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Partial Differential Equations VI

Elliptic and Parabolic Operators

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I. Elliptic Operators on Closed Manifolds

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