acquisition growth: A company acquires a competitor, which leads to duplicate distribution structures. Question: Should warehouses be consolidated and where would be optimal locations?
- Demand shifts: Demand has shifted from one region to another. Questions: Which warehouses should be shut down? Where should new warehouses be opened?

The examples presented above all have in common the question of (a) where should facilities be located, and (b) how many of them at what size in order to achieve a distribution network design at minimum cost fulfilling customer requirements (service times). The costs considered are transportation costs and warehouse setup/operating costs. In this research we are focusing on the transportation cost aspect, considering economies of scale: Transportation from plants to central warehouse and from central to regional warehouse is generally highly consolidated and carried out by full truck loads (with trucks of differing sizes). A transportation cost differentiation through economies of scale is thus not applicable in most practical settings. Transportation on the last mile however, i.e. from regional warehouses to customers, is usually much more scattered with regard to number of delivery addresses and shipment structure. It is typically carried out as general cargo, and not filling up a whole truck.

In practice transportation cost (freight rates) are calculated on the basis of weight and distance. A freight forwarder usually provides its customer a rate sheet, depicting freight rates as a combination of weight and distance. Full truck loads are generally charged with a distance dependent rate, based on an average pre-defined weight and valid for a given time period. General cargo rates are calculated case-by-case based on the valid rate sheet. Thus it is crucial to incorporate not only the hierarchical aspect into the facility location model, but also the economies of scale aspect on the last mile. In this paper we are developing a distribution structure with four echelons: (Virtual) plant/global warehouse, central warehouses, regional warehouses and customers.

The main contributions of the paper are the following: (a) the combination of hierarchical network setup, economies of scale in transportation cost and service time requirements into a single facility location model, (b) the extensive computational experiment indicating problem sizes and solvability with regard to practical application of the model, (c) a regression based approach identifying parameters that contribute to the model's runtime, and (d) the access to real company data, such as freight rate information and demand data, which allowed to test the models' practical applicability and robustness.

The research is constructed as follows: In section 2 we formulate the requirements of the model and review the existing literature on hierarchical production – distribution facility location problems, as well as on location models considering transportation economies of scale. Section 3 provides the mathematical formulation as a linear program, including the transformation of the nonlinear cost function and service time requirements and their respective incorporation into the model formulation. Section 4 shows results of computational experiments with randomly generated test data and a runtime/complexity analysis. Section 5 provides an application of the model to real world data and managerial implications, combining results from the previous computational analysis and the case study. Section 6 concludes our findings.

2. Requirement analysis and literature review

Section 2 is divided into two subsections. In subsection 2.1, we describe the requirements for the hierarchical facility location model to be developed in the course of this paper. Those requirements are derived from problems arising periodically at a strategic level in companies, as described illustratively in the introduction. Following the formulation of the requirements, we provide in subsection 2.2 a literature review and put this into context with our work.

2.1. *Requirements analysis*

As briefly highlighted above, the requirements stem from problems that managers periodically face in strategic planning and strategic reviews of their supply chains. We have presented the main questions to be answered above and additionally have given examples of solutions implemented by actual companies.

In this work we will consider a (production-) distribution network which consists of four layers: Layer one comprises of a (virtual) plant/global warehouse representing the origin of goods to be transported. This layer is optional. However, the (virtual) plant/global warehouse layer is especially important, if it is necessary to determine distribution structures for markets, which do not have production capacity, i.e. the origin of goods is not in the same geographical area as the demand nodes are. In these cases, the origin of goods may affect the location of central warehouses to a great extent (e.g. production/global warehouse in country A, distribution to be planned for country B). The presence of this virtual plant also helps to reduce the model's runtimes significantly. The spatial location of this first layer is fixed. Layers two and three are central and regional warehouses, respectively. They are served by their respective higher-level facilities. The optimal locations of these facilities are part of the model's optimization. Layer four consists of customers. The movement of goods from one layer to another is associated with a shipping cost. Demand is deterministic, originates at the last layer and is aggregated on each of the layers above. There are two important questions in this context: The first one is to determine the appropriate cost incurred for getting from the origin (e.g. plant, warehouse) to the destination (warehouse, customer). Transportation cost in facility location models may refer to distance between depot and demand (e.g. measured in kilometers or time), or freight cost in monetary terms, which again may differ by the way of calculation (e.g. scale effects: FTL vs. LTL). As outlined in the introduction, we are focusing in our model on the freight cost, derived by weight and distance as it is done in most practical applications. Warehouse setup/operating cost is assumed to be fixed in our model. The second question is to decide which information to use as a proxy for demand. Demand may relate to units, weight or volume transported, number of delivery notes (i.e. equivalent to delivery frequency/number of shipments), or revenue distribution among customers. In our research we are using both, weight and frequency, in order to depict customer demand. The average shipment weight is important to determine the exact transportation cost for each shipment and the frequency allows accounting for total cost of customers' shipments.

In the introduction we have outlined the variance of transportation cost with regard to distance and weight. Economies of scale play an important role in freight rates, especially on the last mile to the customers where weight can vary by large. Thus, a requirement of the model is to incorporate these economies of scale into the model. A second important requirement is service time, which plays a critical role in many industries: Promised delivery times or goods availability is offered to customers. It is necessary to have stock located close enough to customer locations in order to serve the demand within a promised time frame. As a last requirement, the model is intended to be part of a decision support system for managerial decision making. Thus an efficient runtime and solution process is important, too. Based on this, a sound runtime analysis examining the complexity of the model and determining which parameters contribute to longer runtimes is essential.

In this paper, we develop a model that incorporates all the above. We will refer to the model as *SiLCaRD*, which is an acronym for "*Simultaneous Location of Central and Regional Distribution Facilities*".

2.2. Literature review

The literature and related work review for our research is focused on papers taking into account hierarchical models in a production – distribution context and location models incorporating economies of scale. For general reviews and surveys on facility location problems and its variations the interested reader is directed to Hamacher and Nickel (1998), Drezner and Hamacher (2002), Snyder (2006), Sahin and Süral (2007), Melo et al. (2009), Farahani et al. (2010), and Arabani and Farahani (2011).

Table 1 provides a summary of the literature review. We characterized the related work according to the following categories: The model itself, indicating the problem solved, hierarchical setup, problem formulation, objective, capacity constraints, number of layers (total and with location decision), economies of scale and service time. Furthermore we point out if computational results are provided, which solution procedures were applied, the size of the largest problem solved and if a complexity/runtime analysis was conducted with regard to model variables influencing the runtime. Runtime assessments of different solution techniques (e.g. algorithm comparisons) were not considered.

The first part of related papers gives an overview of models with a hierarchical setup. In general, transportation variables in hierarchical setups can be formulated in two ways: flow based or assignment based. Flow based formulations have separate decision variables for connecting each layer of the system: y_{ij} denoting transportation from facilities on level *i* to facilities on level *j*, and y_{jk} denoting transportation from facilities on level *j* to customers on level *k*. Assignment based formulations on the other hand have a single decision variable reaching through all levels of the system: y_{ijk} denominating the amount of goods transported from facilities on level *i* via facilities on level *j* to customers on level *k*. Kaufman et al. (1977) state the necessity of considering different levels of plants and/or warehouses in real-world distribution problems. They develop an assignment based model that simultaneously locates plants and warehouses among a set of given candidate sites and propose a branch-and-bound algorithm based on the work of Efroymson and Ray (1966) to solve it with minimal setup and transportation cost. Ro and Tcha (1984) propose a model that locates plants and warehouses in a setup where goods can be delivered directly or indirectly via warehouses. Additional side constraints represent the adjunct relationship of some warehouses to a certain plant. A branch-and-bound procedure is developed to provide a solution. In the same year Tcha and Lee (1984) set up a multilevel model, which indicates the number of facilities to open on each level. Likewise, a branch-and-bound procedure is used to solve the model and a heuristic to improve available integer solutions is introduced. Computational results for two- and small scale three-level problems are presented. Gao and Robinson Jr. (1992) develop a dual-based optimization procedure to solve the two-echelon uncapacitated facility location problem, which is based on the work of Erlenkotter (1978) for the uncapacitated facility location problem. Two years later their work is generalized to single-echelon, two-echelon, and multi-activity uncapacitated facility location problems (Gao & Robinson Jr., 1994). Hindi and Basta (1994) construct a two-stage distribution planning problem in a multi-commodity context with capacity constraints at the warehouse level. A branch-and-bound algorithm is used to solve the problem and determine warehouse locations and shipping schedules. Köksalan et al. (1995) deal with a case study of a beer company: Their multi-period, capacitated model minimizes transportation and inventory holding cost.

Table 1. Literature review summary.

Paper	Year	Model								Comp.	Solution	Size	C/R A	Remarks
		Problem	H	F	O	C	L (T, LD)	EOS	ST	results	procedure			
Kaufman et al.	1977	FLP	X	AB	MC	U	3, 2	-	-	X	BB	5-30-50	-	
Ro and Tcha	1984	FLP	X	AB	MC	U	3, 2	-	-	X	BB	10-20-50	-	
Tcha and Lee	1984	FLP	X	AB	MC	U	k, k-1	-	-	X	BB	4-6-8-50	-	
Gao and Robinson	1992	FLP	X	AB	MC	U	3, 2	-	-	X	BB	25-25-35	X	
Gao and Robinson	1994	FLP	X	AB	MC	U	3, 2	-	-	X	H/BB	20-25-200	X	
Hindi and Basta	1994	FLP	X	FB	MC+	C	3, 1	-	-	X	BB	10-15-30	(X)	
Köksalan et al.	1995	FLP	X	FB	MC+	C	3, 1	-	-	X	-	2-15-300	-	
Pirkul and Jayaraman	1996	FLP	X	FB	MC	C	3, 2	-	-	X	H	10-20-100	X	
Aardal et al.	1996	FLP	X	AB/FB	MP	U	3, 2	-	-	-	H	-	-	
Aardal	1998	FLP	X	AB/FB	MC	C	3, 2	-	-	X	BB	6-16-50	(X)	
Hindi et al.	1998	FLP	X	AB	MC+	C	3, 1	-	-	X	BB	10.10.2010	(X)	
Pirkul and Jayaraman	1998	FLP	X	FB	MC	C	3, 2	-	-	X	H	10-20-100	(X)	
Klose	1999	FLP	X	FB	MC	C	3, 1	-	-	X	H	10-50-500	(X)	
Klose	2000	FLP	X	FB	MC	C	3, 1	-	-	X	H	10-50-500	(X)	
Marin and Pelegrin	1999	FLP	X	AB/FB	MC	C/U	3, 2/1/0	-	-	X	H	60-60-60	-	
Hinojosa et al.	2000	FLP	X	FB	MC	C	3, 2	-	-	X	H	40-40-75	-	
Thanh et al.	2008	FLP	X	FB	MC+	C	4, 3	-	-	X	BB (CS)	15-15-15-160	X	
Gabor and van Ommeren	2010	FLP	X	AB	MC	U	3, 2	-	-	-	3-approx. alg.	-	-	
Zangwill	1968	FLP	-	-	MC	C	2, 1	CF	-	-	Algorithm	-	-	Nonlinear
Soland	1974	FLP	-	-	MC	C/U	2, 1	CF	-	X	BB	25-50	-	Nonlinear
Klincewicz	1990	FTP	X	AB	MC+	U	3, 1	PL	-	X	H	50-6-50	-	Transshipment decision
Fleischmann	1993	NFP	X	-1	MC+	C	4, 3	PL	-	X	H	-2	-	Loc. based on NFP solution
O'Kelly and Bryan	1998	HLP	-1	AB	MC	-	-1	PL	-	X	-	203	-	
Kim and Pardalos	2000	NFP	-1	-1	MC	-1	-1	PL	-	X	H	-	-	
Kim and Pardalos	2000	NFP	-1	-1	MC	-1	-1	PL	-	X	H	-	-	
Syam	2002	FLP	X	AB	MC+	C	3, 2	PX	-	X	H	100-20-??	-	
Klincewicz	2002	HLP	-1	AB	MC	-	-1	PL	-	X	H	253	-	
Lapierre et al.	2004	HLP	-1	AB	MC	-	3, 1	PX	-	X	H	403	-	
Gümüs and Bookbinder	2004	FLP	X	AB	MC+	C	3, 1	PX	-	X	CS	06.05.1942	-	Trucks capacitated
Lin et al.	2006	FLP	X	FB	MC+	U	4, 2	CF	X	X	H	6-6-50-50	-	Nonlinear
Baumgartner et al.	2012	FLP	X	FB	MC+	U	3, 1	PL	-	X	H, CS	15-90-120	-	
Our work	2012	FLP	X	FB	MC	U	4/3, 2	PL	X	X	BB (CS)	1-45-191-661	X	

H = hierarchy, F = formulation, O = objective, C = capacity, L = layers (T = total, LD = layers with location decision), EOS = economies of scale, ST = service time, C/R A = complexity/runtime analysis

1. Not applicable for this NFP / HLP, 2. Computational problem size not mentioned, 3. Total number of locations in the network

Problem: FLP = facility location problem, FTP = freight transport problem, HLP = hub location problem, NLP = network flow problem

Formulation: AB = assignment based, FB = flow based

Objective: MC = minimize cost (transportation and fixed), MC+ = minimize cost (transportation, fixed, and other components, such as variable operating cost, inventory, etc.), MP = maximize profit

Capacity: C = capacitated, U = uncapacitated

EOS: CF = concave function, PL = piece-wise linear approximation, PX = economies of scale through shipment consolidation in model

Solution method: BB = branch & bound, H = heuristics incl. various techniques, such as e.g. Lagrangian relaxation, valid inequalities, factes, etc., CS = comm. solver

Remarks: (X) = limited or only descriptive indications

Pirkul and Jayaraman (1996) present a multi-commodity, tri-echelon plant and warehouse location model, minimizing total transportation and opening/operating cost. The model is solved by employing Lagrangian relaxation and the authors introduce a heuristic to produce an effective feasible solution. Aardal et al. (1995) deal with the two-level uncapacitated facility location problem and examine the relationship to the one-level uncapacitated facility location problem. They consider different problem formulations (assignment and flow based), introduce new families of facets and valid inequalities, and discuss the associated separation problems. Two years later Aardal (1998) focuses on the efficient solution of single- and two level facility location problems, by describing classes of inequalities to obtain lower bounds for branch-and-bound algorithms and explicitly adding these to the problem formulations. Hindi et al. (1998) formulate a two-stage, multi-commodity, capacitated model, with two additions: Each customers' demand for all products must be served by a single distribution center (single-sourcing) and the plant origin of each product quantity must be transparent. The model is solved by a branch-and-bound algorithm, choosing the distribution centers location by minimizing total cost (transport, operation and opening of distribution centers). Pirkul and Jayaraman (1998) build on their previous research and describe a multi-commodity, multi-plant, capacitated facility location model, which they call PLANWAR. The model is solved by a heuristic based on Lagrangian relaxation and minimizes total transportation and opening/operating cost. Klose (1999) develops an LP-based solution procedure for the two-staged capacitated facility location problem (TSCFLP) with single sourcing constraints. The heuristic iteratively refines the LP formulation using valid inequalities and facets for various relaxations of the problem. One year later Klose (2000) presents a Lagrangian relax-and-cut approach to the TSCFLP. Marín and Pelegrín (1999) consider a two-stage capacitated location model with both flow and assignment based formulation. The models are solved by Lagrangian relaxation and results of both formulation techniques are compared to each other in a computational study. Hinojosa et al. (2000) construct a multi-period, two-echelon, multi-commodity capacitated plant location problem, minimizing transportation and opening/operation cost. They solve the model by a heuristic based on Lagrangian relaxation. Thanh et al. (2008) develop a dynamic multi-period, multi-echelon, multi-commodity production–distribution network and solve it with a commercial mixed-integer linear programming solver. Gabor and van Ommeren (2010) suggest a new integer programming formulation for the multilevel facility location problem, assigning demand points to edges rather than paths (assignment based formulation). For solving the problem a 3-approximation algorithm based on LP-rounding is proposed.

The second part of the literature review gives an overview of work done incorporating transportation economies of scale models dealing with facility locations. This includes facility location problems, hub location problems, network flow problems and freight transport problems. Zangwill (1968) recognizes early that economies of scale play an important role for facility location models. In practice, transportation costs are not a linear function, but rather a non-linear (often concave) function of weight and/or distance. He develops algorithms for special cases and applies them to a variation of models, such as a plant location model. Soland (1974) considers a plant location model with concave cost functions for both transportation and construction/operation of warehouses. He proposes a branch-and-bound algorithm to solve the problem. Klincewicz (1990) develops a freight transport model that includes a concave shipping cost function, which is depicted as a piece wise linear function. The model also decides if goods are shipped directly or via a consolidation terminal from distinct sources to destinations. Fleischmann (1993) presents a multi-commodity 3-stage network flow model with arbitrary nonlinear transport and warehouse costs. In contrast to facility location models, his model determines the locational decisions based on the result of the network flow problem, which in turn is solved heuristically by a sequence of linear flow problems obtained from local linearization. O'Kelly and Bryan (1998) set up a hub location model, called FLOWLOC, incorporating economies of scale in transportation when consolidating flows over hubs. Economies of scale are modeled by a concave cost function approximated by a piece wise linearization, rather than a discount factor as in previous works. Kim and Pardalos (2000a) and Kim and Pardalos (2000b) provide solution techniques based on dynamic slope scaling and trust interval techniques for concave piecewise linear network flow problems and a dynamic domain contracting algorithm for nonconvex piecewise linear network flow problems. Syam (2002) presents a multi-commodity, multi-location model, that also accounts for shipment consolidation and shipment cycle times. He proposes two heuristics to solve the model, based on Lagrangian relaxation and simulated annealing. Klincewicz (2002) examines and develops solution procedures for the hub location model FLOWLOC, proposed by O'Kelly and Bryan (1998). He comes up with an optimal enumeration procedure for the model and search heuristics that are based upon tabu search and greedy random adaptive search procedures (GRASP). Lapierre et al. (2004) formulate a hub location model which outputs the number and the location of transshipment centers as well as the best transportation alternative (full truck loads, less than a truckload, parcels, or own fleet) accounting for both weight and volume metrics. The model is solved by two metaheuristics, based on tabu search and variable neighborhood search. Gümüs and Bookbinder (2004) develop a network design model explicitly incorporating cross-docking and shipment consolidation. Decision is made for which cross-docks to open, and if products are shipped direct or consolidated via the opened cross docks. They use a commercial solver to solve the problem. Lin et al. (2006) set up a multi-product multi-echelon distribution system model. They consider economies of scale for transportation, modeling it as a concave function and assuming ever decreasing cost/mile. The model optimizes for the location consolidation and distribution centers, including inventory levels at distribution centers and decisions about routing of shipments (direct vs. consolidation). A heuristic is proposed, finding near-optimal solutions. Baumgartner et al. (2012) develop a three echelon, multi-product supply chain design model with economies of scale for both warehousing and transportation. The model optimizes the locations and sizes of the medium level facilities, material flows, and transportation frequencies. The problems presented are solved by a commercial solver and compared to heuristics they have developed.

To our knowledge, a multi-stage mixed integer facility location model, determining the location of central and regional warehouses simultaneously, accounting for economies of scale on the last mile by a piecewise linear cost function and including service time requirements has not yet been presented. We furthermore examine the effects of selected model variables on complexity/runtime of the model by a multiple regression analysis. The main contribution of this article is to fill this gap by presenting an adequate model. As a result the mathematical formulation of the *SiLCaRD* model is developed in the next section.

3. Problem formulation

In this section, we present the mathematical formulation of the model. In subsection 3.1 we present a general mathematical model formulation of *SiLCaRD* without economies of scale and service time. By building the model gradually and presenting this version as a first step, we are striving to enhance the understanding of how the model is composed for a reader not familiar with (hierarchical) facility location models. Subsections 3.2 and 3.3 introduce economies of scale and service time, including details on how they are implemented in the model. Subsection 3.4 concludes with the mathematical model formulation of *SiLCaRD* integrating economies of scale as a piecewise linear transportation cost function and service time requirements, which is built on the general formulation of subsection 3.1.

3.1. Mathematical problem formulation without economies of scale

In the following we provide the mathematical formulation of the problem as a mixed integer linear program with linear transportation cost. The model optimizes for layer two and layer three locations and goods flow.

The following notation is used:

H = set of plants, indexed by h

I = the set of potential central warehouse sites, indexed by i

J = the set of potential regional warehouse sites, indexed by j

K = the set of demand nodes, indexed by k

Q = the set of sections of the piecewise linear cost function, indexed by q

n_k = number of yearly deliveries/orders at node k

w_k = average demand (weight) per delivery at node k

d_k = demand at node k (defined as w_k times n_k)

D = total demand of all customers ($=\sum n_k\, w_k$)

c^{pc}_{hi} = distance from plant h to central warehouse i

c^{cr}_{ij} = distance from central warehouse i to regional warehouse j

c^{rk}_{jk} = distance from regional warehouse j to demand node k

F^{c}_{i} = fixed cost at central warehouse i

F^{r}_{i} = fixed cost at regional warehouse j

A^{pc} = slope of linear transportation cost function from plants to central warehouses

B^{pc} = intercept of linear transportation cost function from plants to central warehouses

A^{cr} = slope of linear transportation cost function from central to regional warehouses

B^{cr} = intercept of linear transportation cost function from central to regional warehouses

A^{rk} = slope of linear transportation cost function from regional warehouses to customers

B^{rk} = intercept of linear transportation cost function from regional warehouses to customers

p^c = (maximum) number of central warehouses to be located

p^r = (maximum) number of regional warehouses to be located

M^{rk} = Maximum distance from customer k to regional warehouse j

The decision variables of the problem are:

$$y_{hi}^{pc} = \begin{cases} 1 \text{ if central warehouse } i \text{ is assigned to plant } h \\ 0 \text{ if not} \end{cases}$$

z_{hi}^{pc} = amount of goods transported (weight) from plant h to central warehouse i

$$x_i^c = \begin{cases} 1 \text{ if central warehouse } i \text{ is opened} \\ 0 \text{ if not} \end{cases}$$

$$y_{ij}^{cr} = \begin{cases} 1 \text{ if regional warehouse } j \text{ is assigned to central warehouse } i \\ 0 \text{ if not} \end{cases}$$

z_{ij}^{cr} = amount of goods transported (weight) from central warehouse i to regional warehouse j

$$x_j^r = \begin{cases} 1 \text{ if regional warehouse } j \text{ is opened} \\ 0 \text{ if not} \end{cases}$$

$$y_{jk}^{rk} = \begin{cases} 1 \text{ if demand node } k \text{ is assigned to regional warehouse } j \\ 0 \text{ if not} \end{cases}$$

The problem can now be formulated as follows:

Minimize

$$\begin{aligned} & \sum_{h\in H}\sum_{i\in I} c_{hi}^{pc}(A^{pc}z_{hi}^{pc} + B^{pc}y_{hi}^{pc}) && \text{(a)} \\ & + \sum_{i\in I}\sum_{j\in J} c_{ij}^{cr}(A^{cr}z_{ij}^{cr} + B^{cr}y_{ij}^{cr}) && \text{(b)} \\ & + \sum_{j\in J}\sum_{k\in K} y_{jk}^{rk}c_{jk}^{rk}((A^{rk}w_k + B^{rk})n_k) && \text{(c)} \\ & + \sum_{i\in I} F_i^c x_i^c + \sum_{j\in J} F_j^r x_j^r && \text{(d)} + \text{(e)} \end{aligned} \qquad (1)$$

subject to:

$$\sum_{j\in J} y_{jk}^{rk} d_k = d_k \qquad \forall k \in K \qquad (2)$$

$$\sum_{i\in I} z_{ij}^{cr} = \sum_{k\in K} y_{jk}^{rk} d_k \qquad \forall j \in J \qquad (3)$$

$$\sum_{h\in I} z_{hi}^{pc} = \sum_{j\in J} z_{ij}^{cr} \qquad \forall i \in I \qquad (4)$$

$$y_{jk}^{rk} - x_j^r \leq 0 \qquad \forall j \in J, \forall k \in K \qquad (5)$$

$$z_{ij}^{cr} \leq y_{ij}^{cr} D \qquad \forall i \in I, \forall j \in J \qquad (6)$$

$$y_{ij}^{cr} - x_i^c \leq 0 \qquad \forall i \in I, \forall j \in J \qquad (7)$$

$$y_{ij}^{cr} - z_{ij}^{cr} \leq 0 \qquad \forall i \in I, \forall j \in J \qquad (8)$$

$$z_{hi}^{pc} \leq y_{hi}^{pc} D \qquad \forall h \in H, \forall i \in I \qquad (9)$$

$$y_{hi}^{pc} - z_{hi}^{pc} \leq 0 \qquad \forall h \in H, \forall i \in I \qquad (10)$$

$$x_i^c, x_j^r, y_{hi}^{pc}, y_{ij}^{cr}, y_{jk}^{rk} \in \{0,1\} \qquad \forall h \in H, \forall i \in I, \forall j \in J, \forall k \in K \qquad (11)$$

$$z_{hi}^{pc}, z_{ij}^{cr} \geq 0 \qquad \forall h \in H, \forall i \in I, \forall j \in J \qquad (12)$$

The objective function (1) minimizes the total cost consisting of transportation cost from plants to central warehouses (a), central warehouses to regional warehouses (b), regional warehouses to demand nodes (c), and fixed cost for central warehouses (d) and fixed cost for regional warehouses (e). Constraints (2) – (4) are flow conservation constraints. Constraints (5), (6)-(8), (9)-(10) denote that assignment is only possible to open locations and ensure that the binary variables for location opening are assigned correct (based on flow variables). These variables are used to switch on/off the fixed cost portions (B^{pc}, B^{cr}) of the transportation cost function. Constraints (11) are standard integrality constraints, which require assignment and activity to be binary. Constraints (12) are flow constraints, ensuring only positive amounts to be shipped.

Decision variables x^c_i (x^r_j) could also be replaced through a formulation that is summing up the respective y^{pc}_{hi} (y^{cr}_{ij}): $x_i^c = \sum_{h \in H} y_{hi}^{pc}$ ($x_j^r = \sum_{i \in I} y_{ij}^{cr}$). Due to computational reasons – overall model size – we decided to formulate those as separate decision variables.

In some situations it might be required to limit the number of opened warehouses to a predefined number. This can include strategic company decisions, or additional factors, such as stock levels. By adding constraints (13) and (14), the model can be extended to include this requirement.

$$\sum_{i \in I} x_i^c \le p^c \tag{13}$$

$$\sum_{j \in J} x_j^r \le p^r \tag{14}$$

If one wants to limit the assignment to cases, where central warehouses are different from regional warehouses, constraints (15) can be added to the model. Please note that these constraints are only applicable, if potential central warehouse locations are identical to potential regional warehouse locations.

$$x_i^c + x_j^r \le 1 \qquad \forall i = j \tag{15}$$

In the next subsections we want to discuss briefly the two integral elements of the extended model: (a) the economies of scale in transportation cost and how it is implemented in the model, and (b) the consideration of service time requirements by customers.

3.2. Economies of scale in transportation cost

As outlined in the introduction, freight rates are based on a combination of distance and weight/volume. For the cases we examined, volume plays a subordinate role. There might exist applications, where volume is the determining factor rather than weight. An example can be outer packaging made of polystyrene with low weight, but high volume. Full truck loads (FTL) are generally charged a fixed rate for every transport, based on distance from origin to destination. In the transportation world FTL weight is usually not accounted for, as the maximum load capacity is determined by the vehicle. Thus the logistics providers offer a distance based full truck load freight rate, which is applied regardless of total weight. The variance in weight is averaged out by the provider over its whole fleet. Transports which do not require a full truck are charged differently: The majority of freight forwarders provide freight rate matrices, which hold price information dependent on both distance and weight. Fig. 2 illustratively visualizes the 'freight rate landscape' from real-world data of a German logistics provider. In Fig. 3 the three-dimensional data is transformed to a weight-dependent per kilometer pricing scheme. As the data implies, a purely linear transport cost function would either under- or overestimate the actual shipping cost, depending on the level of linear approximation used to model the freight rate. Thus it is necessary to introduce a weight dependent freight rate determination in the model, in order to account for the differences. As will be shown later this is achieved through piecewise linearization of the cost function, which is incorporated into the model's objective function. A question arising from this discussion is whether to incorporate the differentiation to all transport levels, i.e. plant-warehouse (P2W), warehouse-warehouse (W2W), and warehouse-customer (W2C) transportations. From a practical point of view it can be said that P2W and W2W transports are highly consolidated so that FTL transportation schemes are applied (where distance is the rate determining factor and weight playing a subordinate role, as described above). From a modeling perspective the piecewise linearization can be incorporated, but extends the number of decision variables by large. In contrast to that, the piecewise linearization is important on the last mile of transport (W2C), because freight rates vary depending on weight. The weight variance and overall size of shipments, as well as the price sensitivity with regard to weight is higher in the W2C transportation and consequently weight differentiated pricing schemes should be applied.

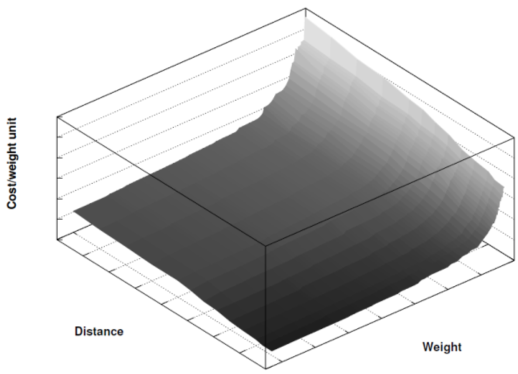

Fig. 2. Freight rate as a function of weight and distance.

