

Contents

Mathematical Epidemiology	1
F. Brauer, P. van den Driessche and J. Wu, editors	
Part I Introduction and General Framework	
1 A Light Introduction to Modelling Recurrent Epidemics ..	3
David J.D. Earn	
1.1 Introduction	3
1.2 Plague	4
1.3 Measles	5
1.4 The SIR Model	6
1.5 Solving the Basic SIR Equations	8
1.6 SIR with Vital Dynamics	11
1.7 Demographic Stochasticity	13
1.8 Seasonal Forcing	13
1.9 Slow Changes in Susceptible Recruitment	14
1.10 Not the Whole Story	15
1.11 Take Home Message	16
References	16
2 Compartmental Models in Epidemiology	19
Fred Brauer	
2.1 Introduction	19
2.1.1 Simple Epidemic Models	22
2.1.2 The Kermack–McKendrick Model	24
2.1.3 Kermack–McKendrick Models with General Contact Rates	32
2.1.4 Exposed Periods	36
2.1.5 Treatment Models	38
2.1.6 An Epidemic Management (Quarantine-Isolation) Model	40

	2.1.7	Stochastic Models for Disease Outbreaks	45
2.2		Models with Demographic Effects	45
	2.2.1	The <i>SIR</i> Model	45
	2.2.2	The <i>SIS</i> Model	52
2.3		Some Applications	55
	2.3.1	Herd Immunity	55
	2.3.2	Age at Infection	56
	2.3.3	The Interepidemic Period	57
	2.3.4	“Epidemic” Approach to the Endemic Equilibrium	59
	2.3.5	Disease as Population Control	60
2.4		Age of Infection Models	66
	2.4.1	The Basic <i>SI*R</i> Model	66
	2.4.2	Equilibria	69
	2.4.3	The Characteristic Equation	70
	2.4.4	The Endemic Equilibrium	72
	2.4.5	An <i>SI*S</i> Model	74
	2.4.6	An Age of Infection Epidemic Model	76
		References	78
3		An Introduction to Stochastic Epidemic Models	81
		Linda J.S. Allen	
	3.1	Introduction	81
	3.2	Review of Deterministic SIS and SIR Epidemic Models	82
	3.3	Formulation of DTMC Epidemic Models	85
		3.3.1 SIS Epidemic Model	85
		3.3.2 Numerical Example	90
		3.3.3 SIR Epidemic Model	90
		3.3.4 Numerical Example	93
	3.4	Formulation of CTMC Epidemic Models	93
		3.4.1 SIS Epidemic Model	93
		3.4.2 Numerical Example	97
		3.4.3 SIR Epidemic Model	98
	3.5	Formulation of SDE Epidemic Models	100
		3.5.1 SIS Epidemic Model	100
		3.5.2 Numerical Example	103
		3.5.3 SIR Epidemic Model	103
		3.5.4 Numerical Example	105
	3.6	Properties of Stochastic SIS and SIR Epidemic Models	105
		3.6.1 Probability of an Outbreak	105
		3.6.2 Quasistationary Probability Distribution	108
		3.6.3 Final Size of an Epidemic	112
		3.6.4 Expected Duration of an Epidemic	115
	3.7	Epidemic Models with Variable Population Size	117
		3.7.1 Numerical Example	119
	3.8	Other Types of DTMC Epidemic Models	121

