

Contents

Preface — V

Introduction: Statistical Computing Algorithms as a Subject of Adaptive Control — 1

Part I: Evaluation of Integrals

1 Fundamentals of the Monte Carlo Method to Evaluate Definite Integrals — 9

- 1.1 Problem setup — 9
- 1.2 Essence of the Monte Carlo method — 10
- 1.3 Sampling of a scalar random variable — 11
 - 1.3.1 The inverse function method — 11
 - 1.3.2 The superposition method — 14
 - 1.3.3 The rejection method — 15
- 1.4 Sampling of a vector random variable — 16
- 1.5 Elementary Monte Carlo method and its properties — 18
- 1.6 Methods of variance reduction — 20
 - 1.6.1 Importance sampling — 20
 - 1.6.2 Control variate sampling — 21
 - 1.6.3 Advantages and relations between the methods of importance sampling and control variate sampling — 21
 - 1.6.4 Symmetrisation of the integrand — 22
 - 1.6.5 Group sampling — 23
 - 1.6.6 Estimating with a faster rate of convergence — 24
- 1.7 Conclusion — 25

2 Sequential Monte Carlo Method and Adaptive Integration — 27

- 2.1 Sequential Monte Carlo method — 27
 - 2.1.1 Basic relations — 27
 - 2.1.2 Mean square convergence — 29
 - 2.1.3 Almost sure convergence — 36
 - 2.1.4 Error estimation — 39
- 2.2 Adaptive methods of integration — 41
 - 2.2.1 Elementary adaptive method of one-dimensional integration — 42
 - 2.2.2 Adaptive method of importance sampling — 44
 - 2.2.3 Adaptive method of control variate sampling — 46

2.2.4	Generalised adaptive methods of importance sampling and control variate sampling —	47
2.2.5	On time and memory consumption —	47
2.2.6	Regression-based adaptive methods —	49
2.2.7	Note on notation —	56
2.3	Conclusion —	57
3	Methods of Adaptive Integration Based on Piecewise Approximation —	59
3.1	Piecewise approximations over subdomains —	59
3.1.1	Piecewise approximations and their orders —	59
3.1.2	Approximations for particular classes of functions —	60
3.1.3	Partition moments and estimates for the variances D_k —	62
3.1.4	Generalised adaptive methods —	63
3.2	Elementary one-dimensional method —	65
3.2.1	Control variate sampling —	66
3.2.2	Importance sampling —	67
3.2.3	Conclusions and remarks —	69
3.3	Sequential bisection —	70
3.3.1	Description of the bisection technique —	70
3.3.2	Control variate sampling —	72
3.3.3	Importance sampling —	76
3.3.4	Time consumption of the bisection method —	78
3.4	Sequential method of stratified sampling —	79
3.5	Deterministic construction of partitions —	80
3.6	Conclusion —	82
4	Methods of Adaptive Integration Based on Global Approximation —	83
4.1	Global approximations —	83
4.1.1	Approximations by orthonormalised functions: Basic relations —	84
4.1.2	Conditions for algorithm convergence —	86
4.2	Adaptive integration over the class S_p —	92
4.2.1	Haar system of functions and univariate classes of functions S_p —	92
4.2.2	Adaptive integration over the class S_p : One-dimensional case —	93
4.2.3	Expansion into parts of differing dimensionalities: Multidimensional classes S_p —	95
4.2.4	Adaptive integration over the class S_p : Multidimensional case —	97
4.3	Adaptive integration over the class E_s^α —	103
4.3.1	The classes of functions E_s^α —	104
4.3.2	Adaptive integration with the use of trigonometric approximations —	104
4.4	Conclusion —	108

5	Numerical Experiments — 111
5.1	Test problems setup — 111
5.1.1	The first problem — 111
5.1.2	The second problem — 112
5.2	Results of experiments — 113
5.2.1	The first test problem — 113
5.2.2	The second test problem — 121
6	Adaptive Importance Sampling Method Based on Piecewise Constant Approximation — 123
6.1	Introduction — 123
6.2	Investigation of efficiency of the adaptive importance sampling method — 123
6.2.1	Adaptive and sequential importance sampling schemes — 123
6.2.2	Comparison of adaptive and sequential schemes — 126
6.2.3	Numerical experiments — 128
6.2.4	Conclusion — 131
6.3	Adaptive importance sampling method in the case where the number of bisection steps is limited — 132
6.3.1	The adaptive scheme for one-dimensional improper integrals — 132
6.3.2	The adaptive scheme for the case where the number of bisection steps is limited — 134
6.3.3	Peculiarities and capabilities of the adaptive importance sampling scheme in the case where the number of bisection steps is fixed — 135
6.3.4	Numerical experiments — 136
6.3.5	Conclusion — 140
6.4	Solution of a problem of navigation by distances to pin-point targets with the use of the adaptive importance sampling method — 141
6.4.1	Problem setup — 141
6.4.2	Application of the adaptive importance sampling method to calculating the optimal estimator of the object position — 143
6.4.3	A numerical experiment — 145
6.4.4	Conclusion — 148