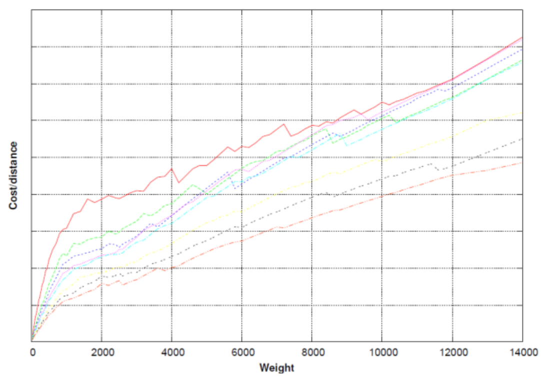

Fig. 3. Cost per distance unit as a function of weight.

An integral part of the model is to incorporate this weight dependent transportation pricing scheme into the objective function. We have chosen the approach of a piece wise linearization of the transportation cost function (Baumgartner et al., 2012; O'Kelly & Bryan, 1998). With this approach the model can still be formulated as a linear program and is approximating the transportation cost function in a better way than a pure linear function. Fig. 4 illustrates the approach. The dotted line represents the cost function; the bold lines represent the linear approximations to different sections of the non-linear cost function. Sections are described by their upper (G^u_q) and lower ($G^l_q = G^u_{q-1}$) bounds and each has its own linear approximation function with a fixed intercept (A_q) and a fixed slope (B_q). These constants are used in the model formulation and the corresponding section is determined by the weight.

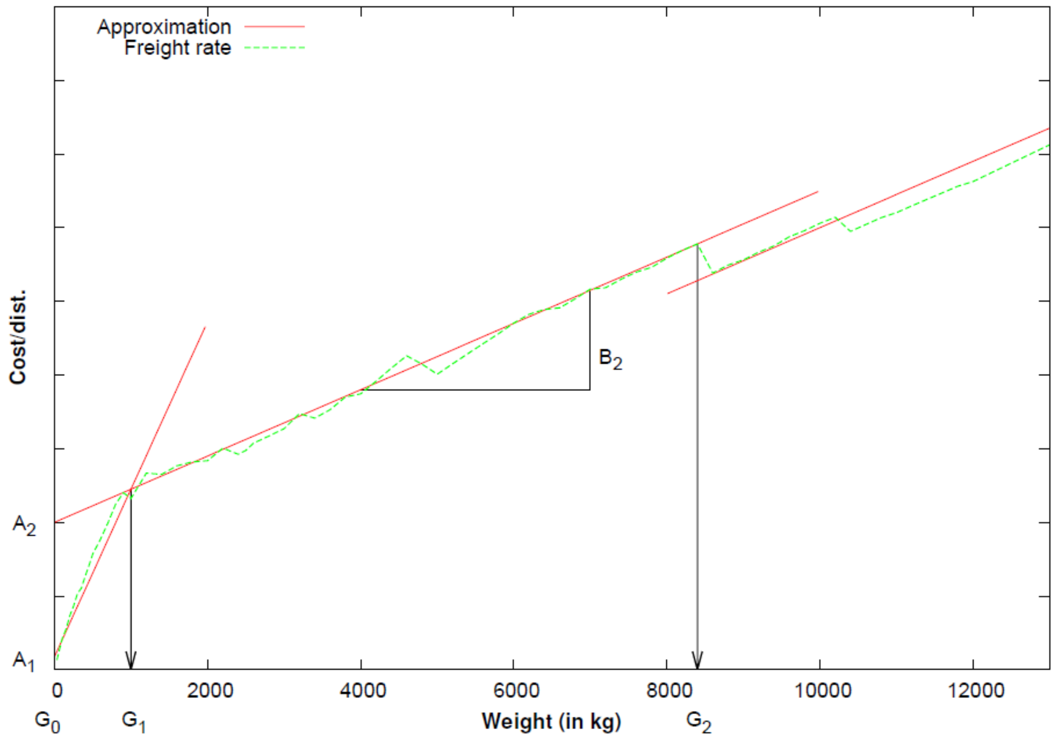

Fig. 4. Piecewise linearization of non-linear freight rate function

3.3. *Customer service times*

In many distribution systems quick customer response times are a crucial element. In plenty of industries pre-defined service times can be found: If the order is placed up to a preset time, the delivery will be carried out within a promised time frame. This requires a certain proximity of the last warehousing stage to the customer. Instead of using drive-time radiuses, a proxy – measured in distance – can be employed, such as a maximum distance allowance from customers to regional facility locations. This requirement is implemented in the model as a constraint which is defining the aforementioned: a maximum distance between any regional warehouse and customer. Travel time may vary for the same distance, as e.g. travelling 50 kilometers through the Alps will most likely take longer than travelling an equal distance in the plane. Nevertheless, the correlation of both is high enough, with a coefficient of correlation of 99%, to use distance as a fair proxy for modeling accessibility. Fig. 5 shows the results of the linear regression analysis. Activating this constraint extends the model from a pure hierarchical location model to an integrated facility location and covering model. Every customer is covered by a regional facility within a certain distance.

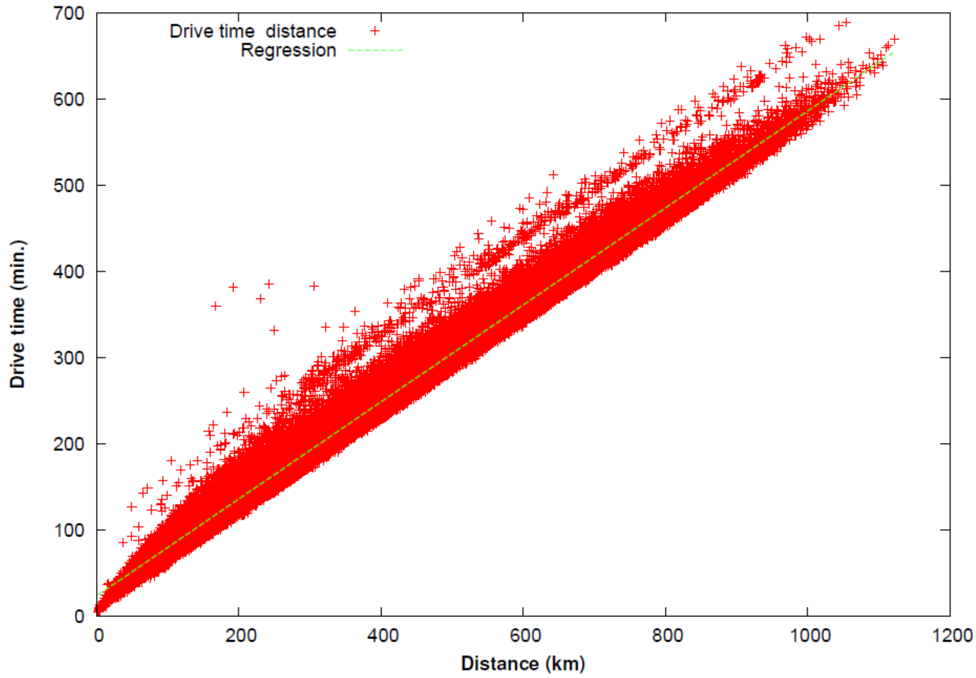

Fig. 5. Correlation of distance and drive time

3.4. Mathematical problem formulation with economies of scale

The model presented in subsection 3.1 uses fixed linear cost functions. The purpose of this paper is – as described in the previous sections – not only to derive a hierarchical facility location model for a production and distribution system, but to also incorporate economies of scale in the transportation cost function and service time requirements. In the following we describe the modifications which have to be applied to the model in order to achieve this. The piecewise linear cost function is applied to the transportation cost of regional warehouses to customers. This formulation could be applied to transportation cost of other levels (e.g. P2W, W2W) in the same manner. We have chosen to modify the transportation cost function for P2W and W2W transports in the formulation below. It is now formulated as distance-based FTL transports, assuming an average load level of full trucks (e.g. 10 tons per full truck load). This reflects the reality in a more accurate way. Thus also some of the constraints could be omitted, because decision variables y^{pc} and y^{cr} are not necessarily needed, as no indicator variable for the piecewise linearization is necessary in the objective function. The notation needs to be extended by:

Q = set of segments in the piecewise linear cost function, indexed by q

A^{rk}_q = slope of segment q in piecewise linear transportation cost function from regional warehouses to customers

B^{rk}_q = intercept of segment q in piecewise linear transportation cost function from regional warehouses to customers

G^u_q = upper bound of segment q piecewise linear transportation cost function from regional warehouses to customers

G^l_q = lower bound of segment q piecewise linear transportation cost function from regional warehouses to customers ($=G^u_{q-1}$)

FTL^{pc} = average FTL weight for transports from plant to central warehouses

FTL^{cr} = average FTL weight for transports from central to regional warehouses

cc^{pc}_{hi} = indicator for cases where plant h equals central warehouse i ($cc^{pc}_{hi}=0$, else 1)

cc^{cr}_{ij} = indicator for cases where central warehouse i equals regional warehouse j ($cc^{cr}_{ij}=0$, else 1)

Following new decision variables are introduced:

$$y^{rk}_{jkq} = \begin{cases} 1 \text{ if demand node } k \text{ is assigned to reg. warehouse } j \text{ in segment } q \text{ of pw.-lin. function} \\ 0 \text{ if not} \end{cases}$$

z^{rk}_{jkq} = amount of goods transported (weight) from regional warehouse j to customer k in segment q of piecewise linear function

The modified model then reads as depicted below. Adapted formulations from the model described previously are characterized by an asterisk. If no adaption is necessary for the piecewise linear model above constraints are still valid. New constraints are presented by a new number, without an asterisk.

Minimize

$$\begin{aligned}
&\sum_{h\in H}\sum_{i\in I}(c_{hi}^{pc}A^{pc}+cc_{hi}^{pc}B^{pc})z_{hi}^{pc}/FTL^{pc} && \text{(a*)}\\
&+\sum_{i\in I}\sum_{j\in J}(c_{ij}^{cr}A^{cr}+cc_{ij}^{cr}B^{cr})z_{ij}^{cr}/FTL^{cr} && \text{(b*)}\\
&+\sum_{j\in J}\sum_{k\in K}\sum_{q\in Q}c_{jk}^{rk}((A_q^{rk}z_{jkq}^{rk}+B_q^{rk}y_{jkq}^{rk})n_k) && \text{(c*)}\\
&+\sum_{i\in I}F_i^c x_i^c+\sum_{j\in J}F_j^r x_j^r && \text{(d*)}+\text{(e*)}
\end{aligned} \qquad (1^*)$$

subject to:

$$\sum_{j\in J}\sum_{q\in Q}z_{jkq}^{rk}=w_k \qquad \forall k\in K \qquad (2^*)$$

$$\sum_{i\in I}z_{ij}^{cr}=\sum_{k\in K}\sum_{q\in Q}z_{jkq}^{rk}n_k \qquad \forall j\in J \qquad (3^*)$$

$$z_{jkq}^{rk}\le y_{jkq}^{rk}D \qquad \forall j\in J,\forall k\in K,\forall q\in Q \qquad (5.1^*)$$

$$\sum_{q\in Q}y_{jkq}^{rk}-x_j^r\le 0 \qquad \forall j\in J,\forall k\in K \qquad (5.2^*)$$

$$y_{jkq}^{rk}-z_{jkq}^{rk}\le 0 \qquad \forall j\in J,\forall k\in K \qquad (5.3^*)$$

$$\sum_{j\in J}\sum_{q\in Q}y_{jkq}^{rk}c_{jk}^{rk}\le M^{rk} \qquad \forall k\in K \qquad (16)$$

$$\sum_{j\in J}\sum_{q\in Q}z_{jkq}^{rk}=w_k \qquad \forall k\in K \qquad (17)$$

$$z_{jkq}^{rk}\le y_{jkq}^{rk}G_q^u \qquad \forall j\in J,\forall k\in K,\forall q\in Q \qquad (18)$$

$$z_{jkq}^{rk}\ge y_{jkq}^{rk}G_q^l \qquad \forall j\in J,\forall k\in K,\forall q\in Q \qquad (19)$$

$$\sum_{q\in Q}y_{jkq}^{rk}\le 1 \qquad \forall j\in J,\forall k\in K \qquad (20)$$

$$x_i^c,x_j^r,y_{hi}^{pc},y_{ij}^{cr},y_{jkq}^{rk}\in\{0,1\} \qquad \forall h\in H,\forall i\in I,\forall j\in J,\forall k\in K,\forall q\in Q \qquad (11^*)$$

$$z_{hi}^{pc},z_{ij}^{cr},z_{jkq}^{rk}\ge 0 \qquad \forall h\in H,\forall i\in I,\forall j\in J,\forall k\in K,\forall q\in Q \qquad (12^*)$$

Constraints (4), and (13)-(15) are still valid for the modified model. Constraints (6)-(8) can be reduced to a single constraint (6*), due to the modeling of FTL transportation cost from P2W.

$$z_{ij}^{cr} \leq x_i^c D \qquad \forall i \in I, \forall j \in J \qquad (6^*)$$

Constraints (9)-(10) can also be omitted due to the aforementioned, as FTL transportation cost is also applied to P2W transports. However, computational tests have shown that leaving the original constraints untouched enhances run times.

The objective function is characterized by equation (1*). Part (a*) and (b*) describe the transport cost from P2W and W2W. Cost is based on FTL transportation which is dependent on distance. Total volume is into FTL, based on an average weight for a full truck. Part (c*) calculates the transport cost from W2C, which is based on the piecewise linear cost function. Average weight of delivery is used to calculate the transportation cost of a single shipment to the customer – based on the right section of the transportation cost function. The total cost is derived by multiplying it with the number of total shipments to this customer. Parts (d*) and (e*) are the setup/operational cost of opened warehouses on central and regional level.

Constraints (2*) and (3*) are again the flow conservation constraints. Constraints (5.1*) – (5.3*) guarantee that assignment is only possible to open locations and that the binary variables for location opening are assigned correct (based on flow variables). Constraint (16) is the service time/maximum distance constraint. Constraints (17) – (20) ensure that the right sections of the piecewise linear cost function is chosen. Constraint (17) is equal to (2*) the flow conservation constraint, but has been presented again in conjunction with (18) – (20) in order to facilitate comprehension of how the piecewise linearization is included. In the optimization it is only listed once. Constraint (17) splits the demand (weight) of customer k among all segments of the piecewise linear cost function. Together with constraints (18) and (19) it is guaranteed, that the assignment to the correct segment is carried out. (18) is describing the upper bound and (19) the lower bound. Thus each demand (weight) w_k is assigned to the correct segment of the piecewise linear cost function. Constraint (20) is necessary that only one y^{rk}_{jkq} is equal to one, as this binary variable turns on/off the intercept part of the linear function in the objective function. It is only equal to one, if z^{rk}_{jkq} (weight in segment q) is greater than zero, i.e. segment *q* of the piecewise linear cost function is active. It can also be omitted in the optimization, as constraint (5.2*) is already satisfying the constraint. Again, we presented it to enhance comprehension of how the piecewise linear segments are introduced into the linear program. Constraints (11*) and (12*) are the standard integrality and non-negativity constraints.

The model is also executable without the virtual plant/global warehouse. As explained before it can be used to provide the optimization with a source of goods, which in turn has a positive effect on the model runtime. If the virtual source is not needed, the first term of the objective function and its corresponding constraints can be omitted.

4. Computational experiments

In this section we test the model with a number of randomly generated problems. We have chosen a two-fold approach: In subsection 4.1 we solve a number of test problems, each containing different random input parameters. In subsection 4.2 we use a regression-based approach to derive analytically which parameters contribute to runtime/complexity. Models in this section are ceteris paribus with regard to input parameters, thus only differing by those parameters under examination, i.e. number of facilities, linear segments, existence of virtual plant/global warehouse, and existence of service time.

4.1. Performance of SiLCaRD model

The randomly created problems are grouped into test instances, which differ by the number of potential central and regional facilities, customer locations and sections of the piecewise linear transportation cost function. Those variables inflate the problem and consequently the runtime solving it optimally. As opposed to section 5, where the model is applied to a real world case study with company data and differing characteristics, this section uses artificially generated test problems. For each instance, ten problems are solved, in order to derive mean results. The randomly generated data follows distributions derived from the analysis of real world freight data. A log-normal distribution for delivery weight is used and a normal distribution for number of deliveries. The virtual plant/global warehouse and potential facility and customer locations are located in a 1,000x1,000 square. The locations are derived by a random generation of x and y coordinates based on a uniform distribution. Distances used in the distance matrix correspond to the Euclidean distance. All other input parameters, such as freight tariffs and fixed cost, are fixed for each of the test problems and modeled with a practical orientation, based on real world data.

Solution procedures for facility location models generally include branch and bound/cut techniques and model relaxations, but also heuristics (such as genetic algorithms, etc.). As the focus of the paper is on the methodological model formulation and its application, we use IBM ILOG CPLEX as a powerful mixed integer linear programming solver. CPLEX makes use of a branch and cut algorithm, solving a series of relaxed sub-problems. Heuristics are used to provide initial solution, which are starting point for the branch and cut search. Problems of the *SiLCaRD* model can be solved optimally with CPLEX, as the consecutive analysis shows. As the problem of locating distribution facilities, which is considered in this paper, is of strategical nature, CPLEX and its associated runtime is an appropriate mean for solving the model. Strategical decisions are generally long-term oriented and thus longer time frames for getting optimal results are accepted, as opposed to other problems, where real-time algorithms are required, such as e.g. critical job scheduling.

Table 2 provides an overview of solved test instances, indicating the models size and respective mean results. Columns denominate the number of central, regional and customer locations. Rows denote the number of linear segments in the transportation cost function. The results provided are the average solving time in seconds, the average optimality gap, and the average objective value of each instance, each consisting of ten problems. The problems were solved on a PC running an Intel® Core™ i5-3320QM CPU at 2.60 GHz with 4.00GB of RAM. The commercial MIP solver used is IBM ILOG CPLEX version 12.3 with standard settings. The problems were modeled with the General Algebraic Modeling System (GAMS), distribution 23.7.3. We have set an upper time limit of twelve hours for the solve process. The optimality gap then results from cases, where the solve process exceeded the preset maximum time allowed to solve the problem. The gap, given in percent, indicates the relative difference of best optimal solution and the best lower bound found after twelve hours solve time. The test runs are set up as follows: 15x3 instances with each consisting of ten problems are solved. Instances vary by (1) facility and customer locations, starting with a number of ten growing up to a number of 200, and (2) linear segments of the piecewise linear transportation cost function, reaching from two to four segments.

Table 2. Average results of solved test problems - Solving time [ST, seconds], optimality gap [OG, percent] and objective value [OV, monetary units].

# linear segments		# of central, regional and customer locations														
		10	20	30	40	50	60	70	80	90	100	120	140	160	180	200
	ST	0	0.1	0.6	2	3	9	32	77	118	136	744	2,284	8,926	10,082	19,783
2	OG	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
	OV	3,433	4,175	4,423	5,184	5,281	6,053	6,136	6,454	6,902	7,247	8,056	8,857	9,132	10,017	10,559
	ST	0	0.1	0.4	1	2	3	16	50	130	154	669	1,242	3,917	8,405	25,722
3	OG	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%
	OV	3,379	4,411	4,699	5,416	6,214	5,962	6,671	7,180	7,093	7,469	8,345	8,839	9,776	10,439	11,219
	ST	0	0.1	0.5	2	6	12	30	81	141	238	1,718	2,718	11,438	21,616	40,365
4	OG	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	1%	3%
	OV	3,734	4,142	4,786	5,284	5,584	5,798	6,478	6,744	6,866	7,251	8,522	8,981	9,395	10,149	11,250

The results of our test problems intimate that problems of realistic size for real world applications can be solved within an acceptable time frame of twelve hours. Furthermore the majority of problems are solved to optimality and the remainder is solved within an optimality gap of less than three percent. As expected, with a growing number of locations the solve time is growing exponentially, as the number of constraints is growing exponentially. The same holds true comparing results for three, respectively four linear segments. Interestingly it can be observed, that the solve time for two linear segments exceeds in 75% of the cases the solve time of instances with three linear segments. We attribute this fact to two reasons: On the one hand artificially created test instances differ and the results presented are mean values of ten problems solved. Thus one *"outlier problem"*, i.e. one of the ten problems requiring a long solve time, affects the average solve time presented negatively. Secondly the IBM ILOG CPLEX solve algorithm reduces the model and decides about the brunch and cut procedure.

4.2. Regression-based runtime/complexity analysis

In a second step, we now want to analyze mathematically which parameters contribute to runtime/complexity of the model. The problems and instances solved and shown in Table 2 all differ by their input variables, such as demand, as each problem was created with random input data. Thus we created new test problems for the runtime/complexity analysis, which only differ by the examined parameters number of facilities (FA), number of linear segments (LS), existence of a virtual plant/global warehouse (VP), and existence of a service time requirement (ST).

Generally a two-fold approach can be used to analyze the models runtime. On the one hand, the model can be assessed using computational complexity theory, which makes a statement about how "hard" it is to solve a problem. On the other hand different instances of the model/problem can be analyzed, making a statement about which parameters/variables of the model contribute positively/negatively to the models runtime.

Garey and Johnson (1979; Current et al., 2002) showed that the p-median model is NP-hard for variable values of p (p = number of facilities to be located). Aardal et al. (1995) state that the two-level uncapacitated facility location problem is also NP-hard, as it generalizes the uncapacitated facility location problem (Aardal et al., 1995; Cornuejols, Fisher, & Nemhauser, 1977; Cornuejols, Nemhauser, & Wolsey, 1990). Our model is basically an extension to those 'general' models, incorporating hierarchy, economies of scale, and service time. It can be concluded that the model belongs to the class of NP-hard problems as it can be reduced to the ones mentioned above: The objective function can be reduced to the classical form of the uncapacitated facility location problem (UFLP) by choosing special cases of the transportation cost functions: Setting intercepts to zero and choosing one linear segment ($q=1$) omits the additional decision variable $y^{rk}{}_{jkq}$ needed for the economies of scale calculation. By leaving out the hierarchical set up in the model (setting variables to zero), the classical objective function of the UFLP is reached: $\sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i y_i$.

The following setup was chosen for our runtime/complexity analysis: In order to obtain which parameters contribute to runtime, again a number of problems were solved. Those results served then as an input to a regression analysis. As in the introduction of this subsection, the problems solved only varied by the input parameters under examination, so as to guarantee the results are not biased by random effects. The parameters under examination varied as follows: Number of facilities reached from 20, 40, 60, up to 80 potential facility and customer locations. Linear segments of the piecewise linear cost function were set to two, three, and four segments. The virtual plant/global warehouse was either existing (1) or non-existing (0), as was the service time requirement. For each combination of the parameters (FA x LS x VP x ST = 4 x 3 x 2 x 2 = 48 problems in total), five demand scenarios were randomly created. The five demand scenarios were identical for each combination of LS x VP x ST, and varied only for the number of facilities, as the demand scenarios are dependent on the number of customer locations. Thus a total of 240 problems were solved.

In order to analyze the effect of these parameters on model's runtime, we applied a log-linear multiple regression analysis to the results. The regressions has the form of *LN(Runtime)* = $\beta_1*FA + \beta_2*LS + \beta_3*VP + \beta_4*ST + k$. We have chosen the log-linear approach, as the results show a higher significance than a model with pure linear dependencies. The total of 240 observations, i.e. problems solved, and four independent variables (FA, LS, VP, ST) translate to 235 degrees of freedom. Thus significance levels of α=0.01 (i.e. a confidence level 99%) and α=0.05 (i.e. a confidence level 95%) lead to critical t-values of 2.60 and 1.97 respectively, and critical F-values of 3.40 and 2.41 respectively. Table 3 shows the results of the regression.

Table 3. Regression results.

Regression Statistics	
R^2	0.836
Adjusted R^2	0.834
F-value	300.514
p-value	2.20E-16

	Coefficients	*t-statistic*	*Significance*[1]	*p-value*
Intercept	-1.191	-3.62	3.61E-04	***
FA	0.1	31.648	1.06E-86	***
LS	0.174	2.004	4.62E-02	*
VP	-1.25	-8.81	2.82E-16	***
ST	-1.547	-10.901	1.16E-22	***

1. significance: *** 99.9%, ** 99%, * 95%; Observations: 240

Since the F-value is larger than the critical F-value we can conclude that the observed relationship between the dependent and independent variables occurs not by chance. Taking a look at the corresponding t- and p-values, it can be deduced that parameters FA, VP, and ST are significant at 99% as the null hypothesis can be rejected. The modulus of their respective t-values is larger than the critical t-value and their p-values are smaller than 0.01. LS is significant at 95%, as the t-value is larger than the critical one and the p-value is smaller than 0.05. Looking at the coefficients, the following effects can be deduced: The inclusion of service time and virtual plant has a positive effect on runtime (i.e. a shorter solve process), whereas an increasing number of facilities and linear segments has a negative effect (i.e. a longer solve process). This also becomes obvious, as the number of constraints increases with an increasing number of facilities. The same is true for an increasing number of linear segments, even though only one segment at a time is active. Let's assume a delivery has a weight of 500kg. Regardless of the number of segments, e.g. three or eight, for both cases only the segment into which the 500kg falls will be active. All other terms of the equation will be equal to zero, as the binary variable denominating the segments will be zero, except for the one where the 500kg falls in.

Summarizing the results above, the findings can be used to develop a guideline allowing distribution networks planners to influence solve time by carefully setting up and selecting variables of the *SiLCaRD* model.

5. Case study model application

In this section we apply the *SiLCaRD* model to a real world problem. During the course of developing the model, real world application was always kept in mind. In the following we are going to describe the process of bringing both together: the *SiLCaRD* model and the multi-faceted real world data. Simplifying assumptions, without sacrificing the models applicability to real distribution planning problems, have to be made. All input data of the solved models in this section is derived from real world sources, in contrast to the previous section. Demand, freight rates and fixed cost are based on company data and distances are based on road distance data. The advantage of our approach is that the model can be directly linked as being part of a decision support system, assisting decision makers in their daily work. Managerial implications are thus not purely based on a hypothetical ground, but on hard figures derived from a business context.

The section is divided into three subsections. Subsection 5.1 gives a brief description of the underlying data and subsection 0 presents the instances solved and their solutions. Subsection 5.3 closes with a view on managerial implications derived from computational experiments of section 4 and the models application to real world data.

5.1. Data

The data used in the case study calculation originates from a company, which is active in the consumer goods sector. It is part of the company's business model to distribute goods and components to end customers and thus it is fulfilling the wholesale function with an own organizational setup. All company centric data, such as demand and cost, originates from the company under consideration. The distances between customers and/or potential facility locations are an integral part for calculating freight rates. As opposed to section 4, where the straight line Euclidean distance was used, in the case study actual road kilometers between origin and destination are obtained. In the case study we used the German market as a reference, as this is one of the best developed markets of the company. The data used is from 2012.

In this subsection we give details on the data used and describe the general approach. We start with a description of the customer and potential facility locations. Customer locations are fixed, with a given spatial reference point, such as a street address or GPS coordinates. Contrary, distribution facilities can be theoretically located at any spatial point of a defined planning area. The facility locations can be placed in- and outside the area in which customer demand is located. In subsection 4.2 we have shown that an increasing number of facility locations leads to an increase in runtime of the solve process. Thus some simplifying assumptions need to be made in order to keep problem size manageable. For our case study, potential locations are within the area in which customers are located, which is Germany. We have chosen to place a grid over the country, denominating potential warehouse locations. Fig. 6 illustrates the grids for central facility locations (squares) and regional facility locations (triangles). A different approach is to choose administrative units and their respective capitals (e.g. counties and county seats). The density of potential facility locations can be defined freely, but the more selected, the bigger the model gets in size, i.e. the number of decision variables is increasing. There are advantages and disadvantages to both approaches: The grid does not account for regional structures, such as population density and infrastructure, but it is easy to implement. The second approach on the other hand may leave ‘white spots’, as administrative units may differ in size. For practical problems both approaches have shown to be valid. A promising approach with regard to a coarse grid chosen, is to refine it in consecutive model runs. The first model run with a coarse resolution gives an indication about regions where to locate facilities. In a second model run, the planner then can only select those regions with a finer resolution. Thus model sizes stay at an acceptable level and the structure (be it the grid or ‘white spots’) become much more granular with higher accuracy.

Customer demand also needs to be aggregated in order to make the data manageable. Today huge amounts of data are available in companies' business intelligence systems. Deliveries can be tracked down to single delivery notes, showing each line item with related information, such as material number, number of pieces delivered, weight, etc. Hence, each delivery note could be assessed in the formulation and optimization, but it would lead to an inflation of the problem size and thus again runtime. The company data used has some 800,000 delivery notes and 28,000 customers. Consequently, in approach (1a) customer delivery data was aggregated to the first three digits of their corresponding postal codes, as is illustrated in Fig. 6 (circles). The German postal code system consists of five digits in total. This structure is fine enough to account for the high number of customers, and still mapping the distribution of population density within Germany, as this is incorporated in the postal code system. In the following case study we have also modeled the customer structure by allocating customers to their corresponding two-digit postal code (1b). A second approach (2) taken in the case study is to allocate customers to the grid of potential regional warehouses (triangles in Fig. 6), where each customer is assigned to its closest (minimal distance) grid point. Demand is defined through number of deliveries and average weight per delivery, as already explained in the section problem formulation.

