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Introduction to Mathematical Methods in Bioinformatics

With 76 Figures and 3 Tables



Springer

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Part I Sequence Analysis

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