

Jerrold E. Marsden

Tudor S. Ratiu

# Introduction to Mechanics and Symmetry

A Basic Exposition of  
Classical Mechanical Systems

Second Edition

With 54 Illustrations



Springer

# Contents

<b>Preface</b>	<b>ix</b>
<b>About the Authors</b>	<b>xiii</b>
<b>1 Introduction and Overview</b>	<b>1</b>
1.1 Lagrangian and Hamiltonian Formalisms . . . . .	1
1.2 The Rigid Body . . . . .	6
1.3 Lie–Poisson Brackets, Poisson Manifolds, Momentum Maps . . . . .	9
1.4 The Heavy Top . . . . .	16
1.5 Incompressible Fluids . . . . .	18
1.6 The Maxwell–Vlasov System . . . . .	22
1.7 Nonlinear Stability . . . . .	29
1.8 Bifurcation . . . . .	43
1.9 The Poincaré–Melnikov Method . . . . .	47
1.10 Resonances, Geometric Phases, and Control . . . . .	49
<b>2 Hamiltonian Systems on Linear Symplectic Spaces</b>	<b>61</b>
2.1 Introduction . . . . .	61
2.2 Symplectic Forms on Vector Spaces . . . . .	66
2.3 Canonical Transformations, or Symplectic Maps . . . . .	69
2.4 The General Hamilton Equations . . . . .	74
2.5 When Are Equations Hamiltonian? . . . . .	77
2.6 Hamiltonian Flows . . . . .	80

2.7	Poisson Brackets . . . . .	82
2.8	A Particle in a Rotating Hoop . . . . .	87
2.9	The Poincaré–Melnikov Method . . . . .	94
<b>3</b>	<b>An Introduction to Infinite-Dimensional Systems</b>	<b>105</b>
3.1	Lagrange’s and Hamilton’s Equations for Field Theory . .	105
3.2	Examples: Hamilton’s Equations . . . . .	107
3.3	Examples: Poisson Brackets and Conserved Quantities . .	115
<b>4</b>	<b>Manifolds, Vector Fields, and Differential Forms</b>	<b>121</b>
4.1	Manifolds . . . . .	121
4.2	Differential Forms . . . . .	129
4.3	The Lie Derivative . . . . .	137
4.4	Stokes’ Theorem . . . . .	141
<b>5</b>	<b>Hamiltonian Systems on Symplectic Manifolds</b>	<b>147</b>
5.1	Symplectic Manifolds . . . . .	147
5.2	Symplectic Transformations . . . . .	150
5.3	Complex Structures and Kähler Manifolds . . . . .	152
5.4	Hamiltonian Systems . . . . .	157
5.5	Poisson Brackets on Symplectic Manifolds . . . . .	160
<b>6</b>	<b>Cotangent Bundles</b>	<b>165</b>
6.1	The Linear Case . . . . .	165
6.2	The Nonlinear Case . . . . .	167
6.3	Cotangent Lifts . . . . .	170
6.4	Lifts of Actions . . . . .	173
6.5	Generating Functions . . . . .	174
6.6	Fiber Translations and Magnetic Terms . . . . .	176
6.7	A Particle in a Magnetic Field . . . . .	178
<b>7</b>	<b>Lagrangian Mechanics</b>	<b>181</b>
7.1	Hamilton’s Principle of Critical Action . . . . .	181
7.2	The Legendre Transform . . . . .	183
7.3	Euler–Lagrange Equations . . . . .	185
7.4	Hyperregular Lagrangians and Hamiltonians . . . . .	188
7.5	Geodesics . . . . .	195
7.6	The Kaluza–Klein Approach to Charged Particles . . . . .	200
7.7	Motion in a Potential Field . . . . .	202
7.8	The Lagrange–d’Alembert Principle . . . . .	205
7.9	The Hamilton–Jacobi Equation . . . . .	210
<b>8</b>	<b>Variational Principles, Constraints, &amp; Rotating Systems</b>	<b>219</b>
8.1	A Return to Variational Principles . . . . .	219
8.2	The Geometry of Variational Principles . . . . .	226