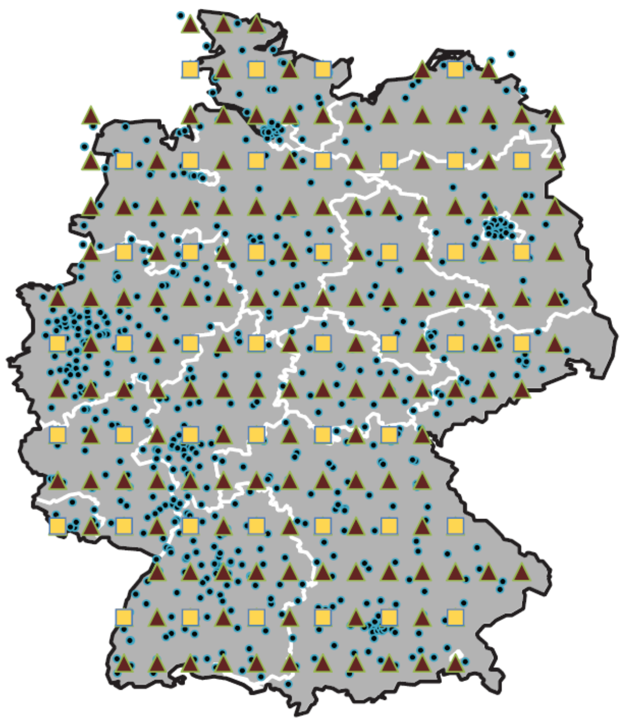

Fig. 6. Potential facility sites and customer locations.

Transportation cost was derived from an actual freight contract of the company with its logistics provider. The rate sheet given was transformed into the piecewise linear form, as described in subsection 3.2. Fixed cost was also extracted from existing contracts.

The service time requirement, which is set to 300km distance in the case study, originates from requirements the company has set towards customer reachability, by promising overnight delivery if orders arrive the day prior until a preset time.

5.2. Problem instances and results

The problems were again solved on a PC running an Intel® Core™ i5-3320QM CPU at 2.60 GHz with 4.00GB of RAM, IBM ILOG CPLEX version 12.3 as MIP solver, and modeled with the General Algebraic Modeling System (GAMS), distribution 23.7.3. We applied a time limit of 43,200 seconds (equals 12 hours) for the solve process. This decision was taken, as from our experience this time frame is the maximum possible that is accepted in a business setting: The optimization can be carried out overnight, with possible results available the next morning.

Table 4 provides an overview of the case study problems we have solved. Problem 1 constitutes a scenario where the planner chooses a low resolution (i.e. coarse grid) of potential facility and customer locations. Problem 2 resembles a medium resolution and problem 3 a high resolution scenario. Problems 1 through 3 are all based on the same cost function with 3 linear segments and all include a virtual plant/global warehouse as well as a service time requirement. In order to show results when the last two input parameters are omitted, problem 2 has been adapted: Problems 2-1, 2-2, and 2-3 denominate scenarios, in which one or both parameters are set to zero, i.e. a virtual plant/global warehouse and/or a service time requirement is non-existent.

Table 5 summarizes the results of the optimization. Runtime, objective value (i.e. total cost) and number of central and regional facilities are indicated. Fig. 7 through Fig. 12 visualize the planning results in a map of Germany. A blue square denotes the central facility and a red triangle regional facilities. For problem 7, 10, 11, and 12 the central and one regional facility coincide.

Table 4. Overview of case study problems.

Problem No. (resolution)	# central facility candidates	# regional facility candidates	# customer locations	Linear segments	Existence of virtual plant	Service time requirement
1 (low)	45 (grid)	45 (grid)	95 (2-digit postal code)	3	1	1
2 (medium)	45 (grid)	191 (grid)	191 (grid)	3	1	1
3 (high)	45 (grid)	191 (grid)	661 (3-digit postal code)	3	1	1
2-1 (medium)	45 (grid)	191 (grid)	191 (grid)	3	1	0
2-2 (medium)	45 (grid)	191 (grid)	191 (grid)	3	0	1
2-3 (medium)	45 (grid)	191 (grid)	191 (grid)	3	0	0

Table 5. Case study problems results.

Problem No.	Runtime (sec.)	Objective value	# central facilities	# regional facilities
1	5	4,767,006	1	4
2	2,645	5,341,278	1	5
3	6,918	5,268,993	1	5
2-1	1,609	4,508,281	1	2
2-2	13,719	4,906,602	1	5
2-3	35,289	4,184,419	1	2

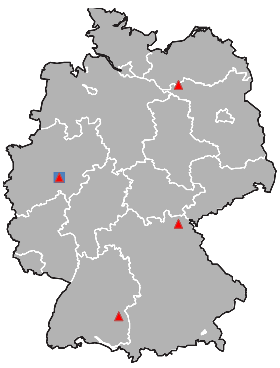

Fig. 7. Planning result for problem 1 (low resolution of customers).

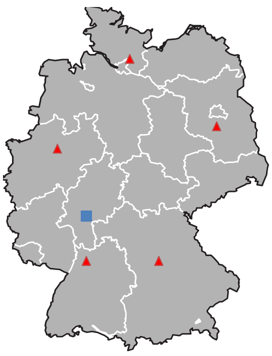

Fig. 8. Planning result for problem 2 (medium resolution of customers).

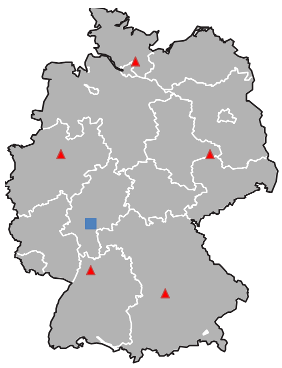

Fig. 9. Planning result for problem 3 (high resolution of customers).

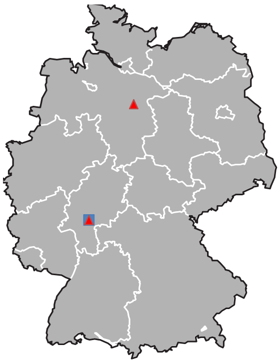

Fig. 10. Planning result for problem 2-1 (medium resolution, ST=0, VP=1).

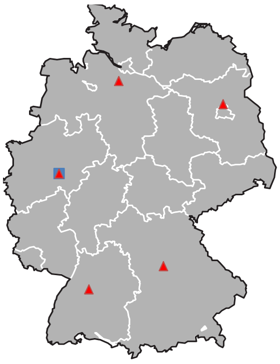

Fig. 11. Planning result for problem 2-2 (medium resolution, ST=1, VP=0).

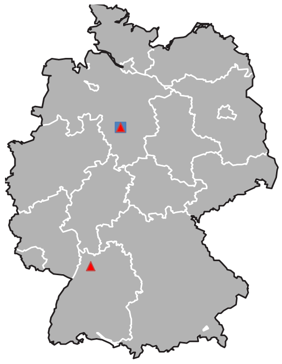

Fig. 12. Planning result for problem 2-3 (medium resolution, ST=0, VP=0).

In the following we are analyzing the results more detailed. In the base scenario – including virtual plant and service time requirement – one central and four, resp. five regional warehouses are established and lead to an optimal cost scenario. Problem 1 is solved optimally within 5 seconds, due to the low potential facility and customer location resolution. Because of the high aggregation of customers to only 95 locations, only four regional warehouses are required. Problems 2 and 3 offer a higher resolution with 191, resp. 661 customer locations and consequently a higher precision. In those scenarios, five regional warehouses lead to an optimal cost structure. The solve time of approximately 44, resp. 115 minutes is adequate, considering the problems size and as a result higher planning accuracy. The difference in total cost of the scenarios is due to the different level of data aggregation.

Problems 2-1 through 2-3 are derivatives of problem 2 and differ in the existence of a virtual plant/global warehouse and service time requirement. If service time is omitted, the number of regional warehouses is reduced to 2, cf. Fig. 10 and Fig. 12. This result is obvious, as no proximity to customers is required to guarantee a promised service time. Total cost is less compared to base case problem 2, as the increase in transportation cost – due to the reduced number of regional facilities – is over-compensated by a reduction in fixed cost, as only two regional facilities are operated. The absence of a virtual plant mainly influences the location of the central warehouses, which moves into the direction of the center of gravity of customer demand (problem 2-3), resp. regional warehouses (problem 2-2). The solve process of the derivative problems 2-1 through 2-3 did also not exceed the allowed time frame of 12 hours, with problem 2-3 taking approximately ten hours.

5.3. Managerial implications

In the course of the paper we have developed a model (*SiLCaRD*) for planning a hierarchical distribution network on a strategic level including economies of scale in transportation cost and service time requirements. The model is of high importance to supply chain/distribution planners as it helps them in their process of strategic decision making. We applied the model to real world problems of reasonable size and were able to obtain satisfactory results within a realistic time frame.

Strategic decisions are an integral part for the future success of businesses. They are in general long-term oriented as opposed to operational, short-term oriented decisions. Against this background strategic decision are not made ad-hoc, but after a longer phase of thorough analysis and scenario evaluation. Having this in mind the maximum allowed time frame of 12 hours for solving the models is adequate for those kinds of decisions, as the results are not needed in real-time. All case study problems, even the ones with a high resolution and consequently a higher number of potential facility and customer locations, were solved within the given time period. Scenarios with a high accuracy, i.e. a high location resolution can be used by supply chain planners. The case study showed that the runtime of all problems was well below the maximum of 12 hours. Thus real world problems can be modeled with adequate accuracy and solved optimally.

We have focused in the paper on the problem formulation, rather than developing efficient heuristics to solve the problem. Instead we have used an available commercial solver using branch and bound techniques to solve the problem instances. In the light of long term strategic decision making, it is again suitable to accept the runtimes of the commercial solver, as laid out above. The problem instances we have considered are furthermore of practice-oriented size: One can model the real world as granular as possible, but the value added is limited at some point. Indications about where to locate distribution facilities are a result of the model and are one input factor amongst others when making strategic decisions about distribution structures. A facility location thus gives an indication of a geographical area where the facility should be located. The micro location again is determined by many other factors: available premises, such as land in case of green-field planning or available warehouses of external logistics providers in case of outsourcing, availability of trained labor, infrastructure, regulatory issues, municipal regulations, etc. to name a few. Thus the granularity we have examined in the paper is fine enough for the majority of business applications. Nevertheless, there might exist cases where a planner wants the potential locations on a finer level. In this case we suggest using a multi-step approach with consecutive model runs: First use a rather coarse grid/setup of potential facility locations. In a second step, refine the grid/setup only at focal points: The focal points in this case are the areas that surround an optimal facility site selected in the first model run. Thus it is possible to keep the overall model size limited, but enhancing the potential facility site resolution.

Our runtime/complexity analysis additionally provides the decision maker with indications on which parameters to adjust or introduce – such as service time or the goods' origin (virtual plant/global warehouse) – in order to enhance runtime, in cases where it is needed (either because results are required quicker or one chooses a higher granularity). The results can serve as a guideline for tolerable problem sizes.

In this section – model application – we deliberately focused on the results of *SiLCaRD* with economies of scale, as this is the core element of our hierarchical facility location model and the paper. In the course testing the model, we also compared results of the *SiLCaRD* model without economies of scale to those presented above including economies of scale. The results of both differ, not only in total cost, but for some instances also in the selection of locations. Our recommendation for real world applications thus is to use the economies of scale version, as it has the ability to model the reality, i.e. non linear freight rates, in a more accurate way and thus providing better results to the decision maker.

6. Conclusions

In this paper we have developed a facility location model called *SiLCaRD*, which is simultaneously locating central and regional facilities, including transportation economies of scale on the last mile and service time requirements. The model was formulated with flow based decision variables. The nonlinear transportation cost was accounted for by transforming the cost function into piecewise linear segments and adapting the model to account for those. Thus it remains still a linear optimization model which can be solved by known techniques. Service time was implemented in form of a covering constraint, setting a maximum distance limit between customers and regional warehouses.

We have shown with computational experiments on randomly generated data that instances of adequate size, applicable to real world problems, can be solved by standard branch and cut techniques. In order to test the model's rigor, we created several instances which varied by the parameters number of facilities, number of linear segments, existence of a virtual plant/global warehouse, and existence of a service time requirement. Based on these instances we conducted a complexity/runtime analysis in order to derive how these parameters influence the models runtime. We can conclude that implementing a service time requirement and/or a virtual plant/global warehouse contribute positively to runtime, i.e. shorten it significantly compared with instances not having those elements. The number of sites – including potential facility sites and customer sites – as well as the number of linear segments have obviously a negative effect on runtime, as the linear problem grows in size (rows, columns, and non-zeroes).

One advantage of this study is that we had access to real world company data, such as freight rates, demand data, distance matrices, etc. Accordingly, we were able to test the model in a business setting and examine its application to a real facility location problem. Results have shown that it is feasible to solve the models within an acceptable time frame, even for bigger sized models with regard to potential facility sites. From a managerial perspective the *SiLCaRD* model provides adequate and sound facility location results. Consequently, it proved to be a powerful tool for distribution network planners and decision makers to plan and optimize their distribution network.

Future work on the model and in the context of hierarchical production-distribution facility location models could include but is not limited to developing powerful algorithms or heuristics which solve the model in even less time than the applied commercial solver. Extensions to the model itself are another research stream: Dynamic aspects allowing to plan over a given time period or multi-commodity settings could be included.

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8. Appendix A. GAMS code of SiLCaRD model

```
* Illustrative example of GAMS code with dummy data

Sets
h depot or plant from which supply comes / 1 /
i central facility locations / 1*10 /
j regional facility locations / 1*50 /
k customers / 1*100 /
q segments of piecewise linear cost function / 1*3 /
;


Table c_pc(h,i) distance from plants h to central warehouse locations i
$include
* Include distance table here
;

Table c_cr(i,j) distance from central warehouse locations i to regional warehouse locations j
$include
* Include distance table here
;

Table c_rk(j,k) distance from regional warehouse locations j to customers k
$include
* Include distance table here
;

Parameters
w(k) average demand (weight) at node k
/ 1 10, 2 20, 3 30, ... /
n(k) number of yearly deliveries or orders at node k
/ 1 10, 2 20, 3 30, ... /
d(k) demand at node k (derived from wk times nk)
DD total demand of all customers
p_c number of central warehouses to be located / 1 /
p_r number of regional warehouses to be located / 4 /
A_rk(q) slope piecewise linear cost function
/1 0.5, 2 0.3, 3 0.2 /
B_rk(q) intercept piecewise linear cost function
/1 1.0, 2 1.5, 3 2.2 /
Gu(q) Upper bounds of segments of pw.-lin. cost function
/0 0, 1 500, 2 1000, 3 20000 /
Gl(q) Lower bounds of segments of pw.-lin. cost function
/0 0, 1 0, 2 500, 3 1000 /
cc_pc(h,i) Indicator parameter for fixed cost portion of FTL cost (distance = 0 -> cc_pc = 0 else cc_pc = 1)
cc_cr(i,j) Indicator parameter for fixed cost portion of FTL cost (distance = 0 -> cc_cr = 0 else cc_cr = 1)
;

d(k) = w(k)*n(k);
DD = sum(k, w(k)*n(k));
cc_pc(h,i)$(c_pc(h,i) eq 0) = 0;
cc_pc(h,i)$(c_pc(h,i) gt 0) = 1;
cc_cr(i,j)$(c_cr(i,j) eq 0) = 0;
cc_cr(i,j)$(c_cr(i,j) gt 0) = 1;
```

```

Scalar
B_pc intercept of linear transportation cost function / 100 /
A_pc slope of linear transportation cost function / 1.0 /
B_cr intercept of linear transportation cost function / 100 /
A_cr slope of linear transportation cost function / 1.0 /
F_r fixed cost for regional warehouse / 500000 /
F_c fixed cost for central warehouse / 1000000 /

Variables
*PLANT and PLANT-to-CENTRAL
y_pc(h,i) binary if plant h delivers to central warehouse i
z_pc(h,i) volume delivered from plant h to central warehouse i

*CENTRAL and CENTRAL-to-REGIONAL
x_c(i) binary variable denominating if central warehouse i is opened
y_cr(i,j) binary variable denominating if regional warehouse j is assigned to central warehouse i
z_cr(i,j) amount of volume delivered from central warehouse i to regional warehouse j

*REGIONAL and REGIONAL-to-CUSTOMER
x_r(j) binary variable denominating if regional warehouse j is opened
y_rk(j,k,q) binary variable denominating if customer k is assigned to regional warehouse j
z_rk(j,k,q) amount of volume delivered from regional warehouse j to customer k

*OBJECTIVE FUNCTION
e total fixed and variable cost;

Binary variable y_pc;
Positive variable z_pc;
Binary variable x_c;
Binary variable y_cr;
Positive variable z_cr;
Binary variable x_r;
Binary variable y_rk;
Positive variable z_rk;

Equations
cost define objective function

volume_regional total volume at regional warehouse
volume_central total volume at central warehouse
volume_plant total volume at plant

open_regional1 only flow from open regional warehouses possible
open_regional2 only flow from open regional warehouses possible
open_regional3 only flow from open regional warehouses possible

open_central1 only flow from open central warehouses possible

piece1 derivation of segment of piecewise linear function
piece2 derivation of segment of piecewise linear function
piece3 derivation of segment of piecewise linear function
piece4 derivation of segment of piecewise linear function
```

```
service_time service time that has to be ensured - proxy in km
;

*OBJECTIVE FUNCTION MINIMIZING TOTAL TRANSPORTATION COST
cost.. e =e= sum((h,i),
(c_pc(h,i)*A_pc+cc_pc(h,i)*B_pc)*(z_pc(h,i)/10000)) +
sum((i,j),(c_cr(i,j)*A_cr+cc_cr(i,j)*B_cr)*(z_cr(i,j)/10000)) +
sum((j,k,q),
c_rk(j,k)*((A_rk(q)*z_rk(j,k,q)+B_rk(q)*y_rk(j,k,q))*n(k))) +
sum(i, F_c*x_c(i)) +
sum(j, F_r*x_r(j));

*CONSTRAINTS
volume_regional(k).. sum((q,j), z_rk(j,k,q)) =e= w(k);
volume_central(j).. sum(i, z_cr(i,j)) =e= sum((k,q),
z_rk(j,k,q)*n(k));
volume_plant(i).. sum(h, z_pc(h,i)) =e= sum(j, z_cr(i,j));

open_regional1(j,q,k).. z_rk(j,k,q) =l= y_rk(j,k,q)*DD;
open_regional2(j,k).. sum(q, y_rk(j,k,q))-x_r(j) =l= 0;
open_regional3(j,k,q).. y_rk(j,k,q)-z_rk(j,k,q) =l= 0;

open_central1(i,j).. z_cr(i,j) =l= x_c(i)*sum(k,d(k));

piece1(k).. sum((q,j), z_rk(j,k,q)) =e= w(k);
piece2(q,k,j).. z_rk(j,k,q) =l= y_rk(j,k,q)*Gu(q);
piece3(q,k,j).. z_rk(j,k,q) =g= y_rk(j,k,q)*Gl(q);
piece4(k,j).. sum(q, y_rk(j,k,q)) =l= 1;

service_time(k).. sum((j,q),y_rk(j,k,q)*c_rk(j,k)) =l= 300;

Model transport /all/ ;

* Include CPLEX Option file
transport.optfile = 1;

solve transport using mip minimizing e;
display z_pc.L, x_c.L, z_cr.L, x_r.L, y_rk.L, z_rk.L;
```

CHAPTER III

A DYNAMIC-HIERARCHICAL FACILITY LOCATION PROBLEM WITH ECONOMIES OF SCALE AND SERVICE TIME FOR A DISTRIBUTION SYSTEM

Abstract

In this paper we develop a planning model for distribution networks, taking decision where and when to locate warehouses. The model simultaneously integrates the hierarchical setup of such systems with regional and central facilities, multi-period planning with opening and closing decisions, economies of scale in transportation cost and service time requirements of customers. These four aspects are considered to be essential in the planning of a distribution network and considering them not at the same time leads to suboptimal results. The problem is formulated as a mixed integer linear program. A computational analysis provides insight into runtimes of models with different sizes. Following this analysis, a preprocessing solution approach is developed, in order to significantly reduce model size and improve solubility, runtime and results. Finally a case study is conducted, showing the applicability of the model to a real-world planning situation with real company data.

Keywords: *Dynamic facility location, hierarchical facility location, distribution networks, economies of scale, service time, decision support systems*

1. Introduction

In today's world, production plants and warehouses are present in almost every spot of the industrialized world. Especially consumer goods require a sophisticated production-distribution network, as many customers have to be supplied according to their demand. Planning such distribution networks and the associated location decisions are a crucial part in the strategic business planning of modern companies. The setup of their distribution network is directly related to cost and customer satisfaction. Decisions are generally long-term oriented, with planning horizons of several years.

Quantitative facility location from a mathematical/ operations research perspective is a well-studied field in academia. The problems can be divided into two problem classes, continuous and discrete location problems. First mentioned problems deal with the location of facilities in the plane, where facilities can be located at any spot in the plane. Last mentioned problems deal with the location of facilities among an a priori known number of potential facility sites. A good introduction into the topic of facility location problems is given by Drezner & Hamacher (2002). In this paper we will focus on discrete location problems. Most basic problems of this kind deal with the location of warehouse facilities in order to minimize the sum of demand-weighted cost (e.g. distance, financial cost) to serve total demand, also known as p-median problems (Hakimi, 1964).

Many extensions of the problem have been studied since then. Extensions include, but are not limited to capacitated setups, hierarchical networks (e.g. three tiers such as plants, warehouses, and customers), multi-product settings, dynamic aspects (i.e. planning several time horizons), different cost functions, and planning under uncertainty. The interested reader is direct to Arabani & Farahani (2011), Drezner & Hamacher (2002), Melo, Nickel, & Saldanha-da-Gama (2009), Melo & Nickel (2007), Owen & Daskin (1998), Revelle & Eiselt (2005), Sahin & Süral (2007), and Snyder (2006) to find overviews and more details of those extensions. In real world applications, many of those extensions appear at the same time. In this paper we want to focus on a combination of some of those aspects, which we feel they should be considered simultaneously while planning a distribution network. The considered aspects are a dynamic (multi-period) setup in a hierarchical context (two warehouse stages) accounting for economies of scale in transportation cost and service time requirements from customers. Especially the link of spatial – where to locate a facility – and temporal – when to locate/close a facility – are an important aspect, as Sheppard (1974) already pointed out in 1974.

Main contributions of this paper are (1) bringing together the dynamics, the hierarchical setup, economies of scale in transportation cost and service time requirements simultaneously in a mixed integer linear programming model, (2) developing a preprocessing approach in order to significantly reduce model size and consequently runtime, and (3) being able to draw upon real world company data to conduct a case study and test the models viability in a business context.

The paper is organized as follows: Chapter 2 reviews related work and introduces relevant papers and concepts. In chapter 3, we will present the mathematical model to the problem and details of the setup along with underlying assumptions. Chapter 4 follows with a computational study of test problems, a solution approach enhancing total runtime and a case study. The problems will be solved with a commercial standard solver. Chapter 5 concludes our findings. Focus of this paper is on the modeling aspect of the problem and the implications from computational results, instead of solution procedures such as algorithms, heuristics, etc.

2. Literature review and related work

The literature on facility location problems is extensive. In the introduction we already pointed out, that many extensions of the original problem have been formulated and studied over the years. Within this section we are focusing on the aspects of facility location problems, which are related to our work. That is, facility location models that incorporate the main aspects we deal with in our model, such as dynamic/multi-period models, hierarchical problem formulations, inclusion of economies of scale, and service time.

The papers of Arabani & Farahani (2011, dynamic) and Sahin & Süral (2007, hierarchical) give a comprehensive overview of existing literature and models in their respective domain of facility location problems. With regard to economies of scale no isolated review exists so far. Work on service time aspects within a distribution context is limited, as this more importantly plays a role in covering models (i.e. to cover all points within a certain/smallest radius). An extensive review of those models can be found in Farahani, Asgari, Heidari, Hosseininia, & Goh (2012). These models are used to locate e.g. emergency facilities such as ambulances or a fire brigade. Table 1 provides a condensed overview of selected work dealing to a big extent with distribution networks. The table is not intended to provide an exhaustive review (please refer to above mentioned papers) but rather models and research which is relevant to or closely related with our work. Those papers review certain aspects of FLPs and a set of combinations of those.

The table provides information about model being presented (P), the objective function (O), dynamic aspects with regard to warehouse opening/closing decision (D), hierarchical setup (H), number of layers (total and with location decision: L(T/LD)), transportation economies of scale (EoS), service time (ST), capacity constraints (C), multi-product setup (MP), and demand uncertainty (DU). We additionally report if experimental results were conducted, which solution procedure was used, and size of the largest problem solved.

In the following we want to present details for a selection of the papers we have reviewed. Those are presented in chronological order, as the combination of considered aspects is overlapping in all directions.

Ballou (1968) was one of the first who recognized in his 1968 paper that in warehouse location it is necessary to not only determine optimal locations for single periods, but to find an optimal plan of warehouse locations and relocations that is cumulatively optimal for a given planning period (maximizing profits in his model). It is thus essential to determine *when* and *where* to locate warehouses based on the analysis conducted in present time with forecasted input parameters. The problem formulated is of continuous nature and solved by techniques of dynamic programming: Optimal locations of the static problems for each period are recursively evaluated in order to find an optimal, profit maximizing location-relocation plan for the whole planning period.

Table 1. Literature review and related work.

Paper		Model										Experimental results (Size largest problem)					Remarks
Authors	Year	P	O	D	H	L (T/LD)	EoS	ST	C	MP	DU	Experi-mental results	Solution procedure	Hierarchy[6]	Periods[7]	Products	
Zangwill	1968	FLP	MC	-	-	2, 1	CF	-	C	(X)	-	-	Algorithm	-	-	-	Nonlinear
Ballou	1968	FLP	MP	X	X	3, 1	-	-	U	-	-	X	H	1-c-6	5	-	Weber problem (euclidean distance)
Wesolowsky	1973	FLP	MC	X	-	2, 1	-	-	U	-	-	X	H	c-5	6	-	Weber problem (rectangular distance)
Soland	1974	FLP	MC	-	-	2, 1	CF	-	C/U	-	-	X	BB	25-50	-	-	Nonlinear
Sheppard	1974	FLP	MC	X	-	2, 1	-	-	C	-	X	-	-	-	-	-	Weber problem (euclidean distance)
Wesolowsky & Truscott	1975	FLP	MC	X	-	2, 1	-	-	U	-	-	X	H/BB	5-10	10	-	
Kaufman, Eede, & Hansen	1977	FLP	MC	-	X	3, 2	-	-	U	-	-	X	BB	5-30-50	-	-	
van Roy & Erlenkotter	1982	FLP	MC	X	(X)[4]	2, 1	-	-	U	(X)[4]	-	X	BB	25-50	10	-	
Ro & Tcha	1984	FLP	MC	-	X	3, 2	-	-	U	-	-	X	BB	10-20-50	-	-	
Tcha & Lee	1984	FLP	MC	-	X	k, k-1	-	-	U	-	-	X	BB	4-6-8-50	-	-	
Balakrishnan & Graves	1985	NFP	MC	-	X	4, 2	PL	-	U	X	-	X	H	10-20-20-10	-	60	
Klincewicz	1990	FTP	MC	-	X	3, 1	PL	-	U	(X)	-	X	H	50-6-50	-	-	Transhipment decision
Gao & Robinson	1992	FLP	MC	-	X	3, 2	-	-	U	-	-	X	BB	25-25-35	-	-	
Daskin, Hopp, & Medina	1992	FLP	MC	X	-	2, 1	-	-	U	-	(X)[4]	X	H	NS	25	-	
Fleischmann	1993	NFP	MC	-	X	4, 3	PL	-	C	X	X	X	H	NS	-	-	Nonlinear, Loc. based on NFP solution
Gao & Robinson	1994	FLP	MC	-	X	3, 2	-	-	U	X	-	X	H/BB	20-25-200	-	40	
Hindi & Basta	1994	FLP	MC	-	X	3, 1	-	-	C	X	-	X	BB	10-15-30	-	3	
Köksalan, Süral, & Ömer	1995	FLP	MC	-	X	3, 1	-	-	C	-	-	X	-	2-15-300	-	-	
Dreszner	1995	FLP	MC	X	-	2, 1	-	-	U	-	-	X	H	c-100	10	-	Weber problem (euclidean distance)
Pirkul & Jayaraman	1996	FLP	MC	-	X	3, 2	-	-	C	X	-	X	H	10-20-100	-	3	
Aardal, Labbé, Leung, & Queyranne	1996	FLP	MP	-	X	3, 2	-	-	U	-	-	-	H	-	-	-	
Hormozi & Khumawala	1996	FLP	MC	X	X	3, 1	-	-	C	-	-	X	H	NS-12-NS	20	-	
Chardaire & Sutter	1996	FLP	MC	X	-	2, 1	-	-	U	-	-	X	H	350-350	5	-	Mixed quadratic program
Canel & Khumawala	1996	FLP	MP	X	-	2, 1	-	-	C/U	-	-	X	CS	11-13	4	-	
Aardal	1998	FLP	MC	-	X	3, 2	-	-	C	-	-	X	BB	6-16-50	-	-	
Hindi, Basta, & Pienkosz	1998	FLP	MC	-	X	3, 1	-	-	C	X	-	X	BB	10-10-10	-	10	
Pirkul & Jayaraman	1998	FLP	MC	-	X	3, 2	-	-	C	X	-	X	H	10-20-100	-	3	
O'kelly & Bryan	1998	HLP	MC	-	-1	-1	PL	-	-	-	-	X	-	202	-	-	
Current, Ratick, & ReVelle	1998	FLP	MC	X	-	2, 1	-	-	U	-	X	X	FE	10-20	-4	-	
Klose	1999	FLP	MC	-	X	3, 1	-	-	C	-	-	X	H	10-50-500	-	-	
Marin & Pelegrin	1999	FLP	MC	-	X	3, 2/1/0	-	-	C/U	-	-	X	H	60-60-60	-	-	
Klose	2000	FLP	MC	-	X	3, 1	-	-	C	-	-	X	H	10-50-500	-	-	
Kim & Pardalos	2000	NFP	MC	-	-1	-1	PL	-	-1	-	-	X	H	-	-	-	
Kim & Pardalos	2000	NFP	MC	-	-1	-1	PL	-	-1	-	-	X	H	-	-	-	
Hinojosa, Puerto, & Fernández	2000	FLP	MC	X	X	3, 2	-	-	C	X	-	X	H	40-40-75	4	3	
Melachrinoudis & Min	2000	FLP/MO	MO	X	X	4, 1	-	X	C	-	-	X	CS	9-7-4-21	5	-	
Tsiakis, Shah, & Pantelides	2001	FLP	MC	-	X	4, 2	PL	-	C	X	X	X	CS	3-6-15-18	-	14	
Canel, Khumawala, Law, & Loh	2001	FLP	MC	X	X	3, 1	-	-	C	X	-	X	H/BB	3-5-15	5	2	
Syam	2002	FLP	MC	(X)[3]	X	3, 2	PX	-	C	X	-	X	H	100-20-??	NS[3]	NS	
Klincewicz	2002	HLP	MC	-	-1	-1	PL	-	-	-	-	X	H	252	-	-	
Wong, Chen, & Xu	2003	FLP	MC	X	X	3, 1	-	-	U	X	-	X	H	50-50-50	20	50	
Lapierre, Ruiz, & Soriano	2004	HLP	MC	-	-1	3, 1	PX	-	-	(X)	-	X	H	402	-	-16	
Gümüs & Bookbinder	2004	FLP	MC	-	X	3, 1	PX	-	C	X	-	X	CS	6-5-42	-	10	Trucks capacitated
Melo, Nickel, & Saldanha da Gama	2005	FLP	MC	X	X	4, 2	-	-	C	X	-	X	CS	5-12-20-75	5	5	
Lin, Nozick, & Turnquist	2006	FLP	MC	-	X	4, 2	CF	X	U	X	X	X	H	6-6-50-50	-	407	Nonlinear
Dias, Captivo, & Clímaco	2006	FLP	MC	X	-	2, 1	-	-	C	-	-	X	H	3-5	3	-	

Table 1 (continued). Literature review and related work.

