

Contents

Part I Background

1 Background	3
1.1 Introduction	3
1.2 Valued Constraint Satisfaction Problems	3
1.3 Complexity of VCSPs	7
1.3.1 Structural Restrictions	7
1.3.2 Language Restrictions	8
1.3.3 Hybrid Restrictions	15
1.3.4 Related Work	15
1.4 Expressibility	16
1.5 Algebraic Properties	20
1.6 Submodularity	25

Part II Expressive Power

2 Expressibility of Valued Constraints	31
2.1 Introduction	31
2.2 Results	31
2.3 Indicator Problem	31
2.4 Weighted Indicator Problem	35
2.5 Fractional Clones	40
2.6 Galois Theory	42
2.7 Summary	46
2.7.1 Related Work	47
2.7.2 Open Problems	47
3 Expressibility of Fixed-Arity Languages	49
3.1 Introduction	49
3.2 Results	49
3.3 Crisp Cost Functions	51
3.4 Finite-Valued Cost Functions	58

3.5	General-Valued Cost Functions	64
3.6	Characterisation of $\text{Mul}(\mathbf{F}_d^{\max})$ and $\text{fPol}(\mathbf{F}_d^{\max})$	73
3.7	Summary	77
3.7.1	Related Work	77
3.7.2	Open Problems	78
4	Expressibility of Submodular Languages	79
4.1	Introduction	79
4.2	Results	79
4.3	Preliminaries	81
4.4	Reduction to (s, t) -MIN-CUT	84
4.5	Known Classes of Expressible Functions	86
4.6	New Classes of Expressible Functions	89
4.7	Summary	93
4.7.1	Related Work	93
4.7.2	Open Problems	94
5	Non-expressibility of Submodular Languages	95
5.1	Introduction	95
5.2	Results	95
5.3	Expressibility of Upper Fans and Lower Fans	96
5.4	Characterisation of $\text{Mul}(\Gamma_{\text{sub},2})$ and $\text{fPol}(\Gamma_{\text{sub},2})$	101
5.5	Non-expressibility of Γ_{sub} over $\Gamma_{\text{sub},2}$	105
5.6	The Complexity of Recognising Expressible Functions	108
5.7	Summary	109
5.7.1	Applications to Artificial Intelligence	109
5.7.2	Applications to Computer Vision	112
5.7.3	Related Work	112
5.7.4	Open Problems	113
Part III Tractability		
6	Tractable Languages	117
6.1	Introduction	117
6.2	Tractable Languages	117
6.3	Boolean Languages	122
6.4	Summary	123
6.4.1	Related Work	124
6.4.2	Open Problems	124
7	Conservative Languages	125
7.1	Introduction	125
7.2	Conservative Languages	125
7.3	Graph Technique	126
7.4	Summary	128
7.4.1	Related Work	129
7.4.2	Open Problems	129

- 8 The Power of Linear Programming** 131
 - 8.1 Introduction 131
 - 8.2 Basic LP Relaxation 131
 - 8.3 Algorithmic Consequences 133
 - 8.4 Summary 135
 - 8.4.1 Related Work 135
 - 8.4.2 Open Problems 135

- 9 Hybrid Tractability** 137
 - 9.1 Introduction 137
 - 9.2 Tractable Triangles 137
 - 9.2.1 CSPs 140
 - 9.2.2 Max-CSPs 141
 - 9.2.3 Finite-Valued VCSPs 142
 - 9.2.4 General-Valued VCSPs 146
 - 9.3 Cross-Free Convexity 146
 - 9.4 Summary 148
 - 9.4.1 Related Work 148
 - 9.4.2 Open Problems 149

- 10 Summary and Open Problems** 151
 - 10.1 Summary 151
 - 10.2 Open Problems 151

- References** 153

- Index** 167