3.8.1 Chain Binomial Epidemic Models 121

3.8.2 Epidemic Branching Processes 124

3.9 MatLab Programs 125

References 128

Part II Advanced Modeling and Heterogeneities

4 An Introduction to Networks in Epidemic Modeling 133
 Fred Brauer

4.1 Introduction 133

4.2 The Probability of a Disease Outbreak 134

4.3 Transmissibility 138

4.4 The Distribution of Disease Outbreak and Epidemic Sizes .. 140

4.5 Some Examples of Contact Networks 142

4.6 Conclusions 145

References 145

**5 Deterministic Compartmental Models: Extensions
 of Basic Models 147**
 P. van den Driessche

5.1 Introduction 147

5.2 Vertical Transmission 148

5.2.1 Kermack–McKendrick SIR Model 148

5.2.2 SEIR Model 150

5.3 Immigration of Infectives 152

5.4 General Temporary Immunity 154

References 157

6 Further Notes on the Basic Reproduction Number 159
 P. van den Driessche and James Watmough

6.1 Introduction 159

6.2 Compartmental Disease Transmission Models 160

6.3 The Basic Reproduction Number 162

6.4 Examples 163

6.4.1 The SEIR Model 163

6.4.2 A Variation on the Basic SEIR Model 165

6.4.3 A Simple Treatment Model 166

6.4.4 A Vaccination Model 168

6.4.5 A Vector-Host Model 170

6.4.6 A Model with Two Strains 171

6.5 \mathcal{R}_0 and the Local Stability of the Disease-Free Equilibrium . 173

6.6 \mathcal{R}_0 and Global Stability of the Disease-Free Equilibrium ... 175

References 177

7	Spatial Structure: Patch Models	179
	P. van den Driessche	
7.1	Introduction	179
7.2	Spatial Heterogeneity	180
7.3	Geographic Spread	182
7.4	Effect of Quarantine on Spread of 1918–1919 Influenza in Central Canada	185
7.5	Tuberculosis in Possums	188
7.6	Concluding Remarks	188
	References	189
8	Spatial Structure: Partial Differential Equations Models ..	191
	Jianhong Wu	
8.1	Introduction	191
8.2	Model Derivation	192
8.3	Case Study I: Spatial Spread of Rabies in Continental Europe	194
8.4	Case Study II: Spread Rates of West Nile Virus	199
8.5	Remarks	202
	References	202
9	Continuous-Time Age-Structured Models in Population Dynamics and Epidemiology	205
	Jia Li and Fred Brauer	
9.1	Why Age-Structured Models?	205
9.2	Modeling Populations with Age Structure	206
	9.2.1 Solutions along Characteristic Lines	208
	9.2.2 Equilibria and the Characteristic Equation	209
9.3	Age-Structured Integral Equations Models	211
	9.3.1 The Renewal Equation	212
9.4	Age-Structured Epidemic Models	214
9.5	A Simple Age-Structured AIDS Model	215
	9.5.1 The Reproduction Number	216
	9.5.2 Pair-Formation in Age-Structured Epidemic Models	218
	9.5.3 The Semigroup Method	220
9.6	Modeling with Discrete Age Groups	222
	9.6.1 Examples	223
	References	225
10	Distribution Theory, Stochastic Processes and Infectious Disease Modelling	229
	Ping Yan	
10.1	Introduction	230
10.2	A Review of Some Probability Theory and Stochastic Processes	231
	10.2.1 Non-negative Random Variables and Their Distributions	231

10.2.2	Some Important Discrete Random Variables Representing Count Numbers	234
10.2.3	Continuous Random Variables Representing Time-to-Event Durations	237
10.2.4	Mixture of Distributions	239
10.2.5	Stochastic Processes	241
10.2.6	Random Graph and Random Graph Process	248
10.3	Formulating the Infectious Contact Process	249
10.3.1	The Expressions for R_0 and the Distribution of N such that $R_0 = E[N]$	251
10.3.2	Competing Risks, Independence and Homogeneity in the Transmission of Infectious Diseases	254
10.4	Some Models Under Stationary Increment Infectious Contact Process $\{K(x)\}$	255
10.4.1	Classification of some Epidemics Where N Arises from the Mixed Poisson Processes	255
10.4.2	Tail Properties for N	258
10.5	The Invasion and Growth During the Initial Phase of an Outbreak	261
10.5.1	Invasion and the Epidemic Threshold	262
10.5.2	The Risk of a Large Outbreak and Quantities Associated with a Small Outbreak	263
10.5.3	Behaviour of a Large Outbreak in its Initial Phase: The Intrinsic Growth	273
10.5.4	Summary for the Initial Phase of an Outbreak	280
10.6	Beyond the Initial Phase: The Final Size of Large Outbreaks	281
10.6.1	Generality of the Mean Final Size	282
10.6.2	Some Cautionary Remarks	283
10.7	When the Infectious Contact Process may not Have Stationary Increment	285
10.7.1	The Linear Pure Birth Processes and the Yule Process	286
10.7.2	Parallels to the Preferential Attachment Model in Random Graph Theory	288
10.7.3	Distributions for N when $\{K(x)\}$ Arises as a Linear Pure Birth Process	288
	References	291