Paper		Model										Experimental results (Size largest problem)					Remarks
Authors	Year	P	O	D	H	L (T/LD)	EoS	ST	C	MP	DU	Experi-mental results	Solution procedure	Hierarchy[6]	Periods[7]	Products	
Thanh, Bostel, & Péton	2008	FLP	MC	X	X	4, 3	-	-	C	X	-	X	CS	15-15-15-160	5	26	
Tsiakis & Papageorgiou	2008	FLP	MC	-	X	3, 2	-	-	C	X	-	X	CS	6-6-8	-	6	
Behmardi & Lee	2008	FLP	MP	X	-	2, 1	-	-	C	X	-	X	CS	14-25	5	5	
Wang, Sun, & Fang	2008	BL	MP	X	X	4, 2	-	X	C	-	-	X	H	1-16-36-500	2	-	
Dias, Captivo, & Clímaco	2008	FLP	MC	X	X	k, k-1	-	-	C/U	-	-	X	H	10-10-10-100	5	-	
Manzini & Gebennini	2008	FLP	MC	X	X	4/3, 2/1	-	-	U	X[5]	-	X	CS	NS	24	1	
Hinojosa, Kalcsics, Nickel, Puerto, & Velten	2008	FLP	MC	X	X	3, 2	-	-	C	X	-	X	H	10-16-125	8	12	
Gebennini, Gamberini, & Manzini	2009	FLP	MC	X	X	3, 1	-	X	(C)	X	X	X	CS	1-5-200	NS	NS	Nonlinear, but transformed to LP
Shui & Ye	2009	FLP	MC	X	-	2, 1	-	-	U	-	X	X	CS	3-4	3	-	
Gabor & van Ommeren	2010	FLP	MC	-	X	3, 2	-	-	U	-	-	-	3-approx. alg.	-	-	-	
Baumgartner, Fuetterer, & Thonemann	2012	FLP	MC	(X)[3]	X	3, 1	PL	-	U	X	-	X	H, CS	15-90-120	(4)[3]	100	
You & Hsieh	2012	FLP	MC	-	X	3, 2	PX	-	C	X	-	X	H/CS	8-8-8	-	8	Truck-load based
Ghaderi & Jabalameli	2013	NFP	MC	X	-	2, 1	-	-	U	-	-	X	H/BB/CS	80-80	20	-	Nonlinear, but transformed to LP
SiLCaRD model - Chapter II of this thesis	2012	FLP	MC	-	X	4, 2	PL	X	U	-	-	X	CS	1-45-191-661	-	-	

Column heading: P = problem, O = objective, D = dynamic, H = hierarchy, L = layers (T = total, LD = layers with location decision), EoS = economies of scale, ST = service time, C = capacity, MP = multi-product, DU = demand uncertainty

Footnotes: 1. Not applicable for this NFP / HLP, 2. Total number of locations in the network, 3. cycle times / transport schedules determined, 4. Problem presented seperated, not integrated with other aspects, 5. Only for three layer case, 6. c = continuous space, 7. Opening/ closing decision over time horizon

Problem: FLP = facility location problem, FTP = freight transport problem, HLP = hub location problem, NFP = network flow/design problem, MO = multi-objective, BL = bi-level (upper- and lower-level) problem

Objective: MC = minimize cost (or distance), MP = maximize profit, MO = multi objective (maximize profit, minimize access time, maximize local incentives)

EoS: CF = concave function, PL = piece-wise linear approximation, PX = economies of scale through shipment consolidation in model

Capacity: C = capacitated, U = uncapacitated

Solution method: BB = branch and bound, H = heuristics incl. various techniques, such as e.g. Lagrangian relaxation, valid inequalities, factes, dynamic programming , genetic algortihms etc., CS = commercial solver, FE = full enumeration

Remarks: (X) = partially fulfilled, not specified, limited or only descriptive indications; NS = not specified

Zangwill (1968) identifies the importance of concave (non-linear) cost functions that arise in a wide variety of situations and practical applications, which had not been studied in network setups so far. He then examines in his seminal paper of 1968 concave cost functions in networks. Algorithms are developed to solve special problems. One application he reports on is the plant location problem.

Soland (1974) studies a plant location problem and assumes constructing/operating cost and distribution cost to be concave functions of total production, respectively total amount transported. Two models are presented, an uncapacitated and a capacitated version. He describes a branch-and-bound algorithm to find optimal solutions and computational experiments are presented.

Kaufman, Eede, & Hansen (1977) were one of the first authors proposing a hierarchical facility location problem, in which the locations of facilities on two levels are determined simultaneously. Hierarchical facility location problems were formulated before, but involving only one layer for which location decisions were made. They noted, with reference to other work, that in many real world distribution systems different levels, such as plants/warehouses or large/small warehouses, are involved. A branch-and-bound algorithm of Efroymson & Ray (1966) is adapted to solve the model and computational experience is reported.

Balakrishnan & Graves (1985) formulate a network flow problem which allows for LTL consolidation of goods, which then are subject to economies of scale. Novel to their approach was to integrate the economies of scale as a piecewise linear cost function into a mixed integer linear program formulation. They propose a composite algorithm to derive good lower and upper bounds for the problem rather than to solve it optimally.

Klincewicz (1990) employs piece-wise linear functions to approximate concave cost functions in a freight transport model. The piece-wise linear function is not integrated into the model, but instead the model is decomposed to a series of linear cost sub-problems for source-to-terminal and terminal-to-destination shipping. A heuristic solving a sequence of those sub-problems iteratively is developed in order to provide a solution to the original problem. Computational results are provided to test the heuristic.

Gao & Robinson Jr. (1992) study the two-echelon uncapacitated facility location problem and develop a dual-based solution procedure. The authors present a novel problem formulation, in which the warehouses (intermediate level) are represented by both their supplying distribution centers and potential locations. The presented algorithm is an extension of the dual ascent and adjustment procedures developed by Erlenkotter (1978) for the uncapacitated facility location problem. Its efficiency is tested in a computational study with over 420 test problems.

Pirkul & Jayaraman (1996)develop a mixed integer programming (MIP) model for the plant and warehouse location problem in a multi-product, three-echelon setup. The model is relaxed by applying Lagrangean relaxation and the authors develop a heuristic to construct an effective feasible solution for the problem. Two years later, Pirkul & Jayaraman (1998) present the PLANWAR model, which is a multi-product, multi-plant, capacitated facility location problem. Again, Lagrangean relaxation is applied to the model and a heuristic solution procedure is developed. The efficiency of the solution procedure is tested in a series of computational experiments.

O'Kelly & Bryan (1998) present the FLOWLOC model, which is a hub location model designed for airline transportation. The authors identify economies of scale in transportation originating by agglomeration of flow on inter-hub links. The nonlinear cost function is transformed and modeled as a piecewise linear function. Economies of scale are based on per mile cost as a function of flow. O'Kelly & Bryan include the piecewise linear function into the linear programming (LP) formulation, retaining the linearity of the objective function and thus the model. The advantage is that they can make use of exact LP solution procedures. An illustrative example demonstrates the FLOWLOC model and its application.

Hinojosa, Puerto, & Fernández (2000) present a facility location model that combines both, dynamic and hierarchical aspects. It is formulated as a mixed integer program and includes multi-commodity setup as well as capacity restrictions on plant and warehouse level. Facility sites are divided into two subsets: One denominating facilities open from the first period on, and the second denominating candidate sites. The model allows open facilities to be closed and candidate sites to be activated once. The authors develop a heuristic based on Lagrangean relaxation and a dual ascent method in order to solve the problem. A computational study with seven test problems over four time periods is conducted and results are compared to a commercial solver (CPLEX). In a consecutive study, Hinojosa, Kalcsics, Nickel, Puerto, & Velten (2008) extend the model and allow outsourcing (i.e. customers are provided from an outside supplier with goods) and inventory carrying.

Tsiakis, Shah, & Pantelides (2001) model a mixed integer linear hierarchical multi-product facility location problem with economies of scale. The network comprises four layers: plants, warehouses, distribution centers, and customers. Economies of scale are incorporated as a piece-wise linear formulation into the model, adhering its linear nature. They extend the model to also account for demand uncertainty. Scenario probabilities are introduced to the objective function, still resulting in a single network design. A case study is presented to provide computational results.

Canel, Khumawala, Law, & Loh (2001) setup a facility location model, which includes a combination of multi-period, hierarchical and multi-product aspects. It is important to note that the objective function is of non-linear nature. They develop an algorithm including branch-and-bound to generate candidate solutions for each period and then use dynamic programming to find an optimal sequence of configurations over the planning horizon. An application of the algorithm is presented to demonstrate the phase in a numerical example.

Melo, Nickel, & Saldanha da Gama (2005) present a mathematical modeling framework for dynamic facility location. Their model includes aspects such as multi-period planning (relocation of facilities and capacity expansion/reduction), hierarchy (generic supply chain structure), external supply, inventory, capacity restrictions, and budget constraints (capital investments for facilities). They solve test instances of the model with a commercial solver.

Thanh, Bostel, & Péton (2008) deploy a planning model for production distribution systems. Their formulation considers a hierarchical, multi-product system in a dynamical context. The model not only accounts for opening and closing decisions in the course of time, but also for capacity extensions of facilities. Supplier selection (first supply chain level), bill of material inclusion, and inventory management are additional features. The status of a facility can only change once in the planning horizon. Computational tests are provided and the instances are solved with a commercial solver.

Manzini & Gebennini (2008) develop a set of dynamic-hierarchic facility location models with different facets: The first model is a single-product, multi-period, two-staged model allowing changing the facilities status only once during the planning horizon. Extensions include the multi-product product case and changing the facility status more often. Another extension is a single-product, multi-period, three-staged model, including plants, a central distribution center, regional distribution centers and customers. A case study of an electronics company was presented in order to demonstrate two of the models and solved with a commercial solver. Gebennini, Gamberini, & Manzini (2009) extends the single-product, multi-period, two-staged model to include uncertainty of demand and safety stock, which results in a non-linear mathematical formulation. The authors bypass this issue by presenting a modified linear version, which is used in a recursive procedure in order to find a global optimum solution for the non-linear model.

Baumgartner, Fuetterer, & Thonemann (2012) use the same approach as Balakrishnan & Graves (1985) or O'Kelly & Bryan (1998) of piecewise linearization of cost in order to model non-linear cost in mixed integer form, in a facility location model with a three-echelon, multi-product setup. The locations of intermediate facilities and the transportation plan are determined. The authors solve test problems with a commercial branch-and-bound solver and with solution heuristics developed in the research.

The literature review showed that there is lots of research done on certain aspects of facility location problems. Highly sophisticated models have been developed, accounting for some of these aspects separately or for specific combinations of those in integrated models. Nevertheless, a model incorporating multi-period and hierarchical aspects including transportation economies of scale modeled as a piecewise linear function and service time requirements in a mixed integer linear program (MILP) has not been presented so far. Formulating it as an MILP, retains the simplicity of using existing linear programming solution techniques.

Considering those four aspects at the same time is novel in our approach and necessary for an integrated distribution network planning. Let us briefly discuss the aspects in detail and why we think it is important to consider them simultaneously in a facility location model:

Dynamics: Facility location is a topic of high strategic importance to a company and decisions usually involve a time horizon over several years. Thus it is essential to incorporate the temporal aspect into the decision making process. As a general rule the dynamics in a facility location problem comprise the simultaneous planning of consecutive time periods. Input parameters, such as e.g. demand patterns, population, market trends, distribution cost, etc. (Arabani & Farahani, 2011), will change over time, which requires an integrated facility location model accounting for those changes. In a static model the optimal locations for each time period are determined, based on the respective input parameters valid for the time period. Compared with that static planning, a dynamic, multi-period model determines the optimal locations in the light of the total planning horizon, i.e. taking into account the parameters and time periods in an integrated manner, rather than solving for optimality isolated by single time periods. Thus not only the optimal set of locations for each time period is determined, but the optimal locations for all time periods. Consequently the result is not only a cost optimal facility network, but additionally involves decisions about opening and closing facilities over the course of time in the planning horizon, also known as location and relocation decisions (Hinojosa et al., 2000).

Hierarchical setup: The hierarchical context of the problem refers to the simultaneous planning of more than two stages. Besides demand nodes and warehousing stage, there exist additional layers, such as plants, and/or a differentiation of warehouses into central and regional facilities. The majority of distribution systems consist of multiple layers. Only a minority of goods is delivered directly from its production site to the final customer. Intermediate warehouses play an important role in consolidating shipments, reducing lead times, and increasing efficiency (e.g. distribution cost); just to name a few advantages of such a setup.

Economies of scale: Economies of scale in transportation cost are a crucial aspect in distribution networks, as cost in practice is often a nonlinear function of distance and weight. The unit price for weight is in most applications a decreasing function. Taking a linear relation into account over- or underestimates the real transportation cost associated with the shipment. In order to formulate a linear program, we have chosen the approach of piecewise linearization of the cost function. As explained later we will introduce piecewise linear transportation cost on the last stage, from regional warehouse to customer. This so called *fine distribution* is the most expensive part of many logistics systems. In all stages above goods flow is highly consolidated, which is not the case in customer distribution.

Service Time: The last requirement, service time, is crucial in many of today's distribution systems in order to meet customers demand within a promised time frame or a predefined service level. The service time requirement will be integrated into the optimization model via a proxy, which is a maximum distance of customers to a distribution center.

As laid out above, we believe that the combination and simultaneous modeling of those aspects is of high practical relevance to supply chain planners. In the following section we will present such an integrated MILP.

3. Mathematical formulation

3.1. Model assumptions

We are now presenting the mathematical model formulation of the dynamic-hierarchical facility location problem with transportation economies of scale and service time. The models objective is to minimize the overall transportation cost (subject to economies of scale) and the warehouse operating fixed cost. The model does not account for variable warehousing cost or inventory holding costs.

The model makes decisions about where and when to open or close central and regional facilities and how many of them. It indicates the assignment of customers to facilities and the transportation plan (goods flow/quantities). In contrast to static models, where generally facility establishment and the transportation plan are decision variables, those decision variables are transformed to time staged ones. All time periods and input parameters are considered simultaneously and the model provides an optimal warehouse setup plan.

Potential facility locations are fixed input variables, which do not change over time. The same holds true for distance matrices between plants, warehouses and customers. Time dependent variables are customer demand, transportation cost functions, and warehousing costs.

The formulation does not account for relocation or setup/closure costs of warehouses, even though they could be easily implemented into the model formulation. Our motivation to not include those costs has a practical background: Many businesses do not establish own warehouses anymore, as it was the case a few decades ago, but rather, they rely on logistics service providers instead. Warehouse space and operations is rented from a third party provider, carrying out all related functions for the contracting company. Cost and expenses are thus being seen as variable parts of logistic costs, without existing sunk cost when relocating/closing a facility. The same is true for new contracts, where the investment is limited compared to the building and operation of an own warehouse. If applicable, setup/closing cost can be implemented into the models notation, but due to the above mentioned we have decided to not do so. Nevertheless, an integrated dynamic model is advantageous over a static model. Although we assume no setup/closure cost, a high organizational complexity does not allow companies to change their infrastructure back and forth from period to period as the no cost assumption would suggest.

The model assumes to have an initial setup of open facilities. Those can either be an actual warehouse setup (if a case study) or the optimal solution of the static first period (if artificially generated data). Facilities can change their status at most once. This means that open warehouses can be closed down and candidate warehouse can be opened, but not closed again, during the planning horizon. This approach has been implemented in many other dynamic facility location problems (Hinojosa et al., 2000; Roy & Erlenkotter, 1982). Reasons to do so include the strategic nature of location decisions, which are generally long-term oriented (i.e. a couple of years). An immense organizational complexity comes with a recurring rapid change of the system setup involving warehouse/stock movements. It involves a long hand planning to function smoothly, without a drop in service level and serving customers at same standard they are used to. Such a shift in warehouses back and forth is not feasible from a practical point of view. And this is the reason why the above limitations on warehouse establishments and closings are being introduced. In the case of own warehouses, which we do not consider in our assumptions, additional high capital investments (setup cost, project cost, IT cost) have to be taken into account, which would also prohibit a constant change of warehouse locations. If planners want to allow constant change of warehouse locations, the model could be reduced to a number of static problems equal to the number of the planning horizon.

3.2. Piecewise linear transportation cost function

The introduction briefly raised the nonlinearity of freight cost. In many practical applications freight forwarders provide rate sheets to their customers with rates as a function of distance and weight. The price per weight unit is a decreasing function of total weight transported. The longer the distance traveled, the higher the per-unit cost. Fig. 1 illustratively depicts such a 'freight rate landscape'. Transforming the three-dimensional data into a two-dimensional space reveals that the resulting function is nonlinear (per distance unit cost as a function of weight, see Fig. 2 for reference). It is also obvious from looking at Fig. 2, that nearly linear segments exist in the functions. Those can be approximated by linear functions with a very good fit. The derived linear segments are then input to the model and are in total referred to as the piecewise linear cost function. A big advantage of this approach is that the mathematical model is still of mixed integer linear nature in the objective function and can be solved with known solution methods.

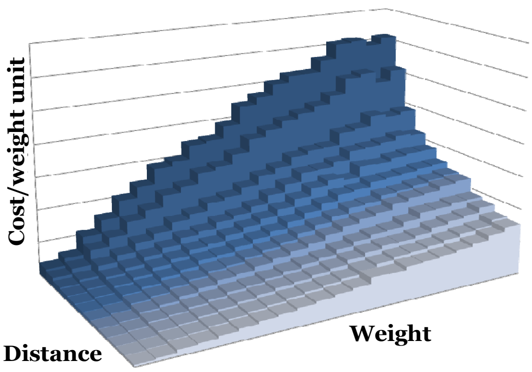

Fig. 1. Illustrative three-dimensional freight rate landscape.

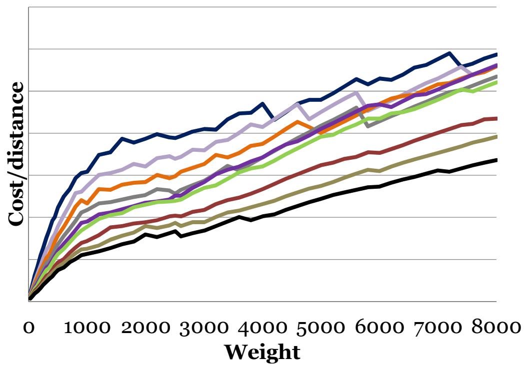

Fig. 2. Illustrative two-dimensional freight rates.

Balakrishnan & Graves (1985) were the first ones who introduced the methodology of modeling a piecewise linear transportation cost function in a mixed integer linear program. Fig. 3 illustratively shows a nonlinear transportation cost function which is approximated with linear functions in different segments. We briefly want to provide the details of this approach. The variables we use are consistent with those used in the models' formulation in section 3. Variable z denominates the weight that is being shipped and which has to be assigned to a segment q of the piecewise linear cost function. Variable z_q determines into which segment q the weight z falls and variable y_q is a binary variable denominating if segment q is active or not. Depending on the number of segments q, only one variable z_q and y_q is unequal to zero (the active segment), all others are equal to zero (inactive segments). Parameters A_q and B_q denote the slope and the intercept of each linear function in segment q. Parameters $G_q{}^u$ and $G_q{}^l$ are the upper and lower bound of each segment q. The mixed integer formulation then reads as follows:

Objective functions' part:

$$\sum_{q \in Q} A_q z_q + B_q y_q \qquad \text{(I)}$$

Constraints:

$$\sum_{q \in Q} z_q = z \qquad \text{(II)}$$

$$z_q \le y_q G_q^u \qquad \forall q \in Q \qquad \text{(III)}$$

$$z_q \ge y_q G_q^l \qquad \forall q \in Q \qquad \text{(IV)}$$

$$\sum_{q \in Q} y_q \le 1 \qquad \text{(V)}$$

$$y_q \in \{0,1\} \qquad \forall q \in Q \qquad \text{(VI)}$$

$$z_q \ge 0 \qquad \forall q \in Q \qquad \text{(VII)}$$

Equation (I) is the part which is included into the objective function and calculates the cost of transportation. Constraint (II) ensures that the weight z is assigned to the right segment q of the piecewise linear cost function, i.e. the sum of all z_q must equal the original shipment weight z. Constraint (III) and (IV) guarantee that the weight assigned to z_q does not exceed the segment's upper bound and is not lower than its lower bound. Constraint (V) states that at only one y_q is equal to one. Constraints (VI) and (VII) are standard integrality and non-negativity constraints.

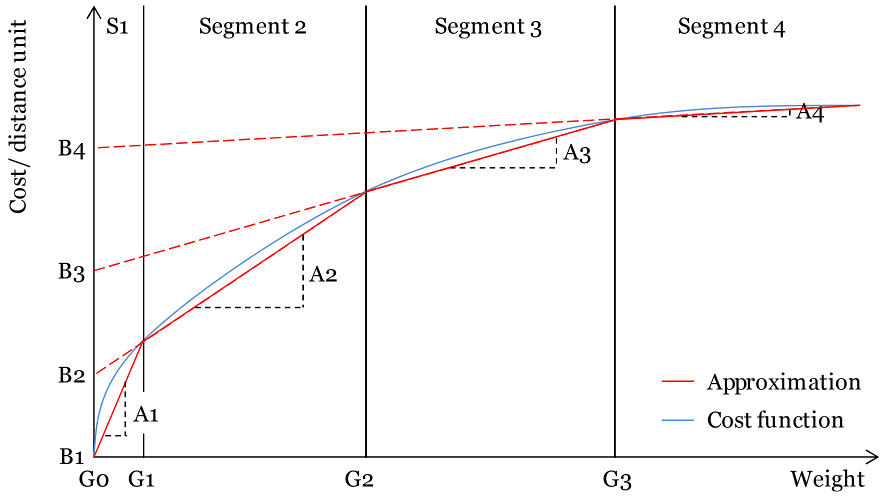

Fig. 3. Piecewise linear approximation of transportation cost function.

In the mathematical model formulation we have only included economies of scale by a piecewise linear cost function for transports from regional facilities to customers. For all other transports, the underlying assumption is that consolidation takes place (orchestrated by the company internally) and transportation is carried out in full truckloads. Yet, the approach of piecewise linearization for the transportation cost can easily be implemented in the model, following the above approach.

3.3. *Service time*

The service time requirement of customers is implemented through a proxy formulation. The reason for doing so is to keep the model as a single objective model. Integrating time into the objective function would result in two different units of measurements. Consequently this requires a method on how to weight those and to find a reasonable trade-off, which can be based on the planners/companies own utility function (trade-off cost vs. service time). The approach we have chosen is simple, but efficient: Bearing in mind, that there is a high correlation of travel time (service time) and travel distance, one can use distance as a proxy for time. Table 2 shows a regression of drive time and distance, with distance as the independent variable and drive time as the dependent variable. From the statistic values R^2, F-value and t-value it can be concluded, that distance is a fair proxy for drive time. Thus a maximum distance of each customer to a warehouse can be defined, guaranteeing a delivery within a certain time frame (assuming the items are on stock).

Table 2. Regression of distance and drive time.

Regression Statistics	
R^2	0.985
Adjusted R^2	0.985
F-value	7,988,252
p-value	2.2E-16

	Coefficients	*t-statistic*	*Significance*[1]	*p-value*
Intercept	0.0165	252	***	0
Distance	0.0004	2,826	***	0

1. significance: *** 99.9%, ** 99%, * 95%; Observations: 125,591

3.4. *Model*

The following notation is being used:

H = set of plants, indexed by h

I = the set of potential central warehouse sites, indexed by i

I_o = subset of potential central warehouse sites, which are open in initial period $t=1$

I_c = subset of potential central warehouse sites, which are candidate sites

J = the set of potential regional warehouse sites, indexed by j

J_o = subset of potential regional warehouse sites, which are open in initial period $t=1$

J_c = subset of potential regional warehouse sites, which are candidate sites

K = the set of demand nodes, indexed by k

Q = set of segments in the piecewise linear cost function, indexed by q

T = set of time periods, indexed by t

n_{tk} = number of deliveries/orders at node k in time period t

w_{tk} = average demand (weight) per delivery at node k in time period t

d_{tk} = demand at node k in time period t (defined as w_{tk} times n_{tk})

D_t = total demand of all customers in time period t (= $\sum_{t \in T} n_{tk} w_{tk}$)

c^{pc}_{hi} = distance from plant h to central warehouse i

cc^{pc}_{hi} = indicator for cases where plant h equals central warehouse i (cc^{pc}_{hi} =0, else 1)

c^{cr}_{ij} = distance from central warehouse i to regional warehouse j

cc^{cr}_{ij} = indicator for cases where central warehouse i equals regional warehouse j (cc^{cr}_{ij} =0, else 1)

c^{rk}_{jk} = distance from regional warehouse j to demand node k

F^{c}_{i} = fixed cost at central warehouse i

F^{r}_{i} = fixed cost at regional warehouse j

M^{rk} = Maximum distance from customer k to regional warehouse j

FTL^{pc} = average FTL weight for transports from plant to central warehouses

FTL^{cr} = average FTL weight for transports from central to regional warehouses

A^{rk}_{q} = slope of segment q in piecewise linear transportation cost function from regional warehouses to customers

B^{rk}_{q} = intercept of segment q in piecewise linear transportation cost function from regional warehouses to customers

G^{u}_{q} = upper bound of segment q piecewise linear transportation cost function from regional warehouses to customers

G^{l}_{q} = lower bound of segment q piecewise linear transportation cost function from regional warehouses to customers ($=G^{u}_{q-1}$)

The decision variables of the problem are:

z^{pc}_{thi} = amount of goods transported (weight) from plant h to central warehouse i in time period t

$$x^{c}_{ti} = \begin{cases} 1 \text{ if central warehouse } i \text{ is opened in time period } t \\ 0 \text{ if not} \end{cases}$$

z^{cr}_{tij} = amount of goods transported (weight) from central warehouse i to regional warehouse j in time period t

$$x^{r}_{tj} = \begin{cases} 1 \text{ if regional warehouse } j \text{ is opened in time period } t \\ 0 \text{ if not} \end{cases}$$

$y_{tjkq}^{rk} = \begin{cases} 1 \text{ if demand node } k \text{ is assigned to regional warehouse } j \text{ in time period } t \\ 0 \text{ if not} \end{cases}$

z_{tjkq}^{rk} = amount of goods transported (weight) from regional warehouse j to customer k in segment q of piecewise linear function in time period t

The sets of candidate facility sites, I and J, are each divided into two subsets: Subsets with index o are sites that are open in the initial time period $t=1$. Subsets with index c are candidate sites which are not active in the initial time period $t=1$. The underlying reason for dividing facility site into subsets was explained above in detail: A facility site which is open from the beginning of the planning horizon is only allowed to be closed within the planning horizon. Candidate sites, on the other hand, are only allowed to be opened during the planning horizon, but not to be closed down again.

