Contents

1 Introduction .................................................. 7
   1.1 Chapter 2 Contents ........................................ 9
   1.2 Chapter 3 Contents ........................................ 11
   1.3 Chapter 4 Contents ........................................ 13

2 DA Stochastic Dynamic Programming with Random Disturbances ................. 15
   2.1 The DA Model ............................................ 16
   2.2 The Certainty Equivalence Principle ...................... 24
   2.3 DA Models as Markov Decision Processes ................. 32
       2.3.1 Markov Decision Processes: Models and Properties ... 33
       2.3.2 DA Models as Markov Decision Processes under Appropriate Assumptions .... 36
           2.3.2.1 The Structure of Decisions within DA Markov Decision Processes ......... 39
           2.3.2.2 An Optimality Criterion for DA MDP and an "Almost-Partial Order" of the States ... 42
       2.3.3 DA Models with Special Properties .................. 51
           2.3.3.1 Notes on DA Models with Distance Properties 51
           2.3.3.2 The Dominant Policy ............................ 64
       2.3.4 Cost-Parametric Analysis of DA Markov Decision Processes ...................... 71
           2.3.4.1 Some Properties of Parametric Markov Decision Processes .............. 72
           2.3.4.2 Cost-Parametric DA Markov Decision Processes ...................... 77
2.3.4.3 Remarks on the Parametrization of DA Markov Decision Processes .................. 92
2.3.5 Remarks on the Solutions of DA Markov Decision Processes .......................... 93

3 The Problem of Stochastic Dynamic Distance Optimal Partitioning (SDDP) .............. 97
3.1 Preliminary Notes on Non-Balanced Transportation Problems .......................... 98
3.2 Model Formulation ........................................ 105
3.3 Properties of SDDP Problems and their Characteristic Quantities and Conversion Numbers ...................................................... 114
3.4 Characterization of Special Cases of SDDP Problems ......................................... 131
  3.4.1 Average Reward Functions of SDDP Problems with Identical Basic Costs ............ 135
  3.4.2 Reduction of SDDP Problems with Identical Basic Costs and Independent and Identically Distributed Requirements ..................... 140
3.5 Notes on the Solution Methods of SDDP Problems ............................................. 145

4 Partitions-Requirements-Matrices .............................................................................. 153
4.1 Arrangement of the Partitions .............................................................................. 155
4.2 Definitions of Partitions-Requirements-Matrices and Initial Properties and Results .............................................................. 163
  4.2.1 General Partitions-Requirements-Matrices .................................................. 164
  4.2.2 Partitions-Requirements-Matrices ................................................................ 166
  4.2.3 Partitions-Requirements-Matrices for Equivalent Lattices of Partitions ............ 173
4.3 The Computation of PRMs by means of Permutations of Perturbed Partitions .......... 176
4.4 Limits of Partitions-Requirements-Matrices ......................................................... 200
  4.4.1 Classification of the Lattices of the Restricted Partitions ............................... 201
  4.4.2 Limits of Partitions-Requirements-Matrices with regard to Sets of Sparse Partitions .................................................. 213
  4.4.3 Limits of Partitions-Requirements-Matrices with regard to Sets of Non-Truncated Heavy Partitions .... 218
4.5 Further Results from Elements of Partitions-Requirements-Matrices .......................................................... 224
4.5.1 Elements of PRMs in Dependence on Variables \( n \)
and \( k_0 \) .................................................................. 225
4.5.2 The Elements of the Last Row and the Last Column
of PRMs in the Case that \( n \geq su \) ......................... 249
4.6 Poisson Equations for PRMs and the
Monotonicity of their Solutions ........................................ 254
4.6.1 Poisson Equations for PRMs .............................. 256
4.6.2 Partial Results .................................................. 257
  4.6.2.1 PRMs for Sets of Restricted Partitions which
     are Equivalent with regard to the Poisson
     Equation ......................................................... 257
  4.6.2.2 The Dominance Condition and PRMs with
     regard to
     \( m \)-Totally Ordered Sets of Partitions ............ 259
  4.6.2.3 PRMs with regard to Sets of Restricted Par-
     titions with at most 4 Partitions ................. 262
4.6.3 The Poisson Equations with regard to Sets of Sparse
  Partitions with Sufficiently Large \( n \) ................. 264
4.6.4 The Poisson Equations with regard to Sets of
  Non-Truncated Heavy Partitions with Sufficiently Large
  \( n \) or \( k_0 \) ......................................................... 272
4.7 Conclusion for SDDP Problems with Identical Basic Cost and
  Independent and Identically Distributed Requirements .... 280

Final observations on SDDP problems 282
References 284
Glossary of Symbols and Abbreviations 287
Index 296