

Contents

1	Introduction	1
1.1	Motivation	1
1.2	Flow Equations: Basic Ideas	2
1.3	Outline and Scope of this Book	7
	References	9
2	Transformation of the Hamiltonian	11
2.1	Energy Scale Separation	11
2.1.1	Potential Scattering Model	12
2.1.2	Kondo Model	19
2.2	Flow Equation Approach	22
2.2.1	Motivation	22
2.2.2	Infinitesimal Unitary Transformations	23
2.2.3	Choice of Generator	25
2.2.4	Flow Equations	28
2.3	Example: Potential Scattering Model	31
2.3.1	Setting up the Flow Equations	31
2.3.2	Methods of Solution	34
2.3.3	Strong-Coupling Case	39
	References	40
3	Evaluation of Observables	43
3.1	Expectation Values	43
3.1.1	Zero Temperature	43
3.1.2	Nonzero Temperature	46
3.2	Correlation Functions	47
3.2.1	Zero Temperature	47
3.2.2	Nonzero Temperature	49
3.2.3	Fluctuation-Dissipation Theorem	50
3.3	Examples	51
3.3.1	Potential Scattering Model	52
3.3.2	Resonant Level Model	54
	References	61

4	Interacting Many-Body Systems	63
4.1	Normal-Ordering	63
4.1.1	Bosons	64
4.1.2	Fermions	69
4.1.3	Important Commutators	70
4.1.4	Normal-Ordered Expansions	73
4.1.5	Normal-Ordering with Respect to Which State?	77
4.2	Kondo Model	78
4.2.1	Expansion in 1st Order (1-Loop Results)	79
4.2.2	Expansion in 2nd Order (2-Loop Results)	83
4.2.3	Nonzero Temperature	88
4.2.4	Transformation of the Spin Operator	91
4.2.5	Spin Correlation Function and Dynamical Susceptibility	95
4.2.6	Pseudogap Kondo Model	98
4.3	Spin-Boson Model	102
4.3.1	Flow of the Hamiltonian	102
4.3.2	Low-Energy Observables	107
4.3.3	Resonant Behavior	109
4.4	Interacting Fermions in $d > 1$ Dimensions	113
4.4.1	Flow Equations and Fermi Liquid Theory	114
4.4.2	Flow Equations and Molecular-Field Type Hamiltonians	121
4.5	Other Applications	123
4.5.1	Construction of Effective Hamiltonians: The Fröhlich Transformation Re-examined	124
4.5.2	Block-Diagonal Hamiltonians	132
	References	133
5	Modern Developments	137
5.1	Strong-Coupling Behavior: Sine-Gordon Model	137
5.1.1	Sine-Gordon Model	138
5.1.2	Flow Equation Analysis	140
5.1.3	Conventional Scaling vs. Flow Equations	146
5.2	Steady Non-Equilibrium: Kondo Model with Voltage Bias	151
5.2.1	Kondo Model in Non-Equilibrium	151
5.2.2	Flow Equation Analysis	153
5.2.3	Correlation Functions in Non-Equilibrium: Spin Dynamics	159
5.3	Real Time Evolution: Spin-Boson Model	163
5.4	Outlook and Open Questions	167
	References	168
	Index	169