Part II: Solution of Integral Equations

7	Semi-Statistical Method of Solving Integral Equations Numerically — 151
7.1	Introduction — 151
7.2	Basic relations — 152
7.3	Recurrent inversion formulas — 154
7.4	Non-degeneracy of the matrix of the semi-statistical method — 155

7.5	Convergence of the method — 161
7.6	Adaptive capabilities of the algorithm — 163
7.7	Qualitative considerations on the relation between the semi-statistical method and the variational ones — 165
7.8	Application of the method to integral equations with a singularity — 165
7.8.1	Description of the method and peculiarities of its application — 165
7.8.2	Recurrent inversion formulas — 168
7.8.3	Error analysis — 168
7.8.4	Adaptive capabilities of the algorithm — 171
8	Problem of Vibration Conductivity — 173
8.1	Boundary value problem of vibration conductivity — 173
8.2	Integral equations of vibration conductivity — 174
8.3	Regularisation of the equations — 180
8.4	An integral equation with enhanced asymptotic properties at small β — 184
8.5	Numerical solution of vibration conductivity problems — 187
8.5.1	Solution of the test problem — 187
8.5.2	Analysis of the influence of the sphere distortion and the external stress character on the results of the numerical solution — 190
9	Problem on Ideal-Fluid Flow Around an Airfoil — 193
9.1	Introduction — 193
9.2	Setup of the problem on flow around an airfoil — 193
9.3	Analytic description of the airfoil contour — 195
9.4	Computational algorithm and optimisation — 198
9.5	Results of numerical computation — 199
9.5.1	Computation of the velocity around an airfoil — 199
9.5.2	Analysis of the density adaptation efficiency — 201
9.5.3	Computations on test cascades — 205
9.6	Conclusions — 207
9.7	A modified semi-statistical method — 208
9.7.1	Computational scheme — 209
9.7.2	Ways to estimate the variance in the computing process — 210
9.7.3	Recommendations and remarks to the scheme of the modified semi-statistical method — 211
9.7.4	Numerical experiment for a prolate airfoil — 211
10	First Basic Problem of Elasticity Theory — 215
10.1	Potentials and integral equations of the first basic problem of elasticity theory — 215

10.1.1	The force and pseudo-force tensors —	215
10.1.2	Integral equations of the first basic problem —	217
10.2	Solution of some spatial problems of elasticity theory using the method of potentials —	218
10.2.1	Solution of the first basic problem for a series of centrally symmetric spatial regions —	219
10.2.2	Solution of the first basic problem for a sphere —	220
10.2.3	Solution of the first basic problem for an unbounded medium with a spherical cavity —	220
10.2.4	Solution of the first basic problem for a hollow sphere —	221
10.3	Solution of integral equations of elasticity theory using the semi-statistical method —	223
10.4	Formulas for the optimal density —	225
10.5	Results of numerical experiments —	227
11	Second Basic Problem of Elasticity Theory —	231
11.1	Fundamental solutions of the first and second kind —	231
11.2	Boussinesq potentials —	234
11.3	Weyl tensor —	235
11.4	Weyl force tensors —	237
11.5	Arbitrary Lyapunov surface —	238
12	Projectional and Statistical Method of Solving Integral Equations Numerically —	241
12.1	Basic relations —	241
12.2	Recurrent inversion formulas —	244
12.3	Non-degeneracy of the matrix of the method —	246
12.4	Convergence of the method —	250
12.5	Advantages of the method and its adaptive capabilities —	253
12.6	Peculiarities of the numerical implementation —	255
12.7	Another computing technique: Averaging of approximate solutions —	257
12.8	Numerical experiments —	259
12.8.1	The test problem —	259
12.8.2	The problem on steady-state forced small transverse vibration of a pinned string caused by a harmonic force —	264

Afterword — 271

Bibliography — 273

Index — 277