Minimize

$$\begin{aligned}
&\sum_{t\in T}\sum_{h\in H}\sum_{i\in I}(c_{hi}^{pc}A^{pc} + cc_{hi}^{pc}B^{pc})z_{thi}^{pc} / FTL^{pc} && \text{(a*)} \\
&+\sum_{t\in T}\sum_{i\in I}\sum_{j\in J}(c_{ij}^{cr}A^{cr} + cc_{ij}^{cr}B^{cr})z_{tij}^{cr} / FTL^{cr} && \text{(b*)} \\
&+\sum_{t\in T}\sum_{j\in J}\sum_{k\in K}\sum_{q\in Q} n_{tk} c_{jk}^{rk}(A_q^{rk} z_{tjkq}^{rk} + B_q^{rk} y_{tjkq}^{rk}) && \text{(c*)} \\
&+\sum_{t\in T}\sum_{i\in I} F_i^c x_{ti}^c + \sum_{t\in T}\sum_{j\in J} F_j^r x_{tj}^r && \text{(d*)} + \text{(e*)}
\end{aligned} \tag{1}$$

subject to:

$$\sum_{j\in J}\sum_{q\in Q} z_{tjkq}^{rk} = w_{tk} \qquad \forall k \in K, \forall t \in T \tag{2}$$

$$\sum_{i\in I} z_{tij}^{cr} = \sum_{k\in K}\sum_{q\in Q} z_{tjkq}^{rk} n_{tk} \qquad \forall j \in J, \forall t \in T \tag{3}$$

$$\sum_{h\in H} z_{thi}^{pc} = \sum_{j\in J} z_{tij}^{cr} \qquad \forall i \in I, \forall t \in T \tag{4}$$

$$z_{tjkq}^{rk} \le y_{tjkq}^{rk} G_q^u \qquad \forall t \in T, \forall j \in J, \forall k \in K, \forall q \in Q \tag{5}$$

$$z_{tjkq}^{rk} \ge y_{tjkq}^{rk} G_q^l \qquad \forall t \in T, \forall j \in J, \forall k \in K, \forall q \in Q \tag{6}$$

$$\sum_{q\in Q} y_{tjkq}^{rk} \le 1 \qquad \forall t \in T, \forall j \in J, \forall k \in K \tag{7}$$

$$y_{tjkq}^{rk} c_{jk}^{rk} \leq M^{rk} \qquad \forall t \in T, \forall j \in J, \forall k \in K, \forall q \in Q \qquad (8)$$

$$y_{tjkq}^{rk} D_t \geq z_{tjkq}^{rk} \qquad \forall t \in T, \forall j \in J, \forall k \in K, \forall q \in Q \qquad (9)$$

$$y_{tjkq}^{rk} \leq z_{tjkq}^{rk} \qquad \forall t \in T, \forall j \in J, \forall k \in K, \forall q \in Q \qquad (10)$$

$$x_{tj}^{r} D_t \geq \sum_{i \in I} z_{tij}^{cr} \qquad \forall t \in T, \forall j \in J \qquad (11)$$

$$x_{tj}^{r} \leq \sum_{i \in I} z_{tij}^{cr} \qquad \forall t \in T, \forall j \in J \qquad (12)$$

$$x_{ti}^{c} D_t \geq \sum_{h \in H} z_{thi}^{pc} \qquad \forall t \in T, \forall i \in I \qquad (13)$$

$$x_{ti}^{c} \leq \sum_{h \in H} z_{thi}^{pc} \qquad \forall t \in T, \forall i \in I \qquad (14)$$

$$x_{ti\_o}^{c} = 1 \qquad \forall t \in 1, \forall i \in I\_o \qquad (15)$$

$$x_{ti\_o}^{c} \geq x_{t+1i\_o}^{c} \qquad \forall t \in T, \forall i \in I\_o \qquad (16)$$

$$x_{ti\_c}^{c} \leq x_{t+1i\_c}^{c} \qquad \forall t \in T, \forall i \in I\_c \qquad (17)$$

$$x_{tj\_o}^{r} = 1 \qquad \forall t \in 1, \forall i \in J\_o \qquad (18)$$

$$x_{tj\_o}^{r} \geq x_{t+1j\_o}^{r} \qquad \forall t \in T, \forall i \in J\_o \qquad (19)$$

$$x_{tj\_c}^{r} \leq x_{t+1j\_c}^{r} \qquad \forall t \in T, \forall i \in J\_c \qquad (20)$$

$$x_{ti}^{c}, x_{tj}^{r}, y_{tjkq}^{rk} \in \{0,1\} \qquad \forall t \in T, \forall i \in I, \forall j \in J, \forall k \in K, \forall q \in Q \qquad (21)$$

$$z_{thi}^{pc}, z_{tij}^{cr}, z_{tjkq}^{rk} \geq 0 \qquad \forall h \in H, \forall i \in I, \forall j \in J, \forall k \in K, \forall q \in Q \qquad (22)$$

The objective function (1) minimize the total cost, separated into transportation cost from plant/global warehouse to central facilities (a), transportation cost from central to regional facilities (b), transportation cost from regional facilities to customers, and fixed operating cost of central (d) and regional (e) facilities. Constraints (2) – (4) are the classical flow conservation constraints. Total customer demand has to be served and in-and outflows of facilities have to match. Constraints (5) – (7) identify and activate the corresponding segments of the piecewise linear cost function. Constraint (8) ensures the service time requirement: A maximum distance of regional facility to customer is employed. Constraints (9) – (10) define y_{tjkq}^{rk} based on z_{tjkq}^{rk}, which activate the intercept of the piecewise linear cost function. Constraints (11) – (12) and (13) – (14) determine x_{tj}^{r} and x_{ti}^{c}, based on the respective z_{tij}^{cr} and z_{thi}^{pc} Constraints (15) – (16) and (18) – (19) ensure that facilities that are open in planning period t=1 can only be closed, but not reopened again. Constraints (17) and (20) guarantee that candidate sites stay open once they were activated. Constraints (21) – (22) are general integrality and non-negativity constraints.

4. Computational analysis and case study

In this section we apply a computational analysis and a case study to the problem introduced above. Testing a mathematical model is an integral part of developing the same. Several techniques to do so exist in literature. In most cases models are tested with randomly generated, artificial data. This data is generally subject to a predefined statistical distribution, as researchers' access to real world data is limited or extreme situations should be tested. A second approach is to use available real world data and test the applicability of the model in a business context. We have chosen both approaches: In section 4.1 we generate random instances of the problem. Those instances are solved and results are analyzed. The data is created artificially, following distributions derived from real world applications. The analysis reveals that problems of bigger size cannot be solved efficiently, due to limited resources, such as memory or time. Consequently, section 4.2 introduces a solution approach, which is able to handle bigger sized problems. In section 4.3 we will apply the model to a case study with real world data.

4.1. Computational analysis

For the computational analysis random problems are generated, solved and analyzed. In the following we refer to instances and problems. An instance is consisting of five problems. Certain parameters remain unchanged for instance, but differ for problems. The reason for doing so is to obtain a random set of solved problems, for which average results can be reported. This is done in order to avoid random bias.

4.1.1. Data generation

Input parameters for our computational analysis can be classified into two categories: Fixed and variable input parameters. Fixed ones can be subdivided into instance fixed and global fixed. Instance fixed parameters are unchanged for each problem of an instance. Global fixed parameters are unchanged for all instances/problems solved. Those parameters are modeled after real world company data. Variable ones are randomly generated for each problem and vary within an instance. They are based on a certain probability distribution, which was selected by analyzing real world data.

Table 3. Input parameters and characteristics.

Input parameter	Category	Remarks/Source/Generation
Customer locations	Variable	1,000 x 1,000 square, uniform distribution of coordinates
Plant/global warehouse	Variable	Location 1 – inside customer area: 1,000 x 1,000 square, identical with customer square, uniform distribution of coordinates Location 2 – outside customer area: 3,000 x 3,000 square, enclosing 1,000 x 1,000 customer square, uniform distribution of coordinates
Central candidate sites	Variable	1,000 x 1,000 square, identical with customer locations
Regional candidate sites	Variable	1,000 x 1,000 square, identical with customer locations
Time periods	Instance fixed	Three or six segments
Number of piecewise linear segments	Instance fixed	Two or three segments
Distance tables	Variable	Euclidian distance based on location coordinates
Demand (weight)	Variable	Log-normal distribution (μ=6, σ=1,2)
Demand (number of deliveries)	Variable	Normal distribution (μ=250, σ=50)
Demand growth	Variable	Uniform distribution (0-17%)
Transportation cost plant – central facilities (slope and intercept)	Global fixed	Derived from real world freight tariff
Transportation cost central – regional facilities (slope and intercept)	Global fixed	Derived from real world freight tariff
Transportation cost regional facilities – customer locations (piecewise linear cost function slope and intercept)	Instance fixed	Derived from real world freight tariff, dependent on piecewise linear segments
Fixed cost central and regional facilities	Global fixed	Derived from real world data

Fig. 4 illustratively shows the spatial setup of customers, facility locations and the plant/global warehouse location. For the computational analysis we have chosen that each customer location coincides with a potential regional and central facility respectively. An advantage of this approach is, that only regions where customers are located are covered with potential facility locations and that distance tables for the facility-facility and facility-customer relations are identical. There are plenty of different approaches that can be taken for the selection of potential facilities. One is to place a regular grid over the planning areas. This allows covering the area holistically and equally. A second one, especially suitable for practical problems, is to make a pre-selection of existing warehouse locations of suitable logistics providers. Consequently, choices are limited to an existing infrastructure.

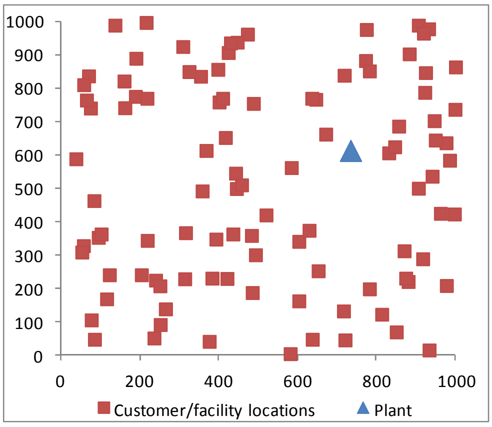

Fig. 4. Illustrative 1,000x1,000 square with customer/facility locations and plant location.

4.1.2. Setup and solution analysis

We used a personal computer with a 64-bit operating system, running with an Intel® Core™ i5-3320M CPU at 2.6GHz with 4 GB of RAM. GAMS (General Algebraic Modeling System) version 23.7.3 was used to formulate the model and IBM ILOG CPLEX version 12.3.0.0 served as the commercial solver to solve the problems. We allowed a runtime of three hours (equals 14400 seconds). If the resource limit was reached, the algorithm terminated and returned the best solution found. In those cases, the optimality gap was reported, which indicates the gap of the mixed integer solution and the best possible solution, i.e. lower bound.

Table 4 provides an overview of the instances considered for the computational analysis. Following the outline above, five problems for each instance were solved and mean runtime and optimality gap results are reported. Parameters time periods, number of segments, location of plant (i=inside customer area, o=outside customer area), number of customer and facility locations varied for each instance. Each dynamic hierarchical facility location problem (DHFLP) was initialized by solving the problem for time period t=1. This solution served as the initial warehouse setup required for the dynamic problem. The initial setup can also be chosen randomly. In a practical setting, the actual warehouse setup marks such an initial solution. Columns "Init." (= initial) and "Dyn." (= dynamic) in Table 4 and Table 5 denote the results for the initial solution and the dynamic solution, respectively. In cases where the resource limit of three hours was exceeded, Table 5 additionally reports the optimality gap.

Table 4. DHFLP problem instances and runtime results.

Time	Seg.	Plant	Facilities									
			20		40		60		80		100	
			Init.	Dyn.	Init.	Dyn.	Init.	Dyn.	Init.	Dyn.	Init.	Dyn.
3	2	I	0.1	0.0	2.9	0.0	17	0.1	194	0.2	295	0.3
		O	0.2	0.0	2.5	0.0	30	0.1	126	0.2	3458	0.2
	3	I	0.1	0.0	2.1	0.0	7	0.1	3308	0.2	426	0.3
		O	0.2	0.1	2.5	0.0	56	0.2	550	0.3	4241	0.5
6	2	I	0.2	5.6	1.1	103	6	7161	2365	14415	814	14398*
		O	0.2	8.3	3.2	899	24	14023	159	14422	574	14396
	3	I	0.2	8.6	4.3	635	32	7431	105	14409	703	14397*
		O	0.2	3.0	3.2	349	27	10412	2980	14430	2058	14401*

*Only four results considered, as for one problem system reported "out of memory status"

Table 5. DHFLP problem instances and optimality gap results in cases where runtime > 3 hours.

Time	Seg.	Plant	Facilities									
			20		40		60		80		100	
			Init.	Dyn.	Init.	Dyn.	Init.	Dyn.	Init.	Dyn.	Init.	Dyn.
3	2	I	-	-	-	-	-	-	-	-	-	-
		O	-	-	-	-	-	-	-	-	0.5%	-
	3	I	-	-	-	-	-	-	0.6%	-	-	-
		O	-	-	-	-	-	-	-	-	0.3%	-
6	2	I	-	-	-	-	-	0.7%	-	3.4%	-	5.5%*
		O	-	-	-	-	-	1.1%	-	2.3%	-	3.5%
	3	I	-	-	-	-	-	1.2%	-	2.6%	-	3.6%*
		O	-	-	-	-	-	0.5%	0.3%	2.7%	-	3.2%*

*Only four results considered, as for one problem system reported "out of memory status"

The results indicate that problems of reasonable size for practical applications can be solved within the allowed time frame of three hours. Nevertheless, the bigger problems get in size – number of facilities or time periods – the higher is the optimality gap. On the other hand, the gap is within a single digit range and only describes the gap between lower bound and actual mixed integer solution. In some cases the optimal solution is already available and only the lower bound is increased by the algorithm. From a practical background, these obtained results are still a valid basis for profound supply chain network planning. Three problems could not be solved due to a lack of memory of the system in use. This issue can easily be bypassed by upgrading the memory. 4 GB, as used in our setup, are standard in today's systems, thus access to additional memory is easily available.

Generally, the initial solutions for bigger problems are obtained quicker than the dynamic ones, due to a reduced overall model size. Accordingly, we are going to develop an approach in the next section which is utilizing this fact in the solution procedure.

4.2. Preprocessing solution approach

The runtime analysis in the previous section showed, that bigger sized dynamic problems are not solved to optimality. Even though the optimality gap is reasonably low in a one digit range, we strived in the course of our research for a way to enhance results. The second reason is that some problems reported a *"out of memory"* status for the system in use. Therefore, we develop a solution approach in this section to circumvent above situations and improve solution quality.

The approach we have chosen is similar to dynamic programming, developed by Bellman in the 1950s (Bellman, 1957). The underlying dynamic hierarchical facility location model has clearly time-varying aspects, as a number of consecutive periods are planned and optimized simultaneously. We utilize this fact and break down the dynamic model into a series of static ones, i.e. one optimization model for each time period considered. The major difference to dynamic programming is that we are not recursively solving the dynamic problem by connecting the static solutions, but rather take the optimal facility locations of each static model as input locations for the dynamic one. Thus, we reduce overall model size of the dynamic program, as the number of potential facility locations is limited and significantly reduced compared to the original dynamic problem. This *'preprocessing'* solution approach enhances consequently the objective value, compared to the objective value with optimality gap of the dynamic solve procedure of the complete problem.

In order to test the performance of the preprocessing solution approach, we created test problems in accordance with the settings and rules defined in the section above. We limited the problems to a set of problems with settings that exceeded the time limit of three hours and showed an optimality gap: 100 facility/customer locations, six time periods, three linear segments and plant located inside and outside the customer area respectively. Table 6 provides an overview of the solved problems. The test setup was as follows: In the first step, the dynamic problems were solved with a preset time of three hours. In the second step, the dynamic problems were transformed to static ones. Each dynamic problem resulted in six (=number of time periods) static problems. Thirdly, these problems were solved individually. In the forth step, the results of the static problems (central and regional facility locations) substituted the potential facility locations of the original dynamic problem. As laid out above, this forth step leads to a drastically reduction in size of the dynamic problem. The last step is to solve the preprocessed dynamic problem.

Table 6 indicates the results of the full dynamic solve compared to the preprocessing dynamic solve processes. Runtime, objective value and optimality gap are reported. Runtime and objective value of the static problems are given in totals, i.e. summing up the individual results. The optimality gap is given as the mean result of all six runs. In order to be able to compare results one-to-one, we solved the full dynamic problem again, allowing the total time required by the preprocessed solve process. The total solve time of the preprocessed approach is then derived by summing up the runtime of all six static problems and the preprocessed dynamic problem. Calculation was only carried out for the seven problems that did not report an *“out of memory”* status.

Comparing the results of the preprocessed dynamic problems with the original dynamic problems in Table 6, we find that the objective values of the former are superior to later ones for all test cases. Improvements of up to 62% of objective values are realized (cf. Table 6, plant = I, problem 2). Furthermore, the original dynamic problems, which reported an *"out of memory"* status, were successfully solved by the preprocessing approach. Therefore, it can be concluded, that the approach of preprocessing the static models to generate a solution space for the dynamic models proofed to be a successful mean of solving the problem.

Table 6. Comparison of results with and without preprocessing.

Plant	Prob.	Full dynamic solve					Preprocessed dynamic solve					
		Solve time: three hours		Solve time: ∑ prepr. dyn.			Static problems (sum of t=1..6)			Dynamic based on static optima		
		OV[1]	G[2]	OV[1]	G[2]	Status	T[3]	OV[1]	G[2]	T[3]	OV[1]	G[2]
I	1	32.5	3.4	32.5	3.1		27,246	33.3	0.4	49	32.4	-
	2	49.9	43.2	-	-	OOM[4]	51,065	31.3	0.7	32	30.8	-
	3	29.1	6.9	29.1	6.8		31,279	29.4	-	37	28.9	-
	4	32.7	3.5	32.6	2.9		32,978	33.0	0.4	18	32.5	-
	5	45.1	40.2	-	-	OOM[4]	26,918	29.8	0.6	7	29.2	-
O	1	62.9	33.2	-	-	OOM[4]	69,474	44.8	1.3	8	44.2	-
	2	54.7	2.5	67.3	21.4		2,228	55.3	-	10	54.6	-
	3	52.0	2.9	51.9	2.6		64,519	52.4	0.2	19	51.9	-
	4	49.8	4.8	49.3	3.4		41,765	49.7	0.4	28	49.2	-
	5	48.6	3.7	48.6	3.6		16,610	49.3	-	18	48.4	-

1. OV = objective value in million, 2. G = gap in percent (average for static), 3. T = time in seconds, 4. OOM = out of memory

4.3. *Case study*

The previous sections dealt with problems that contained random generated data, in order to obtain results regarding performance of the model. In contrast to that, this section applies a real world problem to the model. We first present an overview of the company and the data under consideration, and then continue with a presentation of case study results and implications.

4.3.1. Data

The company under consideration in the case study operates in the consumer goods industry. Activities cover the complete supply chain from sourcing, manufacturing, and distribution to customers. Our focus is on the distribution system. The company runs its own wholesale organization, which is the main contact for customers. Goods are distributed via an existing infrastructure of warehouses at different levels. The case study is limited to the German market, which is a separate legal and organizational unit. Until the 2000's the distribution was solely carried out by the wholesale organization, which comprised some 50 locations spread throughout the country. A major restructuring effort of the distribution system led to a setup comprising central and regional warehouses. Goods are transported from manufacturing plants to the central warehouse, forwarded to regional warehouse and then distributed to customers.

For the case study we were able to use delivery data of the years 2008 and 2012, containing delivery notes of the full year including delivery addresses (5-digit German postal codes) and weight. The access to single delivery notes let us to obtain number of total deliveries to a customer throughout a year and average weight per delivery. The temporal difference allowed us to calculate growth rates of different regions and use that information to project future deliveries.

Table 7 provides an overview of the input data and its characteristics. The customer locations total is 661. It is derived from the first three digits of the German postal code system. Out of the potential 100,000 5-digit postal codes, only approximately 28,500 are in use. Thereof only 8,500 are devoted to cities and key accounts. The remainder is dedicated to PO boxes. All clients that share the same first three digits in postal codes are thus consolidated to the respective 3-digit postal code: The cities *79100 Freiburg* and *79183 Waldkirch* are consequently summarized under the 3-digit postal code *791*.

Table 7. Case study input data and characteristics.

Input parameter	Number	Characteristics
Customer locations	661	Spatial location derived by first three digits of postal code
Plant/global warehouse	1	Actual center of gravity of plants
Central candidate sites	45	Grid evenly distributed over Germany
Regional candidate sites	191	Grid evenly distributed over Germany
Time periods	6	Reaching from 2012 until 2017
Number of piecewise linear segments	2	Below and above 750 kilogram
Distance tables	-	Actual kilometers of German road network
Demand (weight)	-	Delivery notes, aggregated to first three digits of postal code
Demand (number of deliveries)	-	Delivery notes, aggregated to first three digits of postal code
Demand growth	-	Growth from 2008 until 2012 for the 661 customer locations projected into the future
Transportation costs	-	Derived from actual freight contracts of case study company
Fixed cost central and regional facilities	-	Derived from actual contracts of case study company

Fig. 6 gives an overview of the resulting 661 customer locations and their distribution within Germany. Regional and central candidate facility sites are evenly distributed over the planning are in form of a grid, see Fig. 6 and Fig. 7 for reference. We have chosen a time horizon of six years, reaching from the last year with available information of deliveries 2012 until 2017. Demand growth is simulated with historical growth rates from 2008 until 2012 for each of the 661 customer regions. The piecewise linear transportation cost function consists of two segments, below and above 750 kilogram. Demand data, growth rates and cost was derived from available delivery notes and actual contracts.

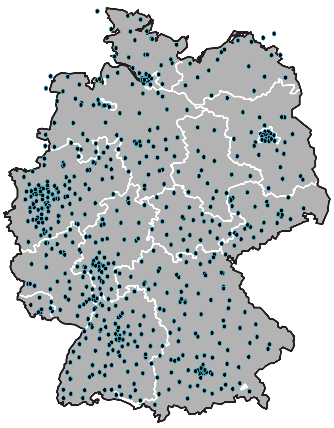

Fig. 5. 661 customer locations

Fig. 6. 191 regional candidate facility sites

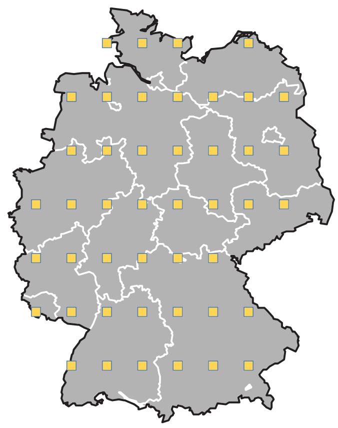

Fig. 7. 45 central candidate facility sites

The actual warehouse setup in 2012, which was established after the restructuring of the distribution system mentioned above, is displayed in Fig. 8. A red square denotes central and a blue triangle regional facilities. One central and six regional warehouses are currently established. As the figure indicates, the central and one regional warehouse are at the same location, but spatially separated under the same roof.

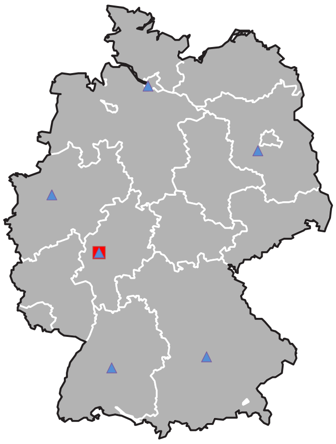

Fig. 8. Actual warehouse setup in 2012

Fig. 9 and Fig. 10 give an overview of the demand distribution of the underlying delivery data in 2008 and 2012, respectively. White areas present no demand, red areas regions with demand. The darker the red, the higher is the demand. It is obvious that demand is concentrated in the west and north of Germany.

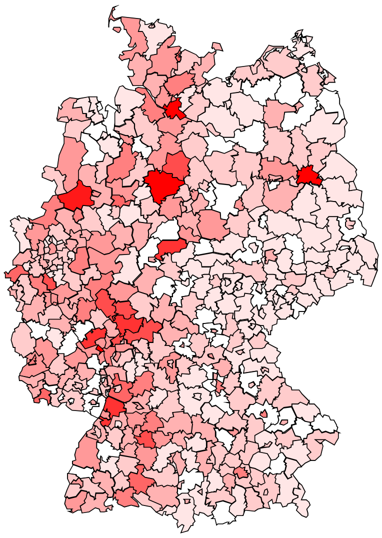

Fig. 9. Delivery distribution (total weight) 2008

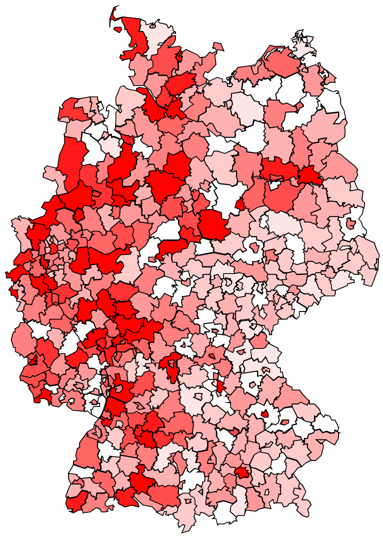

Fig. 10. Delivery distribution (total weight) 2012

4.3.2. Simulation results

In this section the dynamic hierarchical facility location model is applied to the case study with input data as presented above. In a first step we look at the problem in retrospect and compute the optimal results for a distribution network in 2008 (static), in 2012 (static), and from 2008 until 2012 (dynamic). In a second step we calculate the problem forward-looking from 2012 until 2017 by applying historic growth rates. Optimal results for each year planned separately (static) are presented and compared to optimal results for the dynamic problem. The case study is also analyzed with a purely linear cost function and results are backing up our approach of differentiating cost. We additionally provide computational characteristics, such as runtime. The preprocessing approach developed above is also applied to both dynamic problems.

Fig. 11 presents the optimal warehouse setup for the year 2008. In total, one central and five regional facilities need to be established in order to provide a cost and service time optimal structure to customers. Unlike in the actual setup, central and regional warehouse are separated. Fig. 12 illustrates the optimal distribution system for the 2012 data. The setup resembles the actual setup displayed in Fig. 8, with a minor deviation. One central and six regional warehouses mark the optimal result, with central and regional facility coinciding.

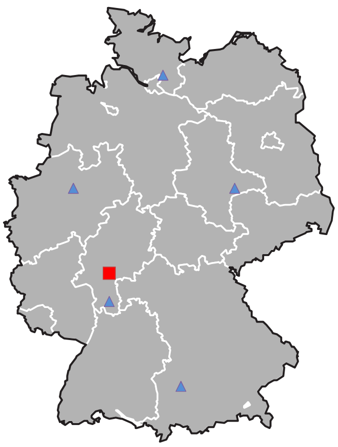

Fig. 11. Optimal warehouse setup static 2008

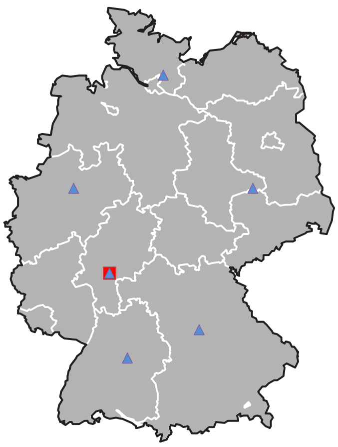

Fig. 12. Optimal warehouse setup static 2012

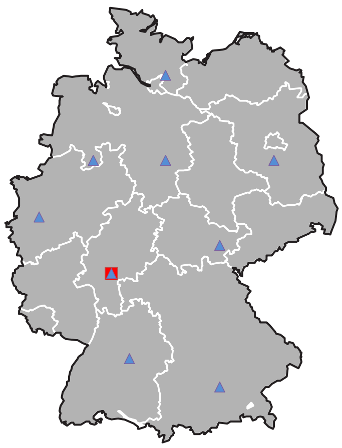

Fig. 13. Optimal warehouse setup static 2017

Combining the five static problems to a dynamic one, with five planning periods from 2008 until 2012, provides information about opening and closing warehouses in the course of time. Demand for 2009 until 2011 is interpolated, based on the available data. Fig. 14 shows how a transition in the past years could have looked like, given a warehouse structure for 2008 as provided in Fig. 11: The center regional warehouse had to move to the central warehouse, and additional two regional warehouses had to be closed, while opening and relocating to three new locations.

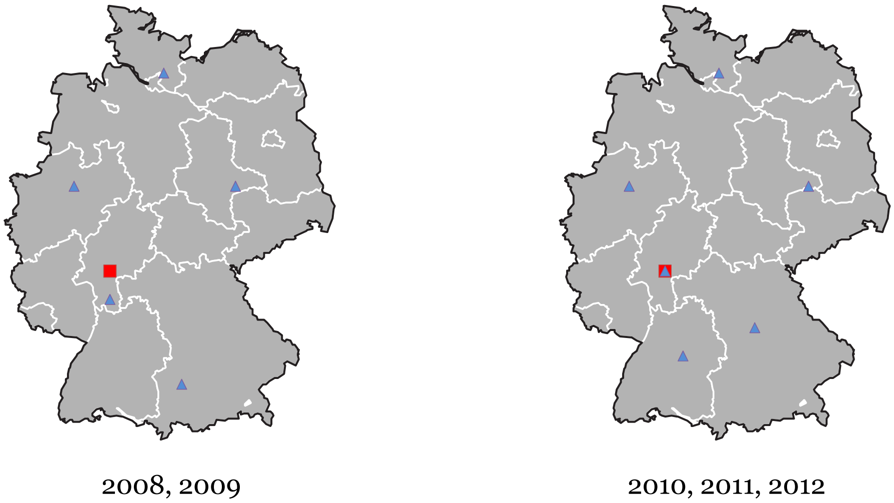

Fig. 14. Dynamic solution providing warehouse transition information for planning period 2008 – 2012

Having analyzed the past with given data, we will assess the future setup of the distribution structure. The problem is structured in three different variations. Variation one are the static (S) problems 2012 until 2017 with linear cost (LIN), in order to compare to variation two, where cost is differentiated, i.e. piecewise linear cost (PW-LIN). Variation three comprises then the developed DHFLP model, solving for the planning horizon dynamically (D). The comparison of above variations illustrates, why it is necessary to consider all aspects simultaneously in model, rather than planning individual periods independently or with a general transportation cost. Table 8 presents the results of above variations, considering the objective value, total runtime and opened/closed regional warehouses. The central warehouse is identical for all settings. Runtime of the DHFLP in total is the solve time of the preprocessed model plus the aggregated solve time of the static piecewise linear results, thus totaling to 20,482 seconds. The results clearly indicate, that a purely linear cost assumption does not reflect the real world differentiated pricing in an adequate manner, and is consequently leading to a different result. Comparing the static problem solutions with piecewise linear cost to the dynamic planning results with piecewise linear cost, it is obvious that a static approach leads to a higher number of warehouse openings and closings. Additionally already closed facilities are reopened in later planning periods again. In contrast to that, the dynamic problem optimizes for a balanced warehouse structure, avoiding unstructured and uncoordinated openings and closings, but anticipating future demand in its decisions. The cost in our case study is approx. 120,000 Euro higher in the dynamic setup, due to adhering longer to opened warehouse and changing locations only if optimal for the overall planning period. Albeit this fact, considering all the organizational effort and project cost coming along with warehouse relocation, as mentioned in the introduction, the cost difference is insignificant.