Part III Case Studies

11	The Role of Mathematical Models in Explaining Recurrent Outbreaks of Infectious Childhood Diseases	297
	Chris T. Bauch	
11.1	Introduction	297
11.2	The SIR Model with Demographics	300

11.3	Historical Development of Compartmental Models	302
11.3.1	Early Models	302
11.3.2	Stochasticity	306
11.3.3	Seasonality	306
11.3.4	Age Structure	307
11.3.5	Alternative Assumptions About Incidence Terms	307
11.3.6	Distribution of Latent and Infectious Period	308
11.3.7	Seasonality Versus Nonseasonality	308
11.3.8	Chaos	309
11.3.9	Transitions Between Outbreak Patterns	310
11.4	Spectral Analysis of Incidence Time Series	310
11.4.1	Power Spectra	311
11.4.2	Wavelet Power Spectra	313
11.5	Conclusions	314
	References	316
12	Modeling Influenza: Pandemics and Seasonal Epidemics	321
	Fred Brauer	
12.1	Introduction	321
12.2	A Basic Influenza Model	322
12.3	Vaccination	326
12.4	Antiviral Treatment	330
12.5	A More Detailed Model	334
12.6	A Model with Heterogeneous Mixing	336
12.7	A Numerical Example	341
12.8	Extensions and Other Types of Models	345
	References	346
13	Mathematical Models of Influenza: The Role of Cross-Immunity, Quarantine and Age-Structure	349
	M. Nuño, C. Castillo-Chavez, Z. Feng and M. Martcheva	
13.1	Introduction	349
13.2	Basic Model	351
13.3	Cross-Immunity and Quarantine	354
13.4	Age-Structure	359
13.5	Discussion and Future Work	362
	References	363
14	A Comparative Analysis of Models for West Nile Virus	365
	M.J. Wonham and M.A. Lewis	
14.1	Introduction: Epidemiological Modeling	365
14.2	Case Study: West Nile Virus	367
14.3	Minimalist Model	368
14.3.1	The Question	368
14.3.2	Model Scope and Scale	368
14.3.3	Model Formulation	370

14.3.4	Model Analysis	372
14.3.5	Model Application	373
14.4	Biological Assumptions 1: When does the Disease-Transmission Term Matter?	374
14.4.1	Frequency Dependence	374
14.4.2	Mass Action	374
14.4.3	Numerical Values of \mathcal{R}_0	377
14.5	Biological Assumptions 2: When do Added Model Classes Matter?	377
14.6	Model Parameterization, Validation, and Comparison	380
14.7	Model Application #1: WN Control	381
14.8	Model Application #2: Seasonal Mosquito Population	382
14.9	Summary	384
	References	386
Suggested Exercises and Projects		391
1	Cholera	395
2	Ebola	395
3	Gonorrhea	395
4	HIV/AIDS	396
5	HIV in Cuba	396
6	Human Papaloma Virus	397
7	Influenza	397
8	Malaria	397
9	Measles	398
10	Poliomyelitis (Polio)	398
11	Severe Acute Respiratory Syndrome (SARS)	399
12	Smallpox	399
13	Tuberculosis	400
14	West Nile Virus	400
15	Yellow Fever in Senegal 2002	400
Index		403