

## **Chapter 1. Introduction and Mathematical Preliminaries**

1.1 Scope .....	1
1.2 Vector Algebra .....	1
1.3 Scalar and Vector Fields .....	3
1.3.1 Definitions .....	3
1.3.2 Gradient .....	3
1.3.3 Divergence .....	4
1.3.4 Curl .....	4
1.4 Indicial Notation .....	4
1.5 Coordinate Rotation .....	5
1.6 Cartesian Tensors .....	7
1.7 Algebra of Cartesian Tensors .....	7
1.8 Operational Tensors .....	8
1.9 Computational Examples .....	10
Exercises .....	10
References .....	11

## **Chapter 2. Traction, Stress and Equilibrium**

2.1 Introduction .....	12
2.2 State of Stress .....	12
2.2.1 Traction and Couple-Stress Vectors .....	12
2.2.2 Components of Stress .....	13
2.2.3 Stress at a Point .....	14
2.2.4 Stress on a Normal Plane .....	16
2.2.5 Dyadic Representation of Stress .....	17
2.2.6 Computational Example .....	18
2.3 Equilibrium .....	20
2.3.1 Physical and Mathematical Principles .....	20
2.3.2 Linear Momentum .....	21
2.3.3 Angular Momentum .....	22
2.3.4 Computational Example .....	23

2.4	Principal Stress . . . . .	23
2.4.1	Definition and Derivation . . . . .	23
2.4.2	Computational Format, Stress Invariants and Principal Coordinates . . . . .	26
2.4.3	Computational Example . . . . .	29
2.5	Stresses in Principal Coordinates . . . . .	30
2.5.1	Stresses on an Oblique Plane . . . . .	30
2.5.2	Stresses on Octahedral Planes . . . . .	31
2.5.3	Absolute Maximum Shearing Stress . . . . .	31
2.5.4	Computational Example . . . . .	32
2.6	Properties and Special States of Stress . . . . .	33
2.6.1	Projection Theorem . . . . .	33
2.6.2	Plane Stress . . . . .	33
2.6.3	Linear Stress . . . . .	33
2.6.4	Pure Shear . . . . .	33
2.6.5	Hydrostatic Stress . . . . .	34
	Exercises . . . . .	34
	References . . . . .	35

### Chapter 3. Deformations

3.1	Introduction . . . . .	36
3.2	Strain . . . . .	36
3.3	Physical Interpretation of Strain Tensor . . . . .	39
3.4	Principal Strains . . . . .	42
3.5	Volume and Shape Changes . . . . .	43
3.6	Compatibility . . . . .	44
3.7	Computational Example . . . . .	46
	Exercises . . . . .	47
	References . . . . .	49

### Chapter 4. Material Behavior

4.1	Introduction . . . . .	50
4.2	Uniaxial Behavior . . . . .	50
4.3	Generalized Hooke's Law . . . . .	52
4.4	Thermal Strains . . . . .	59
4.5	Physical Data . . . . .	59
	Exercises . . . . .	60
	References . . . . .	61

### Chapter 5. Formulation, Uniqueness and Solution Strategies

5.1	Introduction . . . . .	62
5.2	Displacement Formulation . . . . .	62

5.3	Force Formulation .....	63
5.4	Other Formulations .....	65
5.5	Uniqueness .....	66
5.6	Solution Strategies .....	67
	Exercises .....	68
	References .....	69

## Chapter 6. Extension, Bending and Torsion

6.1	Introduction .....	70
6.2	Prismatic Bar under Axial Loading .....	70
6.3	Cantilever Beam under End Loading .....	74
6.3.1	Elementary Beam Theory .....	74
6.3.2	Elasticity Theory .....	78
6.4	Torsion .....	83
6.4.1	Torsion of Circular Shaft .....	83
6.4.2	Torsion of Solid Prismatic Shafts .....	86
6.4.3	Torsion of Elliptical Shaft .....	93
	Exercises .....	96
	References .....	97

## Chapter 7. Two-Dimensional Elasticity

7.1	Introduction .....	98
7.2	Plane Stress Equations .....	99
7.3	Plane Strain Equations .....	101
7.4	Cylindrical Coordinates .....	103
7.4.1	Geometric Relations .....	103
7.4.2	Transformation of Stress Tensor and Compatibility Equation .....	104
7.4.3	Axisymmetric Stresses and Displacements .....	106
7.5	Thick-Walled Cylinder or Disk .....	107
7.6	Sheet with Small Circular Hole .....	110
7.7	Curved Beam .....	115
7.8	Rotational Dislocation .....	118
7.9	Narrow, Simply Supported Beam .....	119
7.10	Clamped Plate with Linear Thermal Gradient .....	122
	Exercises .....	124
	References .....	128

## Chapter 8. Energy Principles

8.1	Introduction .....	129
8.2	Conservation of Energy .....	129

8.3	Strain Energy .....	130
8.4	Work of External Loading .....	131
8.5	Principle of Virtual Work .....	132
8.5.1	Definitions .....	132
8.5.2	Principle of Virtual Displacements .....	133
8.5.3	Principle of Virtual Forces .....	137
8.5.4	Reciprocal Theorems .....	138
8.6	Variational Principles .....	139
8.6.1	Definitions .....	139
8.6.2	Principle of Minimum Total Potential Energy .....	139
8.6.3	Principle of Minimum Complementary Energy .....	141
8.7	Direct Variational Methods .....	142
8.7.1	Motivation .....	142
8.7.2	Rayleigh–Ritz Method .....	142
8.7.3	Torsion of Rectangular Cross Section .....	143
8.7.4	Commentary .....	146
	Exercises .....	146
	References .....	147
	<b>Index .....</b>	<b>148</b>