Table 8. Planning results of static linear, static and dynamic piecewise linear problems.

Problem	Year	Objective	Runtime	Open regional warehouses																		
		(Mio. €)	(sec.)	6	19	23	24	34	49	53	71	77	79	90	93	126	136	137	141	162	172	173
S, LIN	2012	10.4	3,306				X			X	X			X		X				X		
	2013	11.9	2,027	X					X	X	X			X		X						X
	2014	13.8	1,026	X					X	X	X			X		X						X
	2015	16.1	2,489	X		X				X	X			X	X	X						X
	2016	19.1	1,352	X		X				X	X			X	X			X	X			X
	2017	22.8	1,157	X	X	X				X	X			X	X		X		X			X
	Total	**94.0**	**11,357**																			
S, PW-LIN	2012	10.3	2,483				X			X	X			X		X				X		
	2013	11.7	2,117	X					X	X	X			X		X						X
	2014	13.5	1,270	X					X	X	X			X		X						X
	2015	15.7	4,185	X						X	X	X	X				X		X		X	
	2016	18.4	4,000	X						X	X	X	X				X		X		X	
	2017	21.8	6,285	X				X		X	X			X	X		X		X			X
	Total	**91.4**	**20,339**																			
D, PW-LIN (DHFLP pre-processed)	2012	-	-				X			X	X			X		X				X		
	2013	-	-	X				X		X	X			X		X						X
	2014	-	-	X				X		X	X			X		X						X
	2015	-	-	X				X		X	X			X	X	X						X
	2016	-	-	X				X		X	X			X	X		X		X			X
	2017	-	-	X				X		X	X			X	X		X		X			X
	Total	**91.6**	**143**																			

Fig. 15 illustrates the planning results of the individual static problems with piecewise linear cost on a map of Germany. The planning years 2013 and 2014 have the same optimal solution and compared to the 2012 solution, the two regional warehouses located in the central east and west are relocated and an additional one is serving the central region. In the planning years 2015 and 2016 another regional warehouse is established, with four other changing their location. In the last planning year, 2017, again an additional regional warehouse is located. As expected, with considering each planning year isolated of others, warehouse locations are changing rapidly. Taking the northernmost regional warehouse as an example, this location is altered in planning years 2015 and 2016, but relocated to its original spot in the last period.

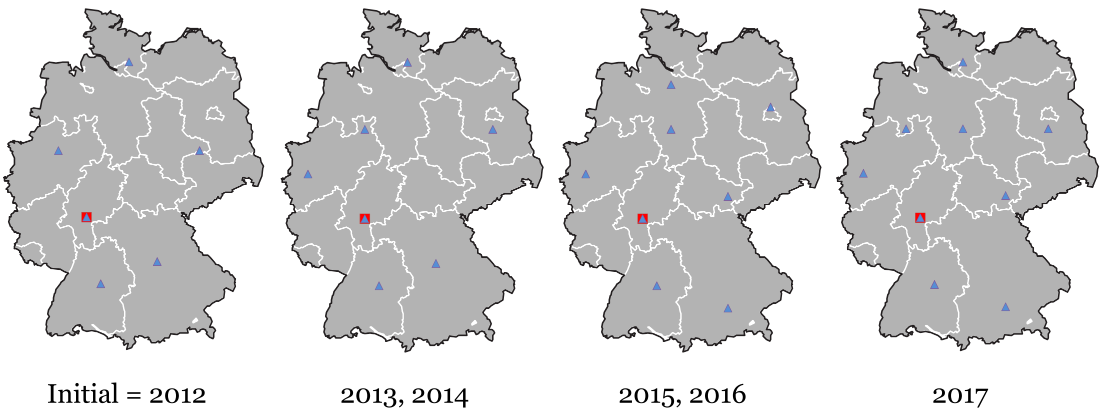

Fig. 15. Static solutions for planning period 2012 – 2017

In contrast to the static consideration of planning periods, Fig. 16 represents the warehouse locations when all planning periods are considered simultaneously. It thus gives a reference for the transition of warehouse locations in the course of time in the dynamic problem. It is obvious that less relocation takes place compared to the individual solutions of the static problems.

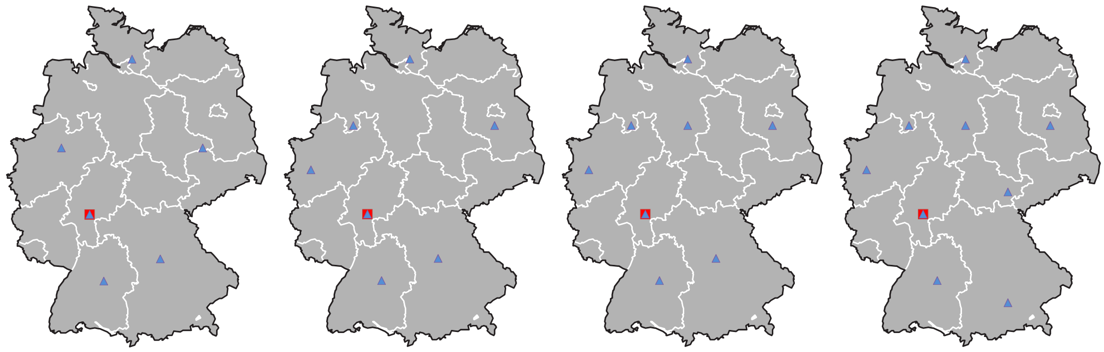

Fig. 16. Dynamic solution providing warehouse transition information for planning period 2012 – 2017

Finally, Table 9 again provides an overview of warehouse openings and closings of the three problems (S, LIN), (S, PW_LIN), and (D, PW-LIN) sorted by years. This table allows comparing the individual results per year more easily, making the difference of all three approaches transparent.

Table 9. Comparison of yearly warehouse openings/closings of regional warehouses for static linear, static and dynamic piecewise linear problems.

Year	Problem	Open regional warehouses (ID)																		
		6	19	23	24	34	49	53	71	77	79	90	93	126	136	137	141	162	172	173
2012	S, LIN				X			X	X			X		X				X		
	S, PW-LIN				X			X	X			X		X				X		
	D, PW-LIN				X			X	X			X		X				X		
2013	S, LIN	X					X	X	X			X		X						X
	S, PW-LIN	X					X	X	X			X		X						X
	D, PW-LIN	X				X		X	X			X		X						X
2014	S, LIN	X					X	X	X			X		X						X
	S, PW-LIN	X					X	X	X			X		X						X
	D, PW-LIN	X				X		X	X			X		X						X
2015	S, LIN	X		X				X	X			X	X	X						X
	S, PW-LIN	X						X	X	X	X				X		X		X	
	D, PW-LIN	X				X		X	X			X	X	X						X
2016	S, LIN	X		X				X	X			X	X			X	X			X
	S, PW-LIN	X						X	X	X	X				X		X		X	
	D, PW-LIN	X				X		X	X			X	X		X		X			X
2017	S, LIN	X	X	X				X	X			X	X		X		X			X
	S, PW-LIN	X				X		X	X			X	X		X		X			X
	D, PW-LIN	X				X		X	X			X	X		X		X			X

5. Conclusion

In this work we have developed a facility location model, which is considering four important aspects with high practical relevance simultaneously. Firstly, the model takes into account the hierarchical setup of a distribution network, including central and regional facilities. Secondly, the transportation cost is modeled with a piecewise linear cost function, reflecting structures of real world freight tariffs. Thirdly, service is implemented, guaranteeing proximity of facilities to customers. Finally, the model allows planning consecutive periods, indicating warehouse openings and closings in the course of time. Most importantly, those decisions are made in a manner considering all periods simultaneously, rather than independent of each other as done in the static models.

The dynamic hierarchical facility location problem (DHFLP) with economies of scale in transportation cost and service time requirement was formulated as a mixed integer linear program (MIP). Necessary adjustments were made to incorporate all aspects into a single MIP: nonlinear transportation cost was implemented in form of a piecewise linear function and service time in form of a distance proxy.

In a computational analysis we demonstrated the performance of the model using randomly generated data. The analysis showed that problems of realistic size could be solved optimal within an acceptable time frame of three hours. Bigger sized problems, including a higher number of potential facility sites, were solved within a single digit optimality gap. We have developed a preprocessing approach in order to downscale problems in size. This preprocessing approach – solving each time period of the dynamic problem statically and using the results as input for the preprocessed dynamic problem – allowed the reduction of overall problem size of the dynamic problem. Solution results and overall runtime performed better in comparison to the original dynamic problem.

Finally the paper benefits from being able to test the model in a business context. We had access to delivery and cost data of a company, which is active in the consumer goods industry. With access to this data, we were able to test the models' applicability and robustness to real world distribution system planning. In the course of the case study, we furthermore showed with a numeric example that it is vital to simultaneously account for above aspects, such as dynamic planning and economies of scale in transportation.

Considering these effects independent of each other, such as static single time periods or only linear transportation cost, leads to suboptimal results.

Overall, it can be summarized, that the developed dynamic hierarchical facility location model including transportation economies of scale and service time requirements has proven to be a beneficial tool supporting supply chain planners. The model can be implemented as being part of a business intelligence system, supporting decision making with respect to distribution system planning. Considering all above aspects simultaneously, the program is a tool that models the real world conditions more precisely.

Future work enhancing the model could consist of, but is not limited to, extending the model to a multi-commodity model or considering uncertainty of future periods. Another research stream is developing efficient heuristics or algorithms being able to solve big instances of the model optimally.

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7. Appendix A. GAMS code of dynamic-hierarchical facility location problem with economies of scale and service time

```
* Illustrative example of code with dummy data

Sets
t time periods / 1*5 /
h depot or plant from which supply comes / 1 /
i central facility locations / 1*10 /
j regional facility locations / 1*50 /
k customers / 1*100 /
q segments of piecewise linear function / 0*3 /
;

Variables
x_ri(j) binary variable denominating if regional warehouse j is opened
x_ci(i) binary variable denominating if central warehouse i is opened
;

Binary variable x_ci;
Binary variable x_ri;

*Load initial status/solution of the problem
execute_load "DHFLP_initial_solution.gdx" x_ci.L x_ri.L

*Subsets denoting open facilities "o" and candidate sites "c"
sets
i_o(i)
i_c(i)
j_o(j)
j_c(j)
;

i_o(i)$(x_ci.l(i)eq 1)=yes;
i_c(i)$(x_ci.l(i)eq 0)=yes;
j_o(j)$(x_ri.l(j)eq 1)=yes;
j_c(j)$(x_ri.l(j)eq 0)=yes;

Table c_pc(h,i) distance from plant h to central warehouse locations i
$include
* Include distance table here
;

Table c_cr(i,j) distance from central warehouse i to regional warehouse j
$include
* Include distance table here
;

Table c_rk(j,k) distance from regional warehouse j to customer k
$include
* Include distance table here
;

```

```
Table w(t,k) average demand (weight) at node k at time t
   1   2   3  ...
1 100 200 50 ...
2 100 200 50 ...
3 200 300 70 ...
4 200 300 70 ...
5 200 300 90 ...
;

Table n(t,k) number of yearly deliveries or orders at node k at time t

   1  2   3 ...
1 20 50  70 ...
2 20 70  70 ...
3 30 90  70 ...
4 30 110 70 ...
5 30 130 70 ...
;

Parameters
d(t,k) demand at node k at time e (derived from w_tk times n_tk)
DD(t) total demand of customers in period t
A_rk(q) slope /1 0.5, 2 0.3, 3 0.2 /
B_rk(q) intercept piecewise linear cost function
/1 1.0, 2 1.5, 3 2.2 /
Gu(q) Upper bounds of segments of pw.-lin. cost function
/0 0, 1 500, 2 1000, 3 20000 /
Gl(q) Lower bounds of segments of pw.-lin. cost function
/0 0, 1 0, 2 500, 3 1000 /
cc_pc(h,i) Indicator parameter for fixed cost portion of FTL cost (distance = 0 -> cc_pc = 0 else cc_pc = 1)
cc_cr(i,j) Indicator parameter for fixed cost portion of FTL cost (distance = 0 -> cc_cr = 0 else cc_cr = 1)
;

d(t,k) = w(t,k)*n(t,k);
DD(t) = sum(k, d(t,k));
cc_pc(h,i)$(c_pc(h,i) eq 0) = 0;
cc_pc(h,i)$(c_pc(h,i) gt 0) = 1;
cc_cr(i,j)$(c_cr(i,j) eq 0) = 0;
cc_cr(i,j)$(c_cr(i,j) gt 0) = 1;

Scalar
B_pc intercept of linear transportation cost function / 100/
A_pc slope of linear transportation cost function / 1.0 /
B_cr intercept of linear transportation cost function / 100/
A_cr slope of linear transportation cost function / 1.0 /
F_r fixed cost for regional warehouse / 500000 /
F_c fixed cost for central warehouse / 1000000 /

Variables
*PLANT and PLANT-to-CENTRAL
z_pc(t,h,i) volume delivered from plant h to central warehouse i

*CENTRAL and CENTRAL-to-REGIONAL
x_c(t,i) binary variable denominating if central warehouse i is opened
```

```
z_cr(t,i,j) amount of volume delivered from central warehouse i to regional warehouse j

*REGIONAL and REGIONAL-to-CUSTOMER
x_r(t,j) binary variable denominating if regional warehouse j is opened
y_rk(t,j,k,q) binary variable denominating if customer k is assigned to regional warehouse j
z_rk(t,j,k,q) amount of volume delivered from regional warehouse j to customer k

*OBJECTIVE FUNCTION
e total fixed and variable cost;

Positive variable z_pc;
Binary variable x_c;
Positive variable z_cr;
Binary variable x_r;
Binary variable y_rk;
Positive variable z_rk;

Equations
cost define objective function

piece2 derivation of segment of piecewise linear function
piece3 derivation of segment of piecewise linear function
piece4 derivation of segment of piecewise linear function

central_begin_open Central facilities open from initial period
central_no_reopen Central facilities: stay closed if closed
central_stay_open Central candidate sites: stay open if opened

regional_begin_open Regional facilities open from initial period
regional_no_reopen Regional facilities: stay closed if closed
regional_stay_open Regional candidate sites: stay open if opened

volume_regional total volume at regional warehouse
volume_central total volume at central warehouse
volume_plant total volume at plant

constraint1 determines y_rk(t j k q) on basis of z_rk
constraint2 determines y_rk(t j k q) on basis of z_rk

constraint3 determines x_rk on basis of z_cr
constraint4 determines x_rk on basis of z_cr

constraint5 determines x_cr on basis of z_pc
constraint6 determnies x_cr on basis of z_pc

service_time service time that has to be ensured - proxy in km
;

*OBJECTIVE FUNCTION MINIMIZING TOTAL TRANSPORTATION COST
cost.. e =e= sum((t,h,i), (c_pc(h,i)*A_pc+cc_pc(h,i)*B_pc)*(z_pc(t,h,i)/10000)) +
sum((t,i,j), (c_cr(i,j)*A_cr+cc_cr(i,j)*B_cr)*(z_cr(t,i,j)/10000)) +
sum((t,j,k,q), n(t,k)*c_rk(j,k)*(A_rk(q)*z_rk(t,j,k,q) + B_rk(q)*y_rk(t,j,k,q))) +
```

```
sum((t,i), F_c*x_c(t,i)) +

piece2(t,q,k,j).. z_rk(t,j,k,q) =l= y_rk(t,j,k,q)*Gu(q);
piece3(t,q,k,j).. z_rk(t,j,k,q) =g= y_rk(t,j,k,q)*Gl(q);
piece4(t,k,j)..   sum(q, y_rk(t,j,k,q)) =l= 1;

central_begin_open(t,i_o)$(ord(t)=1)..   x_c(t,i_o) =e= 1;
central_no_reopen(t,i_o)$(ord(t) lt 5).. x_c(t,i_o) =g= x_c(t+1,i_o);
central_stay_open(t,i_c)$(ord(t) lt 5).. x_c(t,i_c) =l= x_c(t+1,i_c);

regional_begin_open(t,j_o)$(ord(t)=1)..   x_r(t,j_o) =e= 1;
regional_no_reopen(t,j_o)$(ord(t) lt 5).. x_r(t,j_o) =g= x_r(t+1,j_o);
regional_stay_open(t,j_c)$(ord(t) lt 5).. x_r(t,j_c) =l= x_r(t+1,j_c);

volume_regional(t,k).. sum((q,j), z_rk(t,j,k,q)) =e= w(t,k);
volume_central(t,j)..  sum(i, z_cr(t,i,j)) =e= sum((k,q), z_rk(t,j,k,q)*n(t,k));
volume_plant(t,i)..    sum(h, z_pc(t,h,i)) =e= sum(j, z_cr(t,i,j));

constraint1(t,q,k,j).. y_rk(t,j,k,q)*DD(t) =g= z_rk(t,j,k,q);
constraint2(t,q,k,j).. y_rk(t,j,k,q) =l= z_rk(t,j,k,q);

constraint3(t,j).. x_r(t,j)*DD(t) =g= sum((i), z_cr(t,i,j));
constraint4(t,j).. x_r(t,j) =l= sum((i), z_cr(t,i,j));

constraint5(t,i).. x_c(t,i)*DD(t) =g= sum((h), z_pc(t,h,i));
constraint6(t,i).. x_c(t,i) =l= sum((h), z_pc(t,h,i));

service_time(t,j,k,q)$(ord(t) gt 1).. y_rk(t,j,k,q)*c_rk(j,k) =l= 300;

Model transport /all/ ;

* Include CPLEX Option file
transport.optfile = 1;

solve transport using mip minimizing e;
display z_pc.L, x_c.L, z_cr.L, x_r.L, y_rk.L, z_rk.L;
```

CHAPTER IV

OPTIMAL LOCATION OF CHARGING STATIONS IN SMART CITIES: A POINTS OF INTEREST BASED APPROACH

This chapter is an extended version of the paper:

Wagner, S., Götzinger, M., Neumann, D. (2013). Optimal Location of Charging Stations in Smart Cities: A Points of Interest Based Approach. *International Conference on Information Systems (ICIS) 2013*

Abstract

Electric vehicles (EV) have become one of the most promising transportation alternatives in recent years. Due to continuously increasing gas prices and CO2 taxes, while at the same time subsidies of electrified cars run into millions, many countries such as the USA, UK, and Germany intend to bring large amounts of EVs onto their roads in the near future. As a prerequisite, an adequate charging infrastructure is needed in order to supply these vehicles with electrical fuel. In this paper we present a point of interest based business intelligence system to determine the optimal locations for charge point stations. The underlying methodology is exploiting the potential of Big Data by analyzing and evaluating real charging sessions on the one hand and urban trip destination for vehicle owners on the other hand creating smart cities. On this basis, we formulate schemes to calculate an optimal charging infrastructure, which are based on covering facility location problems. A case study for Amsterdam and Brussels validates our results. In addition to the case study, a computational analysis is presented. The analysis includes a parameter sensitivity analysis, which enables city planners to adjust model settings to their planning requirements.

Keywords: *Electric vehicles, charging infrastructure, points of interest, covering problems, facility location, decision support systems, infrastructure planning, big data*

1. Introduction

Electric vehicles (EV) have received a great deal of attention recently, not only in broad public, but also in academia. With CO2 emissions in focus for traditional means of transportation (gas fueled cars, busses, etc.), EVs are sometimes even seen as the "*green*" savior. Apparently e-mobility is on the rise, as the investments for battery technologies and charge point infrastructures in various countries such as the USA, UK and Germany run into billions. Projects within these countries intend to bring large amounts of EVs onto the roads by 2020 (Cabinet of Germany (Die Bundesregierung), 2009; HM Government, 2009; The White House Office of the Press Secretary, 2008). However, the sales figures for EVs are far away from achieving the above desired goal (Electric Drive Transportation Association, 2013). The main reasons for this lie in high acquisition costs of EVs on the one hand, and also short driving ranges, due to insufficient battery technologies, on the other hand. *"Range anxiety"* as described by Eberle and von Helmolt (2010) is one of the reasons, why EV adoption in the mass market is limited. Furthermore, handling new transportation technology and the distrust in electricity as fuel, is one of the main reasons people are not willing to change their habits. The anxiety of running out of power, also caused by a small number of charging opportunities, reinforced this attitude. Eventually, theses flaws currently predominate the advantages of electrified cars, like CO2 free emissions or cheap fuel. In order to take the *"range anxiety"* concern from people's minds, city and infrastructure planners are focused on providing an adequate charging infrastructure to a planning area.

Millions of Euros are invested to expand the current *charge point* (CP) infrastructure and to enable the opportunity to supply each customer with electrical fuel at any given place. In this context, the European Commission launched a clean fuel strategy, proposing high targets for a minimum level of infrastructure, in order to promote electric mobility (European Commission, 2013). Such agreements are one of many steps towards a new CO2 free transportation system, as combustion engines are substituted with electric motors. Additionally, a comprehensive charge point infrastructure is one of the most important investments to reduce peoples' anxiety and to increase the sales figures of EVs for the long term. Planning the locations and the spatial setup of EV charging infrastructure is thus a central theme to foster EV adoption in the mass market.

In this paper we develop a business intelligence system to support city planners to find the optimal location to set up a predefined number of CPs based on Big Data. As such our work on smart city planning is targeting related objectives as IBM's smarter planet initiative at the interface between business intelligence, e-government and urban economics.

Our approach is mainly user-centric and correspondingly data driven. More specifically we relate optimal CP locations with urban infrastructure buildings, such as restaurants, stores, parks, and all other possible *points of interest* (POIs), as these constitute potential trip destinations for users of EVs. Since charging of EVs consumes more time than traditional gas refueling, people will most likely recharge their vehicles, while running errands. As a study by Sommer (2011) points out, longest car parking times are over night or during work. While charging infrastructure at home can be realized individually, at work people might use public stations or ones being provided by their employers. Thus, activities such as private errands, recreational activities, or shopping require the use of public infrastructure. As people spend time is this area, while their vehicles are parking, it is natural that POIs should be an integral part in the planning of EV charging infrastructure. Additionally, different categories, such as restaurants, stores, parks, etc., allow taking a differentiated view on various areas in a city. This is also consistent with the land use theory introduced by Giuliano (2004) and with urban economics by McDonald and McMillen (2011). Fig. 1 presents the positioning of E-Transportation within urban land use theory. As the available CP infrastructure influences the land usage, i.e. the visit frequency of individual POIs, it also determines the charging activity. Hence, to derive optimal locations for new CPs it is important to analyze the influence of POIs on the actual charge point usage behavior – an area where city planners can tap into Big Data information on traffic and user behavior. Consequently, we have collected usage data from one of the best constructed cities regarding CP infrastructure worldwide – Amsterdam. The acquired set is composed of more than 100,000 data points. From the usage data of each individual CP, such as daily utilization and number of users, we can infer on the importance of POIs, which are in the sphere of influence of each CP. To explain the usage behavior we further acquire a set of POI containing more than 30,000 individual data points in the city of Amsterdam. The analysis of CP usage linked with POI location provides a ranking of POI categories. This ranking then can be applied to a green field planning of CP infrastructure, in order to deduce an optimal CP setup.

Hence, the research introduced in this paper provides the following contributions:

- Determining the importance of points of interests, concerning the establishment of a charge point infrastructure.
- Deriving optimal locations for CPs based on actual charging infrastructure usage.
- Developing an algorithm to calculate an optimal charge point infrastructure, based on urban economics and big data.
- Developing a tool for city planners.

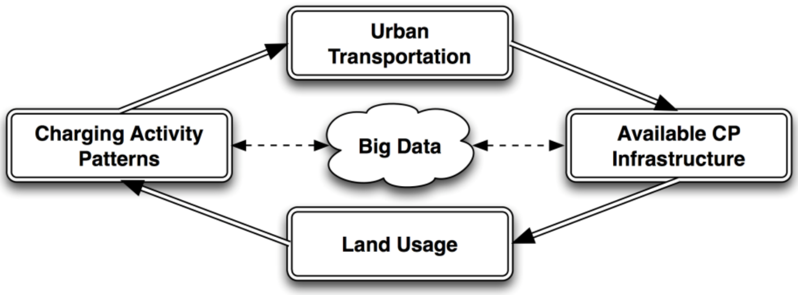


Fig. 1. Land use theory in context of E-Transportation.

The paper is structured as follows. In the next section, we provide an overview on research and case studies related to our work. This is followed by introducing the methodology and formal aspects of our optimal planning schemes. Afterwards, we validate our simulation results with case studies for Amsterdam and Brussels. A computational study and with a sensitivity analysis provides insight into how parameters effect planning results. An iterative algorithm is developed in order to obtain fast results for parameter calibration. Finally, we conclude by summarizing this paper and provide an outlook on our future research.

2. Related Work

Since EVs are entering more and more the mass market, academic work to determine the optimal EV charging infrastructure becomes more important these days. In the following we want to present an overview of related work in this area.

As aforementioned, studies on urban transportation as a subarea of urban economics deal with a general perspective of land usage. Rodrigue, Comtois, & Slack (2013) illustrates the basic principles of land rent theory. It is assumed that the rent of land is a function of the availability of a specific area. As we move away from the center of this area the rent drops substantially since the amount of available land increases exponentially. Further, a recent mobility study of Sommer (2011) indicates that private transportation aims primarily for getting to work, shopping, recreational activities, private errands and private transport. The study indicates that parking time varies between one and seven hours.

However, the actual development of charging infrastructures for EVs is discussed extensively in recent years, especially under consideration of governments' budget constraints. Various case studies in e.g. Beijing, Stockholm or Taiwan have been realized to plan an urban charge point infrastructure using programming and optimization schemes, in order to minimize investments and operation cost: Jia, Hu, Song, & Luo (2012) build a mixed integer quadratic programming model that makes decision about siting and sizing of charge point locations. The objective function minimizes investment and operation cost. Input to the model is demand, defined by time spent by vehicles at a node, and a transportation network model build by graph theory. The model is validated through a case study for Stockholm, Sweden. Liu, Zhang, Ji, & Li (2012) present a non-convex, non-linear model that minimizes the investment, operational and charging cost of the overall charge point infrastructure to be planned. Demand is derived by traffic flow. The authors propose an adaptive particle swarm optimization algorithm to solve the problem and use a district in Beijing, China as a case study. Meng & Kai (2011) use a game theory approach to make a decision among competing project alternatives for charge point locations. Influencing factors are determined and the alternatives make up the strategy set. The game model is transformed to a linear programmed and solved. A small case study is presented. Wang (2008) investigates the location and number of battery exchange stations for electronic scooters used by tourists in Taiwan in an integer programming model. The model minimizes the cost of battery exchange stations. The author also conducts a sensitivity analysis with regard to capacity of service stations in the Taiwanese case study.

Chen, Kockelman, & Khan (2013) present a model which combines regression to predict parking demand (based on trip data from a survey in the Seattle region) and a facility location problem formulated in mixed integer form, which assigns optimal charge point locations. The objective function minimizes total access cost as a function of walk distance between zones weighted by parking duration. The number of charge point locations is an exogenous variable.

Ge, Feng, & Liu (2011) present a model for locating and sizing charge point infrastructure. The model minimizes the user's loss on his/her way to the charge point. Charging demand is determined by traffic flow in the planning area. Grid partitioning is used to zone each station's coverage area. A genetic algorithm is used to optimize locations. In a study building on this previous work, Ge, Feng, Liu, & Wang (2012) combine aspects of the road network, traffic flow, structure and capacity constraints of the distribution network and minimization of total cost for all stakeholders into a planning model for a charge point infrastructure. These costs consist of the investment and operation costs, power station loss, charging cost and power loss cost of the user on his/her way to the charging point. The service area of each charge point is determined by a Voronoi diagram. The authors present a case study to validate their model. In another study, Feng, Ge, & Liu (2012) propose a model which minimizes the users power loss on his/her way to a charge point. The EV charging demand is derived by traffic flow in the planning area. Initial charging locations are determined by the capacity of conventional charging stations. These locations then are optimized using a weighted Voronoi diagram approach. A case study is used to illustrate the model. Tang, Liu, & Wang (2011) present a model, which's objective function maximizes the annual operating income of charge points in the planning area. Distribution of electric vehicles, power grid structure and transport network are assumed to be given input parameters. The approaches of weighted Voronoi diagram, in order to partition the service area, and particle swarm optimization are used to simultaneously optimize the locations and service areas. A computational study is presented to illustrate the model.

Feng, Ge, Liu, Wang, & Feng (2012) design a model, which is locating a charge point infrastructure on trunk roads. The location decision is derived by maximizing the expectation of electric vehicles that need to be charged at a charge point, whereas the sizing of the charge points (i.e. number of chargers available) is achieved by minimizing the station's service cost and the user's waiting fees. Queuing theory is used to determine the service level of each charging station. A small case study provides computational results.

