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Plane Answers to Complex Questions

The Theory of Linear Models

Second Edition

With 34 Illustrations



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