

Helmut Rieder

Robust Asymptotic Statistics



Springer-Verlag

New York Berlin Heidelberg London Paris
Tokyo Hong Kong Barcelona Budapest

Contents

Preface	vii
Notation	xvii
Chapter 1: Von Mises Functionals	1
1.1 General Remarks	1
1.2 Regular Differentiations	1
1.3 The Delta Method	6
1.4 M Estimates	8
1.5 Quantiles	16
1.6 L Estimates	25
Chapter 2: Log Likelihoods	39
2.1 General Remarks	39
2.2 Contiguity and Asymptotic Normality	39
2.3 Differentiable Families	47
2.3.1 Differentiable Arrays	47
2.3.2 Smooth Parametric Families	56
2.3.3 Other Differentiability Notions	60
2.4 Linear Regression	61
Chapter 3: Asymptotic Statistics	71
3.1 General Remarks	71
3.2 Convolution Representation	71
3.3 Minimax Estimation	79
3.3.1 Normal Mean	79
3.3.2 Asymptotic Minimax Bound	90
3.4 Testing	99
3.4.1 Simple Hypotheses	99
3.4.2 Passage to the Normal Limit	102
3.4.3 One- and Two-Sided Hypotheses	103
3.4.4 Multisided Hypotheses	109

Chapter 4: Nonparametric Statistics	123
4.1 Introduction	123
4.2 The Nonparametric Setup	124
4.2.1 Full Neighborhood Systems	124
4.2.2 Asymptotically Linear Functionals	129
4.2.3 Asymptotically Linear Estimators	134
4.3 Statistics of Functionals	138
4.3.1 Unbiased Estimation	139
4.3.2 Unbiased Testing	146
4.3.3 Remarks and Criticisms	156
4.4 Restricted Tangent Space	160
Chapter 5: Optimal Influence Curves	165
5.1 Introduction	165
5.2 Minimax Risk	166
5.3 Oscillation	170
5.3.1 Oscillation/Bias Terms	170
5.3.2 Minimax Oscillation	178
5.4 Robust Asymptotic Tests	184
5.5 Minimax Risk and Oscillation	194
5.5.1 Minimum Trace Subject to Bias Bound	195
5.5.2 Mean Square Error	207
5.5.3 Nonexistence of Strong Solution	209
5.5.4 Equivariance Under Reparametrizations	211
Chapter 6: Stable Constructions	219
6.1 The Construction Problem	219
6.2 M Equations	221
6.2.1 Location Parameter	223
6.2.2 General Parameter	227
6.3 Minimum Distance	235
6.3.1 MD Functionals	235
6.3.2 MD Estimates	242
6.4 One-Steps	246
6.4.1 Functionals	251
6.4.2 Estimators	255
Chapter 7: Robust Regression	261
7.1 The Ideal Model	261
7.2 Regression Neighborhoods	261
7.2.1 Errors-in-Variables	262
7.2.2 Error-Free-Variables	262
7.2.3 Translation Invariance	265
7.2.4 Neighborhood Submodels	266

7.2.5	Tangent Subspaces	267
7.3	Conditional Bias	268
7.3.1	General Properties	269
7.3.2	Explicit Terms	270
7.4	Optimal Influence Curves	273
7.4.1	Optimization Problems	273
7.4.2	Auxiliary Results	278
7.4.3	Contamination Optimality	285
7.4.4	Total Variation Optimality	297
7.4.5	Hellinger Optimality	304
7.5	Least Favorable Contamination Curves	309
7.5.1	Hellinger Saddle Points	310
7.5.2	Contamination Saddle Points	314
7.6	Equivariance Under Basis Change	319
7.6.1	Unstandardized Solutions	321
7.6.2	M Standardized Equivariant Solutions	325
7.6.3	Robust Prediction	329
Appendix A: Weak Convergence of Measures		331
A.1	Basic Notions	331
A.2	Convergence of Integrals	336
A.3	Smooth Empirical Process	339
A.4	Square Integrable Empirical Process	344
Appendix B: Some Functional Analysis		355
B.1	A Few Facts	355
B.2	Lagrange Multipliers	356
B.2.1	Neyman–Pearson Lemma	361
Appendix C: Complements		365
C.1	Parametric Finite-Sample Results	365
C.2	Some Technical Results	368
C.2.1	Calculus	368
C.2.2	Topology	372
C.2.3	Matrices	373
Bibliography		377
Index		383