Sweda & Klabjan (2012) develop an agent based model, which is identifying patterns in residential EV ownership and driving activities. The authors take social interaction into account, which influences EV purchasing decisions. The model is tested with a case study using data from the Chicagoland area. The core of the research is an EV adoption model, simulating transition of drivers to EVs and the effects of charging infrastructure on EV adoption, capturing the recharging behavior of EV users. Nevertheless, different charge point scenarios in the simulation allow the user to take the model for strategic deployment of new charging infrastructure.

Moreover, there is additional research considering geographical and environmental constraints regarding trip and charging times of EVs. Frade, Ribeiro, Gonçalves, & Antunes (2011) formulate a discrete maximum covering model with decay and capacity restrictions to determine charge point locations. The model was tested in a case study for a neighborhood in Lisbon, Portugal, with the number of charge points to be located as an exogenous number. Based on census data, refueling demand is determined, differentiated by daytime (related to employment) and nighttime (related to residents) demand. Xi, Sioshansi, & Marano (2013) develop a charge point location model for plugin hybrid electric vehicles (PHEV) that consist of two modules. Module one is a simulation model which determines the relationship between charging service levels and the number of chargers at each station. Module two is a linear integer programming model that makes decision about where to locate charging stations and the number of chargers at each station. The objective function maximizes the total charging service over all candidate sites. A case study for the central-Ohio region is conducted. Traffic and demographic data is used to model demand (volume of PHEV flows).

Andrews, Dogru, Hobby, Jin, & Tucci (2012) develop a mixed integer programming (MIP) model that determines locations for charging stations. The authors use trip data from a travel survey in order to classify vehicle tour data into two categories, i.e. tours that are able to be performed by an EV and tours that are not able to be performed by an EV (e.g. trip distance too long for battery charge level). Input to the model are those "failed" vehicles. The number of charging stations to be opened is given and the model assigns each failed vehicle to a station by minimizing the total distance that needs to be traveled to and from the charging station by all vehicles, which can be considered as a proxy for aggregate inconvenience. Data from Chicago and Seattle is used to show the optimization process in case studies.

Hess et al. (2012) set up a genetic programming model, which is determining charge point locations by minimizing average trip time of electric vehicles. The trip time not only includes the time to a charging station, but rather the total trip time, including time from origin to charge point, queuing and recharging time, and time to final destination. The siting of charge points is determined by the expected mobility of EVs and the approach includes a depletion and charging model. Furthermore, a general mobility (car following) model is integrated, accounting for an EV route adaption decision logic, which changes the route if recharging is required. A case study shows the application of the model. Ip, Fong, & Liu (2010) formulate a two-staged model to optimize charge point allocation. In stage one, road traffic information is prepared and aggregated into demand clusters through hierarchical clustering analysis. Stage two is a linear programming model that assigns charging stations to demand clusters. Three scenarios are considered: one-to-one assignment by minimizing running cost, assigning stations to clusters by considering their corresponding unique demands and capacities, and assigning stations to demand clusters based on further constraints, such as limited capital. Hanabusa & Horiguchi (2011) develop an analytical method for charge point facility location. The model consist of two objectives, which are (1) minimizing total trip time, including possible detour to charge point and charging time, and (2) the equalization of the electric demand for each charging station.

He, Wu, Yin, & Guan (2013) show a game theoretical approach that examines the interactions among availability of public charging opportunities, destination and route choices of EVs and price of electricity in coupled transportation and power networks, which leads to a equilibrium modeling framework. Optimal allocation of charge points is then conducted by a mathematical program which builds on the equilibrium model. As the model is of strategic nature, it does not optimize exact locations and capacities of the allocated charging stations, but indicates metropolitan areas. The authors leave the exact decision to planners on a tactical level. A numerical example illustrates the allocation model.

Wirges, Linder, & Kessler (2012) formulate a dynamic spatial EV charging infrastructure model. It is divided into four sub-models, resulting in time-spatial scenarios of the development of charging infrastructure. The sub-modules determine the development of EV ownership, the refinancing of charging infrastructure, the mobility of EV owners, and the demand and supply for charging infrastructure. The authors use the model to generate scenarios of the development of a charging infrastructure in the Stuttgart (Germany) region until 2020.

Finally, Nakano, Miyakita, Sengoku, & Shinoda (2011) examine the tradeoff between extra waiting time for recharging a vehicle, in cases where errand time is smaller than recharging time, and the possibility of running out of battery in a network model. The density of charging stations at points of interest (shops, restaurants) and the number of outlets at each station in order to keep a sufficiently high probability of finishing a trip and minimizing waiting time is additionally considered. Numerical results are provided.

As can be seen in the literature review above, manifold approaches exist in order to locate and optimize EV charge point locations. Most studies focus on demand modeled by demographic, traffic or individual trip data. In our study we use the reference city Amsterdam as basis with an existing, well developed public EV charging infrastructure. By referring to the respective city, we derive from available charge point usage data the attractiveness of the charge point based on its surrounding POIs. It is assumed that the POIs, which represent trip destinations of EV users, have an influence on charge point usage. Matching POI information and charge point usage enables us to rate and rank different POI categories. This information is subsequently used to determine the *"charge point attractiveness"* of a spatial area based on its POIs.

3. Methodology and Charge Point Infrastructure Optimization

This section describes the methodology we have developed in order to obtain an optimal charge point infrastructure for a planning area. The section is divided into four subsections. Subsection 1 presents how we have derived the importance of individual charge points in an existing charge point infrastructure in the city of Amsterdam. Subsection 2 establishes the link between points of interest and the CP importance. We demonstrate which impact the POIs have on the CP usage. In subsections 3 and 4 two planning models are established that allow city planners to design a charge point infrastructure based on available POI data. The maximum coverage location model in subsection 3 maximizes the covered demand in the planning area based on a given number of CPs to be established. The minimum charge point location model in subsection 4 minimizes the number of CPs needed in order to satisfy a given demand percentage.

3.1. Charge Point Importance

This section presents the analysis of CP usage data and how the CP importance factor is derived. Therefore, we collected charge point usage data of the urban infrastructure of the city of Amsterdam. Since Amsterdam is known for its pioneering role with regard to EVs and has set itself high targets: *Amsterdam Electric*, an e-mobility initiative, aims at eliminating CO2 emissions of the entire transport system of the city by 2040, targeting 200,000 EVs (Government of Amsterdam (Gemeente Amsterdam), 2013). The present charging infrastructure is among the best developed ones in the world. As Amsterdam already operates a quite reasonable number of EVs on their roads, we have chosen this city as a reference and collected over 100,000 data point regarding the usage of more than 230 CPs. The collected data holds information about the average utilization per day and the usage frequency, i.e. how many customers use a specific CP at a given day. The data was collected over a period of several months with a number of more than 32,000 charging sessions in total, including only sessions with a duration above 5 minutes. Fig. 2 illustrates a histogram of the charging session durations at current Amsterdam's CPs. The graphical representation shows that approximately one third of all performed sessions are shorter than 30 minutes, even due to the fact that this amount of time is insufficient to fully charge a EVs' battery using a normal outlet of 230 volts and 15 ampere.

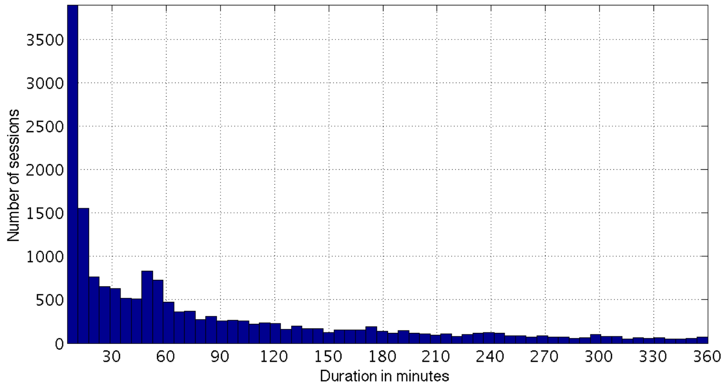


Fig. 2. Charging session duration histogram.

Based on the collected data points we define a set of charge points $\mathbb{C}$, located in Amsterdam, as

$$\mathbb{C} = \{C_1, \dots, C_n\}, \tag{1}$$

where each charge point $C_i \in \mathbb{C}$ is defined as the following 5-tuple

$$C_i = \{c_i^{lat}, c_i^{lon}, c_i^{fq}, c_i^{dur}, c_i^{rank}\}, \tag{2}$$

with c_i^{lat} as the CP's latitude, c_i^{lon} as the CP's longitude, c_i^{fq} as the average daily usage frequency and c_i^{dur} as the average daily usage duration of the respective charge point C_i. From the average daily usage frequency and duration we derive an overall CP importance factor c_i^{rank}, calculated as

$$c_i^{rank} = \sum_{d=1}^{n} \frac{c_i^{fq} \cdot c_i^{dur}}{n}, \tag{3}$$

for a CP $C_i \in \mathbb{C}$ and $d_1, \dots, d_n$ as the respective data collection days. In the above Equation 3 we imply that, the longer a CP is used and the more users patronize it in the course of a day, the more important it is.

In the further course of this section, the introduced CP rank is applied to POIs in a predefined radius, in order to derive their importance. Subsequently, we use the empirical analysis to design optimal CP locations for smart cities.

3.2. *Points of Interests and their Impact on Charge Points*

Points of interests represent potential trip destinations of vehicle owners in general, as previously mentioned. The underlying assumption is that POIs exhibit an influence on the CP usage. Two important factors determine this influence: On the one hand, the influence of POIs is diminishing with distance, being dependent on a certain proximity to the CP. On the other hand, not all POIs will require the same duration of stay, i.e. the time available for recharging the EV. It can be assumed that dining at a restaurant will most likely consume a longer time period than withdrawing money from an ATM and spending time at a beauty salon will take longer than a purchase in a pharmacy. Thus, different POI categories have a varying influence on CP usage.

The POIs are clustered in categories such as e.g. restaurants, bars, banks or parks. To use these kind of data in our model, we define a set of points of interests $\mathbb{P}$, located in the planning area, as

$$\mathbb{P} = \{P_1, \dots, P_n\}, \tag{4}$$

while each $P_i \in \mathbb{P}$ consists of a 4-tuple defined as follows

$$P_i = \{p_i^{lat}, p_i^{lon}, p_i^{type}, p_i^{rank}\}. \tag{5}$$

The POI properties p_i^{lat} and p_i^{lon} are the GPS-coordinates of the respective POI location and p_i^{type} declares a specific category as mentioned above. The POI rank value p_i^{rank} declares an individual importance factor, based on the charge point rankings c_i^{rank} of the surrounding CPs $C_i, \dots, C_j \in \mathbb{C}$ within a predefined range. To decide whether a point of interest $P_j \in \mathbb{P}$ is in the predefined range of a charge point $C_i \in \mathbb{C}$, we calculate the geographic distance $dist(C_i, P_j)$ between these two points using the following *haversine* formula (Gellert, Küstner, Hellwich, & Kästner, 1977)

$$dist(C_i, P_j) = 2 \cdot r \cdot \arcsin\left(\sqrt{\sin^2\left(\frac{p_j^{lat} - c_i^{lat}}{2}\right) + \cos(c_i^{lat}) \cdot \cos(p_j^{lat}) \cdot \sin^2\left(\frac{p_j^{lon} - c_i^{lon}}{2}\right)}\right), \tag{6}$$

with an earth mean radius of $r = 6{,}371$ kilometer.

We calculate the distances between CPs and POIs using the geographical distances in contrast to the Euclidean distances, because even for short ranges the deviation is often above one meter. Since differences of only a few meters can lead to calculation deviations, we prefer geographical distances.

In the following, we have carried out a regression analysis using the CP and POI data of Amsterdam. The Amsterdam POI database comprises 92 categories (e.g. food, store, health) with over 30,000 POI locations. We used the CP importance factorc_i^{rank}as the dependent variable. The POIs and the boroughs of Amsterdam denominate the independent variables. In order to avoid an endogeneity bias, we considered all POIs which already existed when the CP was first placed. This way, it is assured that the POIs determine the attractiveness of CP locations and not vice versa. Fortunately, most CPs have been established quite recently so that endogeneity is not an issue for our analysis.

For the regression we have chosen the main POI categories such as finance, food, store, etc. The POI input factors consist of the number of POIs of each category within a distance of 500 meters, (cf. Equation 6). Fig. 3 for instance illustrates a CP (blue marker) and a number of POIs (red markers) at a given radius of 500m, located at the public *Vondelpark* in Amsterdam.

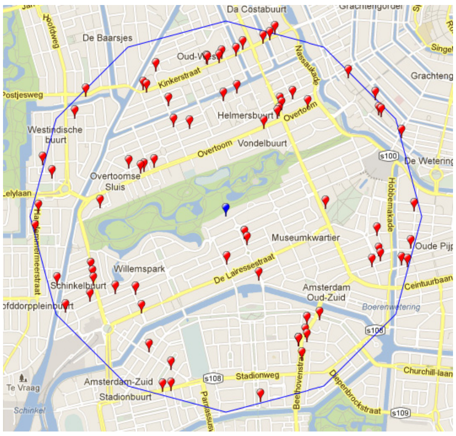


Fig. 3. POIs within a given radius of a CP in Amsterdam.

The CP usage data was limited to weekday data (Monday – Friday) in order to account for POI availabilities. To control for regional differences, we have clustered the six boroughs of Amsterdam as control variables. Table 1 summarizes the regression results. As can be deduced from the t-values of the coefficients the influence of the POI categories, such as food, store, health, bus station, museum, and school on the charge point usage is significant. We have additionally conducted the regression for coverage radiuses of 250m and 750m, in order to test for the radius of influence of POIs on CPs. The results were less significant. Thus, we can conclude that a radius of 500m is adequate as a walking distance from the CPs to the stores.

These results lead to two conclusions. Firstly, the empirical analysis suggests that we can use the CP usage reference data of Amsterdam in order to measure the importance of different POI categories on CP usage behavior. Secondly, we can further rate the POIs of a city in order to derive a proxy for CP infrastructure demand. As Table 1 shows, the adjusted R-squared value is at 0.1, which is acceptable in social studies. In our result we focus on the t-values to find out which factors we need to incorporate in our subsequent optimization step. Furthermore, the category types *food*, *health*, and *museum* show a significance value of more than 95%, with a t-statistic value above 2. This indicates a positive influence on charge point usage. Since, visiting a museum is a time consuming activity, the charging session durations of EVs will be substantially higher in contrast to withdrawing money from an ATM. This is also the reason, why e.g. the *finance* category type provides no significance at all. Overall, POIs have a significant influence on the usage of charge points and, thus, have to be considered when developing a future charging infrastructure for smart cities.

Table 1. POI Regression.

Regression Statistics	
R^2	0.164
Adjusted R^2	0.101
F-value	2.593
p-value	0.001

	Coefficients	t-statistic	Significance[1]	p-value
Intercept	0.31	5.72	***	0.000
Food	0.55	2.14	**	0.034
Store	-0.66	-2.21	**	0.028
Health	0.56	2.55	**	0.012
Finance	-0.18	-1.47		0.144
Bus station	-0.12	-1.68	*	0.095
Museum	0.35	2.15	**	0.033
School	-0.30	-1.92	*	0.056
Church	-0.02	-0.16		0.874
Travel agency	-0.03	-0.19		0.850
Hair care	-0.03	-0.26		0.794

1. significance: *** 99%, ** 95%, * 90%, blank <90% Observations: 229

From this information we can then build a location model which is able to provide city planners the optimal locations for new CP infrastructure based on POI locations, which again are trip destinations of EV users.

Eventually, we calculate for each POI $P_i \in \mathbb{P}$ its ranking value p_i^{rank}, based on the subset of surrounding CPs $\dot{\mathbb{C}} = C_k, \dots C_l$ as follows

$$p_i^{rank} = \sum_{j=1}^{n} c_j^{rank} \mid dist(C_j, P_i) \leq \rho, \tag{7}$$

with $\dot{\mathbb{C}} \subseteq \mathbb{C}$ and ρ as a predefined range. For our calculation we assumed a ρ value of 500m, which is equivalent to a six minutes' walk, as this is a natural threshold an individual is willing to walk to get to the desired destination (POI). Based on each individual POI rank p_i^{rank} , we then derive a POI category index. Thus, we define a set of POI categories in such a way that each subset includes all POIs of the same category as

$$\mathbb{P}T = \{PT_1, \dots, PT_n\}, \tag{8}$$

with $\mathbb{P}T \subseteq \mathbb{P}$. Further each category $PT_i \in \mathbb{P}T$ is defined as the following tuple

$$PT_i = \{pt_i^{type}, pt_i^{rank}\}, \tag{9}$$

while pt_i^{type} is the respective POI category type. The ranking values pt_i^{rank} for each POI category $PT_i \in \mathbb{P}T$ are derived from the sum of all individual POI ranks p_i^{rank} from a category divided by all POIs of this category as follows

$$pt_i^{rank} = \left(\frac{\sum_{j=1}^{n} p_j^{rank}}{n}\right) \mid p_j^{type} = pt_i^{type}. \tag{10}$$

Hence, the ranking index of each POI category $PT_i \in \mathbb{P}T$, calculated using Equation 10, will be used to further assess POIs in a predefined planning area to finally derive optimal CP locations. After we have calculated a specific ranking for each category, based on the usage behavior of Amsterdam's charge points, we are now also able to use this data to derive the POI rankings in other cities without a given CP infrastructure.

3.3. Maximum Coverage Facility Location Model with Fixed Number of Charge Points

Having established a relationship between the *attractiveness* of CPs on POIs, we can subsequently exploit it to locate EV charge points optimally – i.e. in terms of the POI category index – in a planning area. Above results, the POI category index, can now be used to allow for green field planning of charge point infrastructure.

Preliminaries for the optimization are a predefined spatial planning area $\mathbb{A}$ and a database holding relevant POI information i.e. spatial location and category. In a first step the planning area is divided into a set $\mathbb{B}$ of subareas defined by

$$\mathbb{B} = \{B_1, .., B_n\}. \tag{11}$$

The size and shape of the subareas are subject to the city planner preferences and can be defined independently. However, a simple and effective approach is to divide the planning are into a grid with boxes of same edges length l. The models introduced later in this section are to support the strategic planning process of city planners and thus it is sufficient to provide an optimal box area, e.g. a 100x100m square, as opposed to an exact micro location, such as e.g. street address. The planning of the exact CP location is usually carried out in a consecutive planning step, in which non-quantitative, so called "soft" factors play an important role. These factors include land and parking space availability or power supply. Furthermore, each subarea $B_i \in \mathbb{B}$ is defined as the following triple

$$B_i = \{b_i^{lat}, b_i^{lon}, b_i^{rank}\}, \tag{12}$$

with b_i^{lat}, b_i^{lon} as the GPS coordinates of the center of the subarea and b_i^{rank} as the total *box factor* (BF). The center of each box serves as its reference point. The box factor is determined by summing up all POI category ranks of all POIs within a radius of influence σ calculated by

$$b_i^{rank} = \sum_{j=1}^{n} pt_j^{rank} \mid dist(B_i, P_j) \leq \sigma, \tag{13}$$

with $b_i \in \mathbb{B}$ and $pt_j \in \mathbb{PT}$. Thus all POIs that fall into the radius σ of an individual box contribute to its BF. A radius of influence greater than the box itself mitigates the risk of dividing the planning area in unfortunate ways e.g. split up a conglomerate of POIs into four adjacent boxes, and thus weakening its combined weight. Hence, the BF denominates the charge point attractiveness of a spatial area based on its surrounding POIs. The higher the weight, the more important is a box for a charge point. The BF thus can be seen as a proxy for EV charging demand.

Moreover, each box $B_i \in \mathbb{B}$ of the grid is a potential location area for a charge point. Fig. 4 illustrates the grid placed over a section in Amsterdam. For the center box we have visualized the radius $\sigma = 100m$ and illustratively the POIs of one category that contribute to the BF of this box. Due to the above mentioned, the radius is intentionally selected bigger than the box itself.

Fig. 4. Illustrative box factor calculation: 100x100 meter grid with box center as reference point and POIs within 100m radius.

Based on the demand proxy we develop a maximum coverage facility location model (MCFL). It is formulated as a linear program, which is maximizing the demand (i.e. box factor) served by a given number of CPs. Depending on the models parameters, the demand covered by a CP can include demand of surrounding boxes. CP locations are selected simultaneously by the program in order to maximize total demand covered.

In the following we introduce the mathematical formulation of the linear program. The maximum coverage facility location model is based on the one presented in Drezner & Hamacher (2001). The following listing of sets and symbols is required for the general notation of the MCFL.

$\mathbb{B}$ = set of grid's boxes, indexed by i

$\mathbb{B}'$ = set of potential facility locations, indexed by j (equivalent to the grid's boxes $\mathbb{B}$)

$d_{i,j}$ = distance between box i and potential facility site j, calculated as $dist(B_i, B'_j)$

dc = coverage distance, i.e. all boxes within this distance are covered by located CPs

$N_i = \{j \mid d_{i,j} \leq dc\}$ = set of all potential facility locations that cover demand of box i

p = number of charge points to be located

$$x_j = \begin{cases} 1 & \text{if location (box) is selected to locate CP} \\ 0 & \text{otherwise} \end{cases}$$

$$z_i = \begin{cases} 1 & \text{if demand node } i \text{ is covered} \\ 0 & \text{otherwise} \end{cases}$$

The mathematical formulation of the MCFL is the following:

Maximize $\sum_{i \in \mathbb{B}} b_i^{rank} \cdot z_i$ (14)

Subject to $\sum_{j \in N_i} x_j - z_i \geq 0 \qquad \forall i \in B_i$ (14.1)

$\sum_{j \in \mathbb{B}'} x_j = p$ (14.2)

$x_j \in \{0,1\} \qquad \forall j \in B'_j$ (14.3)

$z_i \in \{0,1\} \qquad \forall i \in B_i$ (14.4)

Objective function (14) maximizes the total demand coverage. Constraint (14.1) ensures that demand of box B_i is not counted as covered unless a CP is located at a candidate site which covers box B_i. Constraint (14.2) guarantees the number of facilities to be sited. Constraint (14.3) and (14.4) ensure the binary nature of variables x and z. The sets N_i of candidate locations that cover demand of box B_i can be set individually by the city planner. In the case study we will use a simple approach to determine the CP coverage: Around each CP to be located, a circle with a predetermined radius r is drawn. All boxes that fall into that radius are covered by the located CPs. The linear program is NP-hard (Drezner & Hamacher, 2002), but can be solved effectively with the application of Lagrangean relaxation and branch-and-bound algorithms. For our case study we are using the *General Algebraic Modeling System* 23.7.3 (GAMS) to formulate the program and IBM ILOG CPLEX 12.3.0.0 to solve it optimally.

3.4. *Minimum Charge Point Location Model with Fixed Demand Coverage*

The objective of the MCFL presented above is to maximize demand covered with a given number of CPs. This approach serves well in cases, where city planners know (approximately) the number of charge points to be installed. Reasons include e.g. budget constraints that allow only a given number of CPs to be installed or self-imposed targets of city councils stating *"150 charge points in the inner city by the end of 2013"*. The problem of charge point infrastructure planning can also be considered from another point of view. The central question can be phrased as follows: "How many charge points need to be established in order to cover X% of total demand". The program to be established then has the objective of minimizing the number of CPs with a constraint guaranteeing that the predefined demand coverage is reached. We call the model minimum charge point location (MCPL) model subsequently.

The mathematical formulation of the problem then reads as follows. The notation of the MCPL is being used with following additions:

c = proportion of total demand that needs to be covered by CPs to be installed

Minimize $\sum_{j \in \mathbb{B}'} x_j$ (15)

Subject to $\sum_{j \in N_i} x_j - z_i \geq 0 \quad \forall i \in B_i$ (15.1)

$\sum_{i \in \mathbb{B}} b_i^{rank} \cdot z_i \geq c \cdot \sum_{i \in \mathbb{B}} b_i^{rank}$ (15.2)

$x_j \in \{0,1\} \quad \forall j \in {B'}_j$ (15.3)

$z_i \in \{0,1\} \quad \forall i \in B_i$ (15.4)

Objective function (15) minimizes the total number of CPs to be established. Constraints (15.1) fulfill the same requirements as (14.1). Constraint (15.2) ensures that the preset proportion c of total demand is covered by the located CPs. Constraints (15.3) and (15.4) guarantee the binary nature of decision variables x and z.

4. Case Study

Having outlined our optimization schemes we present a case study for locating EV charging stations in different cities with different CP infrastructures. Overall, we conduct two case studies, where the first pertains to results for our reference city Amsterdam, whereas the second addresses a different city – Brussles – with inferior CP infrastructure, so that we can apply our approach as a kind of green field planning experiment.

4.1. Amsterdam

The first case study we have conducted with POI data of our reference city, Amsterdam. Coming from the usage data of Amsterdam's existing EV charging infrastructure, we have derived individual POI category ranks for e.g. restaurants, stores, banks, etc. These result are used to calculate an EV charging demand proxy based on given POI data, using above methodology.

First we divide the city of Amsterdam into a grid. We have chosen an edges length l of 100m for the case study, which proved to be the best value in our test runs with regard to granularity and run time. For each of the grid's boxes, the *box factor* has been calculated according to above methodology. The *BF* is determined by a radius of 100 meters around the boxes center, in order to account for surrounding boxes and POIs, as mentioned above. Fig. 5 shows the box factors (i.e. EV charging demand proxy) in a heat map for both methodologies, the MCFL and the MCPL. The darker the boxes are, the higher is the box factor and thus the anticipated charging demand. The calculated demand per box is then passed to the MCFL and the MCPL respectively as an input variable.

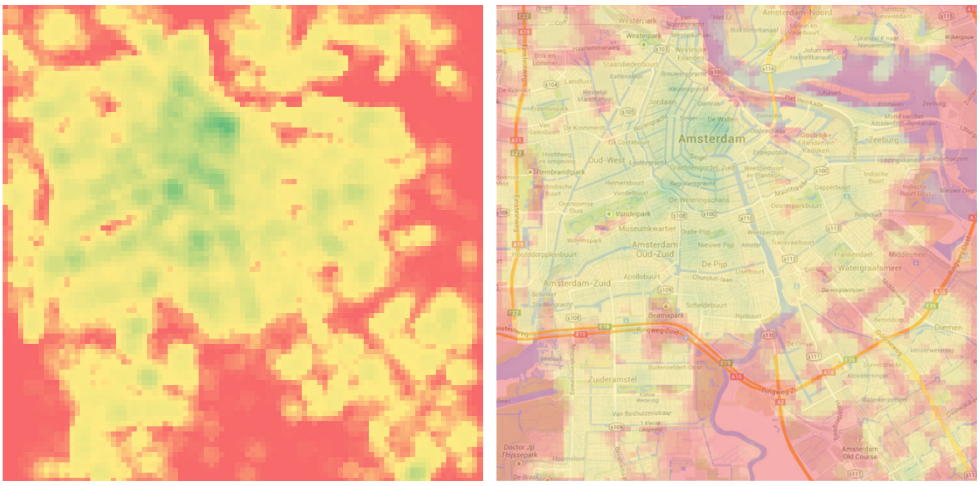
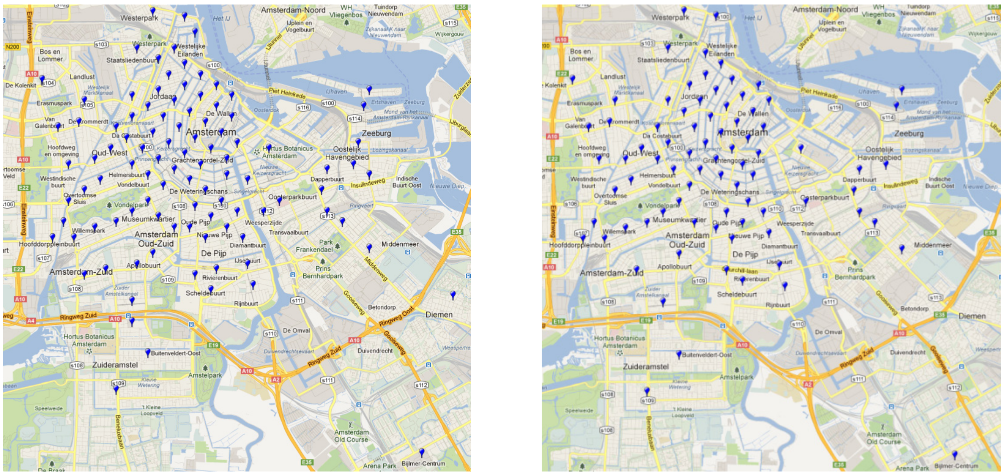


Fig. 5. Heat map of Amsterdam showing box factors with coverage σ = 100m.

Fig. 6 (a) shows the result of the MCFL with p=100 CPs to be located and a CP coverage radius r of 200m. Again, the coverage radius defines how many boxes of the grid are covered by a CP. The objective of the MCFL is to maximize demand covered by located CPs. With p=100 CPs to be located, 56% of the total demand will be covered. This approach is especially suitable for cities where no or less CP infrastructure exists, as a widespread area is covered with CPs. As a consequence this counteracts the range anxiety of EV users, which fear to run out of battery in a place without charging infrastructure. The coverage radius *r* can be used by planners for sizing the coverage of a CP infrastructure in a city accordingly. As the approach supports strategic planning aspects, city planners can decide in a consecutive planning step, which qualitative factors such as land availability etc. play a role. They can then also make decision about where to locate the charge points in the selected 100x100m boxes and how many plugs. The solution time for CPLEX was 18.61 seconds.

