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Adaptive Algorithms and Stochastic Approximations

Translated from the French by Stephen S. Wilson

With 24 Figures



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