8.3	Constrained Systems . . . . .	234
8.4	Constrained Motion in a Potential Field . . . . .	238
8.5	Dirac Constraints . . . . .	242
8.6	Centrifugal and Coriolis Forces . . . . .	248
8.7	The Geometric Phase for a Particle in a Hoop . . . . .	253
8.8	Moving Systems . . . . .	257
8.9	Routh Reduction . . . . .	260
<b>9</b>	<b>An Introduction to Lie Groups</b>	<b>265</b>
9.1	Basic Definitions and Properties . . . . .	267
9.2	Some Classical Lie Groups . . . . .	283
9.3	Actions of Lie Groups . . . . .	309
<b>10</b>	<b>Poisson Manifolds</b>	<b>327</b>
10.1	The Definition of Poisson Manifolds . . . . .	327
10.2	Hamiltonian Vector Fields and Casimir Functions . . . . .	333
10.3	Properties of Hamiltonian Flows . . . . .	338
10.4	The Poisson Tensor . . . . .	340
10.5	Quotients of Poisson Manifolds . . . . .	349
10.6	The Schouten Bracket . . . . .	353
10.7	Generalities on Lie–Poisson Structures . . . . .	360
<b>11</b>	<b>Momentum Maps</b>	<b>365</b>
11.1	Canonical Actions and Their Infinitesimal Generators . . . . .	365
11.2	Momentum Maps . . . . .	367
11.3	An Algebraic Definition of the Momentum Map . . . . .	370
11.4	Conservation of Momentum Maps . . . . .	372
11.5	Equivariance of Momentum Maps . . . . .	378
<b>12</b>	<b>Computation and Properties of Momentum Maps</b>	<b>383</b>
12.1	Momentum Maps on Cotangent Bundles . . . . .	383
12.2	Examples of Momentum Maps . . . . .	389
12.3	Equivariance and Infinitesimal Equivariance . . . . .	396
12.4	Equivariant Momentum Maps Are Poisson . . . . .	403
12.5	Poisson Automorphisms . . . . .	412
12.6	Momentum Maps and Casimir Functions . . . . .	413
<b>13</b>	<b>Lie–Poisson and Euler–Poincaré Reduction</b>	<b>415</b>
13.1	The Lie–Poisson Reduction Theorem . . . . .	415
13.2	Proof of the Lie–Poisson Reduction Theorem for $GL(n)$ . . . . .	418
13.3	Lie–Poisson Reduction Using Momentum Functions . . . . .	419
13.4	Reduction and Reconstruction of Dynamics . . . . .	421
13.5	The Euler–Poincaré Equations . . . . .	430
13.6	The Lagrange–Poincaré Equations . . . . .	440

<b>14 Coadjoint Orbits</b>	<b>443</b>
14.1 Examples of Coadjoint Orbits . . . . .	444
14.2 Tangent Vectors to Coadjoint Orbits . . . . .	451
14.3 The Symplectic Structure on Coadjoint Orbits . . . . .	453
14.4 The Orbit Bracket via Restriction of the Lie-Poisson Bracket . . . . .	459
14.5 The Special Linear Group of the Plane . . . . .	465
14.6 The Euclidean Group of the Plane . . . . .	467
14.7 The Euclidean Group of Three-Space . . . . .	472
<b>15 The Free Rigid Body</b>	<b>481</b>
15.1 Material, Spatial, and Body Coordinates . . . . .	481
15.2 The Lagrangian of the Free Rigid Body . . . . .	483
15.3 The Lagrangian and Hamiltonian in Body Representation	485
15.4 Kinematics on Lie Groups . . . . .	489
15.5 Poincot's Theorem . . . . .	490
15.6 Euler Angles . . . . .	493
15.7 The Hamiltonian of the Free Rigid Body . . . . .	495
15.8 The Analytical Solution of the Free Rigid-Body Problem .	498
15.9 Rigid-Body Stability . . . . .	503
15.10 Heavy Top Stability . . . . .	507
15.11 The Rigid Body and the Pendulum . . . . .	512
<b>References</b>	<b>519</b>
<b>Index</b>	<b>553</b>