Fig. 6 (b) shows the results of the MCPL with c = 50%, i.e. 50% of the total demand has to be covered with CPs to be located. The CP coverage radius was also set to r = 200m. As outlined above, this approach is especially useful, if city planner want to cover a certain amount of demand with a minimum number of CPs. The planning result of MCPL with c = 50% is to locate 82 CPs. The solution time for CPLEX was 7.52 seconds. One advantage of both, the MCFL and the MCPL, is that CP locations are optimized simultaneously. Thus the programs find the optimal locations, and in case of the MCPL also number, of CPs by optimizing for all CPs at the same time. Selecting charge points iteratively, e.g. by taking the maximum demand region as CP location and repeat that step in descending order, would lead to inferior results compared to MCFL and MCPL.



(a) MCFL result with p = 100 (b) MCPL result with c = 50%

Fig. 6. Optimal CP locations for Amsterdam based on MCFL and MCPL.

4.2. *Brussels*

In the second case study, we apply the methodology and POI category ranks we have derived from our reference city Amsterdam to the city of Brussels. The results show, that our approach is applicable to other planning areas and, thus, allows for green field planning of EV charging infrastructure. Fig. 7 shows again the demand distribution for both approaches in a heat map.

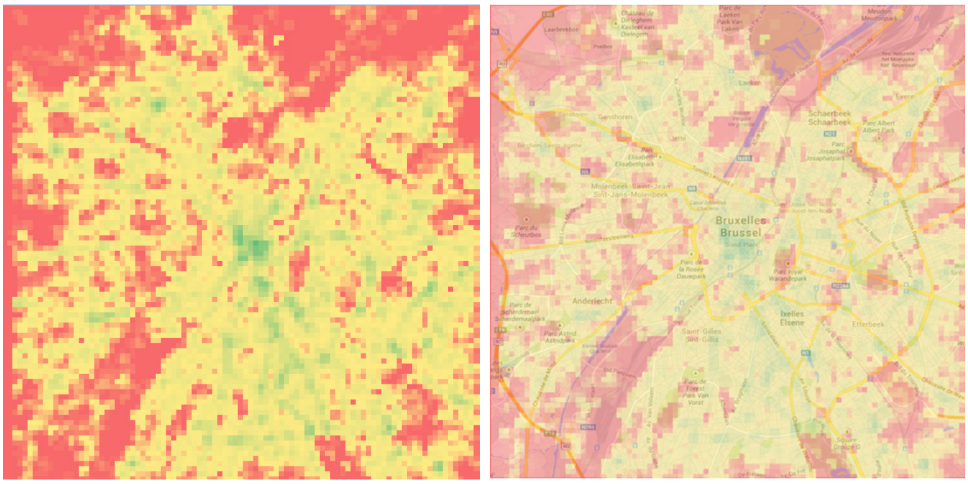
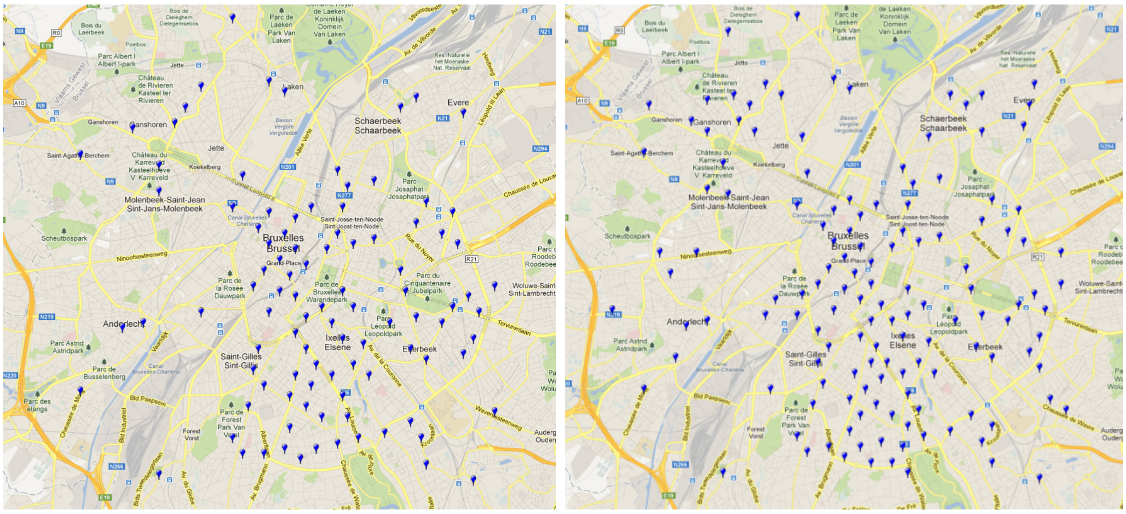


Fig. 7. Heat map of Brussels showing box factors with coverage = 100m.

Fig. 8 shows the optimal planning result for the MCFL and the MCPL, respectively. Input parameter settings are identical to the ones used for the Amsterdam case study. Total demand covered for the MCFL is 41% with p=100 charge points. In the MCPL 138 CPs are needed to cover c=50% of total demand. Runtime for the MCFL was 1.69 seconds and 6.71 seconds for the MCPL.

Comparing the results of Amsterdam and Brussels, it is obvious, that demand in Amsterdam is much more concentrated on the city centers. This is reflected by demand distribution (cf. Fig. 5, heat map), as well as the planning results of CP locations. In contrast, Brussels demand is much more scattered at different spots of the city. Charge points are densely located in the city center, but much more spread out to regional demand peaks. This is also reflected in the total coverage: In Amsterdam cover 100 CPs X% of the total demand, whereas in Brussels it is only X%. The MCPL results demonstrate this effect much more: Only 82 CPs are required to cover 50% of the total demand, whereas it is 138 CPs to cover the same in Brussels.



(a) MCFL result with p = 100 (b) MCPL result with c = 50%

Fig. 8. Optimal CP locations for Brussels based on MCFL and MCPL.

4.3. Managerial implications

The case studies showed that our approach provides a valuable methodology to city planners in order to configure a charging infrastructure for a planning region. One advantage of the planning tool is that city planners have various options to customize the planning approach to their individual needs and city specificities. Four general and additional approach specific parameters exist in total, which allow adjusting results. The general ones are POI category ranks, definition of the planning area (total grid size and edge length of grid boxes l), box factor calculation, and CP coverage radius r. Specific ones consist of number of CPs to be located p for the MCFL and proportion of demand to be covered c for the MCPL. POI category ranks can be adjusted in order to account for local specificities. Planners can weight individual categories higher or lower, depending on local conditions. In some cities people spend e.g. more time in a restaurant and less time at a hairdresser or vice versa. The planning area $\mathbb{A}$ can also be divided individually: The higher the number of subareas areas $\mathbb{B}$ is chosen, the more precise is the CP location information. On the other hand runtime increases. This aspect is certainly acceptable to some degree as the tool supports strategic planning and is not a real time application. Box factor calculation is an integral step of the planning approach: Selecting a high radius σ smoothens the influence of POIs, as an individual POI contributes to the box factor of many boxes. Choosing a lower radius allows taking a more differentiated look at the expected EV charging demand in a city based on POI data, as city regions/boxes set themselves apart from each other more sharply. The coverage radius *r* allows city planners to define how dense the CP infrastructure network is to be planned. A high coverage radius will result in a spacious layout of CPs in the planning area, whereas a low coverage leads to a denser result. In the MCFL, the input parameter *p* – number of charge points to be located – permits the city planner to select how many CPs to be located in each planning run. In contrast to selecting the number of CPs as an input parameter and get the proportion of demand covered as a result, in the MCPL the city planner defines the proportion of demand to be covered *c* and gets the number and location of CPs as a result. As range anxiety is one of the biggest concerns among potential EV users, the last three parameters – coverage radius *r,* number of CPs to be located *p,* and covered demand *c* – allow city planners to configure their charging infrastructure networks accordingly, i.e. obtaining a broad coverage of the planning area.

5. Computational Study and Parameter Sensitivity Analysis

In this section we conduct a computational study and a sensitivity analysis with regard to the parameters presented in the managerial implications in the section above. The study is conducted based on the POI data available for Amsterdam and the MCFL model.

5.1. Parameter sensitivity analysis

We have chosen the MCFL model to conduct a sensitivity analysis with regard to different planning parameters. As outlined in the section managerial implications, these parameters allow city planners to adjust the models to their specific needs, respectively city/planning area specificities. The parameters which will be examined in the further course of this section can be seen in Table 2. The parameter POI category ranks is deliberately not part of the sensitivity analysis as this is a specific parameter subject to city planners' preferences (i.e. giving e.g. cafes more weight than banks). No general conclusions can be derived in contrast to the parameters under examination.

Table 2. Parameter variation for sensitivity analysis.

Case	Parameter	Variation	Total # of problems	Fixed parameters
A	Number of CPs to be located p	p = 10, 20, 30, …, 200	20	r=200m, σ=100m, l=100m
B	CP coverage radius r	r = 100, 200, …, 1000	10	p=100, σ=200m, l=200m
C	BF calculation with radius σ	σ = 200m and 500m	4 (combined)	p=100, r=200m
	Edge length of grid boxes l	l = 100m and 200m		

We have divided the analysis into three cases: Case A is a variation of number of CPs to be located, reaching from 10 to 200 in steps of 10. Case B is a variation of CP coverage radius r, reaching from 100m to 1000m in steps of 100m. Case C is a scenario evaluation consisting of a combination of BF calculation radius σ and the edge length of grid boxes l.

Table 3 provides results of the problems solved. With an increasing number of charge points p the solve time is increasing accordingly. Fig. 9 illustrates the results with natural logarithm of CPU time. A log-linear relation between number of CPs to be located and CPU time can be implied. A linear regression shows an R^2 of 98%. Keeping in mind that the models support the strategic planning aspects of infrastructure planning, solves times observed are at a reasonable level. Table 3 additionally shows the percentage of total demand covered and the marginal demand gain as compared to the problem with ten CPs less. As expected the marginal demand gain is decreasing with more CPs to be added. This kind of analysis also helps city planners to assess the marginal demand gain, by setting in contrast the additional investment cost for installing additional CPs.

Table 3. Problems solved for variable number of CPs *p*.

# CPs (p)	CPU time (sec.)	Demand covered (objective value)	% of total demand	Marginal demand gain in %
10	0	39,891	11%	-
20	0	68,226	19%	7.9%
30	2	91,267	26%	6.5%
40	1	112,250	31%	5.9%
50	6	131,575	37%	5.4%
60	4	148,407	42%	4.7%
70	8	163,370	46%	4.2%
80	11	176,600	49%	3.7%
90	23	188,697	53%	3.4%
100	19	199,798	56%	3.1%
110	81	210,034	59%	2.9%
120	145	219,553	62%	2.7%
130	208	228,399	64%	2.5%
140	227	236,750	66%	2.3%
150	708	244,574	69%	2.2%
160	1,688	251,965	71%	2.1%
170	6,607	258,926	73%	2.0%
180	3,932	265,509	74%	1.8%
190	7,090	271,740	76%	1.7%
200	6,096	277,574	78%	1.6%

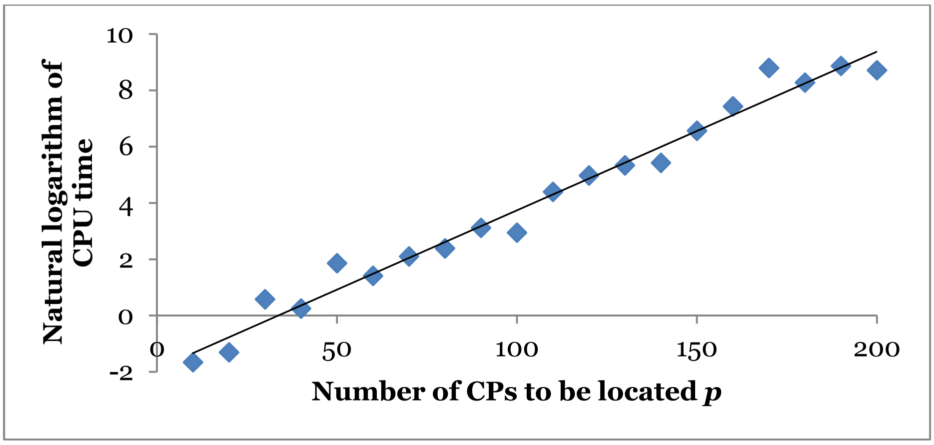


Fig. 9. CPU time (natural logarithm) as a function of CPs to be located.

The results for variable CP coverage radius *r* are shown in Table 4. Results include CPU time used to solve the problem, with a maximum time allowed of 12 hours. If the 12 hours are exceeded, the solve process terminates. The optimality gap gives an indication of the relative gap between the best integer and the best possible (lower bound) solution found at that time. Demand covered presents the total demand covered by located CPs. Maximum boxes coverable indicates how many boxes of the grid can (hypothetically) covered for each problem with the set CP coverage radius. Two general findings can be derived from the results: The closer the percentage of total demand with set parameter *r* is approaching total demand coverage (i.e. 100%), the higher is the CPU time to solve the model optimally. As soon as total coverage is reached with set parameter *r*, the CPU time is within seconds. The reason is obvious: With a predefined number of CPs to be located, as soon as total demand/box coverage is reached with a coverage radius r, all values greater than that "overcover" the demand in the planning area, as CP coverage areas overlap. In those cases the planner is advised to use the MCPL model, as he/she can then optimize for the lowest number of CPs possible to cover total demand. Generally the total demand coverage cannot be stated prior to solving a problem. Thus a proxy for city planners to assess the influence of parameter *r* on runtime is the maximum number of boxes of the grid that can (hypothetically) be covered with the set parameter. Identically to demand covered, the closer the percentage of total boxes that can be hypothetically covered is approaching total coverage of boxes (i.e. 100%), the higher is CPU time consumed to solve the model.

Table 4. Problems solved for variable number CP coverage radius *r*.

CP coverage radius (*r*)	CPU time (sec.)	Optimality gap (%)	Demand covered (objective value)	% of total demand	Max. boxes coverable	% of total boxes
100	0.0	0.0%	89,509	25.0%	100	4%
200	3.0	0.0%	236,449	66.0%	500	22%
300	0.3	0.0%	301,776	84.2%	900	39%
400	1,003.8	0.0%	333,914	93.2%	1300	56%
500	43,224.7	0.2%	356,943	99.6%	2100	91%
600	3.5	0.0%	358,234	100.0%	2304	100%
700	1.2	0.0%	358,234	100.0%	2304	100%
800	1.0	0.0%	358,234	100.0%	2304	100%
900	0.9	0.0%	358,234	100.0%	2304	100%
1000	1.1	0.0%	358,234	100.0%	2304	100%

Fig. 10 and Fig. 11 give an overview of the results – heat map and planning results – for the scenario analysis, varying parameters box factor calculation with radius σ and edge length of grid boxes l. From the heat map it can be directly observed, that a higher radius σ for box factor calculation smoothens the individual weights of the POIs. With σ=200m much more local demand peaks are visible, where POIs are concentrated in specific areas. The heat map wit σ=500m shows less local demand peaks, as single POIs contribute to the BF of much more boxes and thus transition between boxes is less "sharp", but smoother. The edge length of grid boxes l influences in how many squares the planning area is divided and thus the granularity of the planning results. With l set to 100m, the planning area is divided into factor 4 more boxes than with l set to 200m. Especially with regard to how exactly a planner wants to know the micro location for setting up a CP, the parameter l is of great importance.

Assessing the planning results, similar conclusions can be drawn: a lower radius σ takes local demand peaks into account and thus plans a more differentiated infrastructure. CP locations for a planning with high radius σ are more concentrated at the city center, as the agglomeration of POIs in combination with the large radius of influence favors the city center. Local agglomerations of POIs lose importance, as no other surrounding POIs contribute to the weight of their according box/boxes. The edge length of grid boxes l influences in this context (a) the granularity of the result with regard to exact location, as explained above, and (b) also how dense/spacious the network to be planned is going to be.

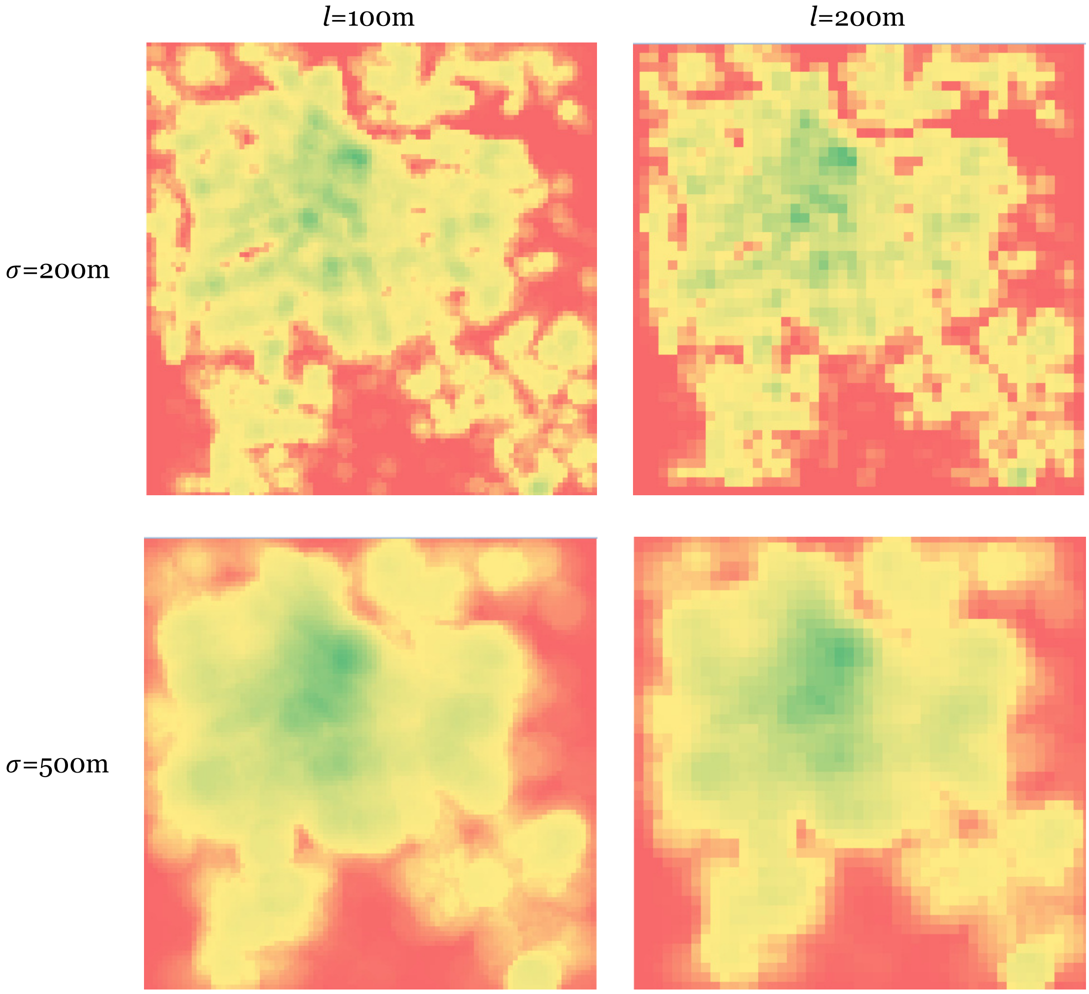


Fig. 10. Heat map for variation of σ and l.

l=100m l=200m

σ=200m

σ=500m

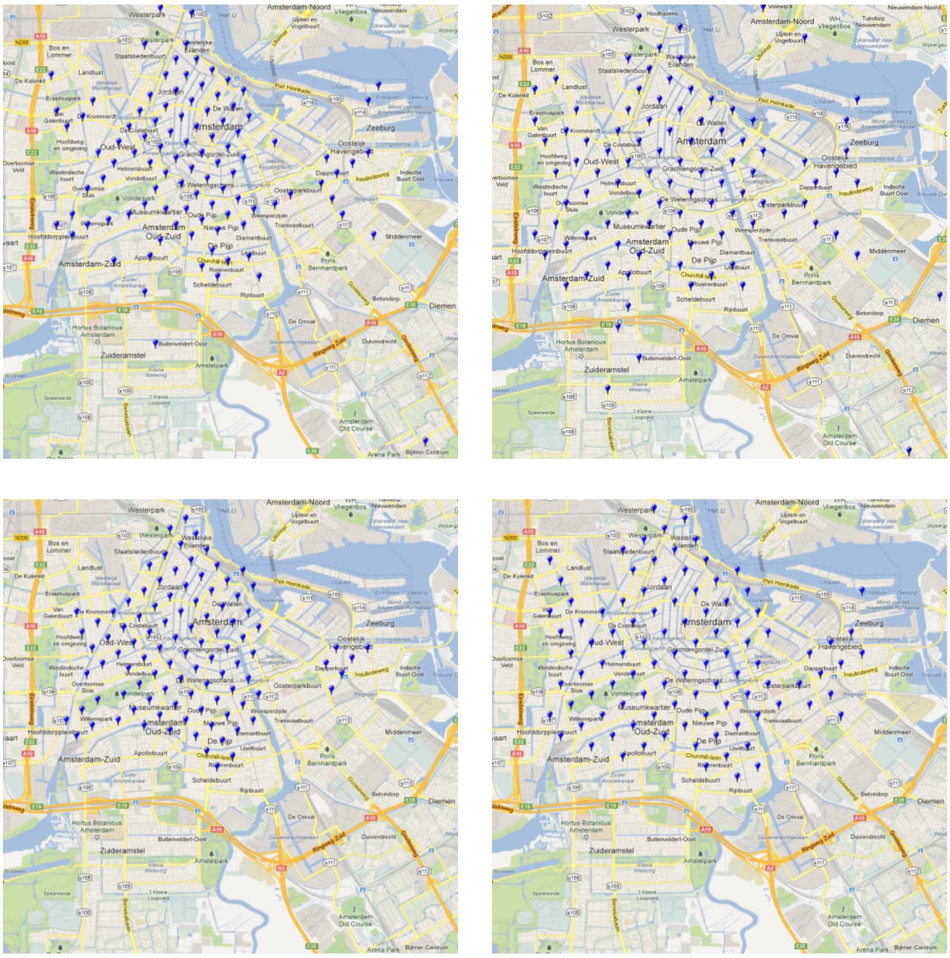


Fig. 11. Planning results MCFL for variation of σ and l.

5.2. *Iterative algorithm*

The computational study showed that bigger and more complex problems require a higher amount of time to be solved optimally. As the models support the strategic planning aspect of a charging infrastructure, runtime generally is not as important as in real time applications. Nevertheless there might be situations, such as calibrating parameters to specific planning areas, where quick, indicative results are needed. Those results then can be used to specify parameters quickly, before actually calculating scenarios with MCFL and MCPL. Consequently we have developed an iterative algorithm (IA), providing city planners with indicative results for e.g. parameter calibration. Fig. 12 shows the comparison of results for the MCFL with parameters equal to the Amsterdam case study and variable p from 1 to 100. The relative gap between the objective value (i.e. demand covered) of the algorithm and the

MCFL reaches from 0% to 3.5%. The percentage gap of total demand covered only from 0% to 2%. Thus it can be concluded that the iterative algorithm with a runtime of 50.45 seconds for 100 CPs is a fair method to obtain quick results within an acceptable range of inaccuracy compared to optimal results. As the algorithm calculates CP locations iteratively, the results for every $p<100$ are additionally available.

The algorithm works as follows: The box with maximum consolidated demand is selected for the first charge point to be located. Consolidated demand refers to the box factor (demand) of the box itself and the box factor of surrounding boxes which are covered by the CP coverage radius *r*. The demand of all covered boxes is then set to zero and new consolidated demand for each box is calculated. This step is only done for those boxes, where the coverage radius overlaps with a box where demand was set to zero. The second charge point is then again placed in the box with maximum consolidated demand. Steps are repeated until *p* charge points are located.

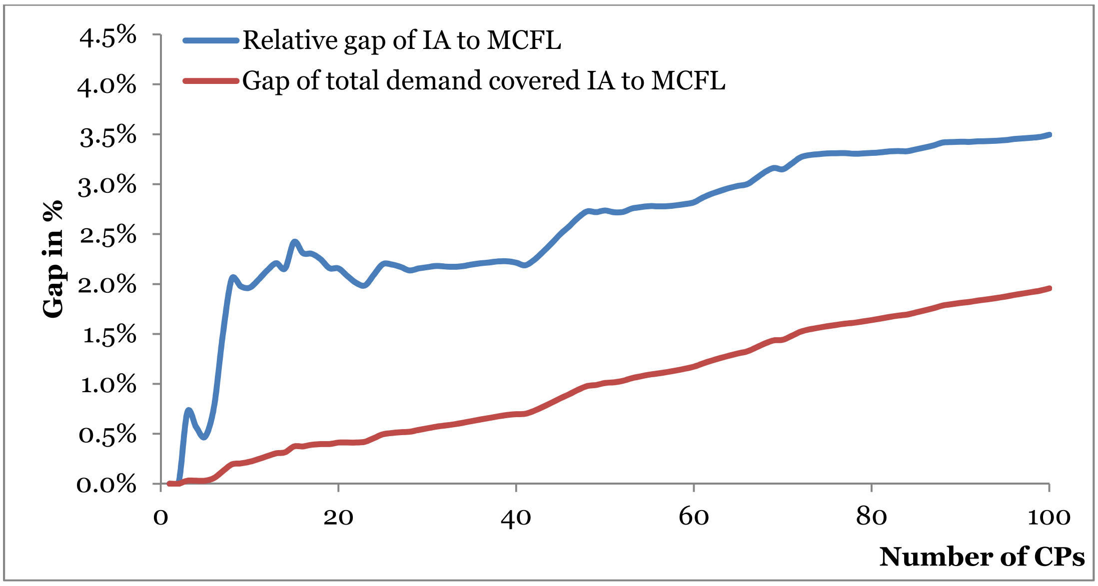


Fig. 12. Comparison of iterative algorithm and MCFL.

6. Conclusion

In this paper we have presented a business intelligence system for city planners incorporating a novel methodology in order to plan an optimal EV charging infrastructure in an urban setting. As a data basis, we evaluated more than 32,000 charging sessions, including daily usage frequency and actual demands from one of the best developed charging infrastructure in the world, Amsterdam. Further, we investigated the influence of possible local trip destinations of EV owners on CP usage. The destinations, so called "*points of interest"*, are grouped in 92 different categories, such as restaurants, stores or banks. We show that these POIs have a significant influence on the actual charging behavior of EV owners in Amsterdam by performing a linear regression. On this basis, we defined a ranking procedure to rate individual POIs based on the surrounding charge point usage behavior. The individual ranking then contributed to the POI category ranks, which in turn can be used to assess the "charge point attractiveness" of selected urban areas. This EV charging demand proxy served then as an input to our location models.

We developed two different approaches to derive optimal charge point locations for urban green field planning – city planners are able to choose among the approaches depending on their preferences. The first approach is a maximum coverage facility location problem, which maximizes the total demand covered based on a given number of charge points. The location of CPs is optimized simultaneously. This approach is best suitable for planners, which have a given budget constraint for CPs to be located or have requirements from city councils, such as "establishing 100 additional charge points by the end of 2014". The second approach is formulated a minimum charge point location problem, which is minimizing the number of charge points needed for a give coverage ratio. In cases where no constraints are imposed on planners, or simply no given number of CPs to be established is given, this approach is best suited. Hence planners can define the demand to be covered by the planned charging infrastructure and the MCPL will provide the minimum number of CPs needed to achieve this requirement.

We have proven the application of both models in a case study for the cities of Amsterdam and Brussels. Parameters to the models, which allow infrastructure planners to calibrate and adjust the models to their specific situation, have been discussed in depth. Altering POI category ranks allows planners to adjust for local specificities by weighing single categories stronger or weaker. The definition of the planning area and its granularity has an effect on how detailed the infrastructure is planned with regard to exact location. Box factor calculation, i.e. a proxy for expected EV charging demand, allows to smoothen/delimit demand more precisely. The CP coverage radius enables planners to influence the density of the planned network. Additionally specific parameters exist for the MCFL – number of CPs to be located – and the MCPL – demand to be covered. The last three parameters are integral ones to counter the range anxiety observed by many (potential) EV users, as they allow to adjust the charge point infrastructure with regard to broad coverage of the planning area. A computational study showed the sensitivity of the parameters and compared results. An iterative algorithm was developed to allow for quick results with regard to parameter calibration. Even the results are not optimal, compared to MCFL and MCPL, they are within a gap of 2% to optimal results and thus precise enough for parameter calibration. Hence, city planners are able to modify the introduced methods depending on individual city characteristics. This can be achieved by adjusting the parameters discussed above.

Finally, we succeeded in developing a methodology to derive future charge point infrastructures for smart cities, by analyzing real charging behaviors. As electric vehicles are not yet part of the everyday life it is important to find such an opportunity to determine CP locations based on daily routines. Accordingly, the presented approaches use real charging behaviors as a benchmark to develop a generally applicable method. We show that these approaches are practicable for any desired city and, moreover, the results are self-adapting as soon as the usage of EVs increases in the near future.

While our approach realized the benefits, but only scratched the surface of Big Data, the full potential for smart city planning is enormous. In our future work we will include more structural and environmental data (e.g. traffic flows, walking patterns, urban population) into our analysis. This way we aim at developing a guideline for city planners helping them to select the right input parameters with regard to grid size, POI category ranks, box factor calculation, coverage radius r and specific parameters, such as the number of CPs p for the MCFL and demand to be covered c for the MCPL.

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Curriculum Vitae

The vita includes personal data, which have been removed from the online version.

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Götzinger, M., Brandt, T., & Neumann, D. (2012). Green Facility Location – A Case Study. *Proceedings of the 18th Americas Conference on Information Systems (AMCIS 2012)*

Qiu, X., **Götzinger, M.**, & Neumann, D. (2012). The Position-Aware-Market: Optimizing Freight Delivery for Less-Than-Truckload Transportation. *Proceedings of the 18th Americas Conference on Information Systems (AMCIS 2012)*

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(Underlined publications are included or partially included in the thesis)