Comparison of Different Radiative Heat Transfer Models and their Applicability to Coal-Fired Utility Boiler Simulations

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Abstract - Thermal radiation is the dominant mode of heat transfer in coal-fired utility boilers. In this paper the implementation of six radiative heat transfer models - Monte Carlo model, discrete transfer model, discrete ordinates method, finite volume method, flux model and moment method - into the simulation program AIOLOS is presented to compare their accuracy as well as their computational economy and numerical stability. All six models are applied to two different test cases. In the first test case the program is validated simulating thermal radiation in an elongated box in which the computed radiation source-terms and wall heat fluxes are compared with the existing exact solution presented by Selçuk (1985). In the second case the code is applied to a 252 MW pulverised coal-fired utility boiler. The geometry is discretised on a Cartesian grid. A comparison of temperature profiles is done on the first burner level and at the top of the furnace for the radiation heat transfer models mentioned before.

Key Words: Radiative Heat Transfer, Mathematical Modelling, Coal-Fired Utility Boilers

NOMENCLATURE

A,B,C coefficients of the flux model
F cell face area
I intensity
Q radiation energy
S source-term
T temperature
V cell volume
k_a absorption coefficient
n unit vector normal to a wall
r position vector
s unit vector in direction of a ray
w weight factor
x,y,z Cartesian coordinates

Greek Symbols

Δ finite difference
Ω solid angle
α absorption
ε emission
μ, η, ξ direction cosines
σ Stephan-Boltzman constant

Subscripts

P cell centre
b black body
d downstream
f face
nb neighbours
u upstream
w wall
m outgoing directions
m’ incoming directions

INTRODUCTION

The equation governing radiative transfer is of an integro-differential type. Only a few numerical exact solution procedures exist for this equation, e.g. the zone method and the Monte Carlo method. These methods are excessively computing-time demanding and therefore currently not suitable for an incorporation in an economical simulation program for the prediction of fluid flow, chemical reactions and heat transfer. Approximate and rapid solution methods combining accuracy and computational economy are preferred especially for three-dimensional simulations. In the following, six of the most common approximate radiative heat transfer models are described and their computational results are compared for different test cases.

RADIATIVE HEAT TRANSFER

In contrast to the convective and diffusive transport of flow and combustion variables, thermal radiation is an electromagnetic phenomenon and in so far not bound to molecular transport. In a physical space discretised by the finite volume method, this means that each volume is not only in interaction with its direct neighbours but with all visible elements. As a rule, an analytical solution of the equation describing the process of radiation is not possible. However, numerous numerical methods describing the radiative heat transfer have been developed fulfilling the main aim of radiation modelling to compute a radiation source-term for the energy conservation equation and subsequently the temperature distribution in the furnace.

Radiation Transfer Equation

Assuming a grey medium and neglecting the change of radiation intensity due to in- and out-scattering the simplest form of the radiation transfer equation valid in furnaces can be written as follows:

\[
\frac{d}{d\xi}(I(\xi,\eta)) = k_\alpha(I_b(\xi) - I(\xi,\eta))
\]  

(1)

The left hand side of equation (1) describes the change of radiant intensity I at the location \( \xi \) along the path length ds in the direction of the vector s. \( I_b \) indicates the blackbody intensity. The intensity is changed by the terms on the right hand side of equation (1) representing an increase through emission and a decrease due to absorption.

Boundary Conditions

The boundary conditions of the radiation transfer equation define the intensity emerging from the enclosure of the radiation domain at the location \( \xi \) in the direction \( \eta \). Under the assumption of a grey, diffusive radiation of the surface the boundary condition of the radiation intensity is defined as follows:
\[ I(r_w) = \frac{\epsilon(r_w) \sigma T^4}{\pi r_w^2} + \frac{1 - \epsilon(r_w)}{\pi} \int_{\Omega} I(r_w, \Omega') \frac{d \Omega'}{\Omega'} \]  
(2)

The fact that rays do not perpendicularly incident onto the wall is taken into consideration on the right hand side of equation (2) by inserting the dot product \( \frac{d \Omega'}{\Omega'} \) into the integral. The integration over all solid angles \( d \Omega \) sums up to the area of a hemisphere \( 2 \pi \).

Radiation Source-Term

As mentioned before the principle intention of solving the radiation transfer equation is not to compute the intensity distribution in the considered domain but a source term \( S_{\text{radiation}} \) for the energy conservation equation

\[ S_{\text{radiation}} = \kappa_s \left( \int 1 d \Omega - 4 \sigma T^4 \right) \]  
(3)

which results from integrating the radiation transfer equation over all solid angles \( d \Omega \).

**NUMERICAL METHODS**

In general there are two different kinds of approximate radiation models: ray-tracing and differential methods. The essential difference of these two groups are the directions in which the transport equations are formulated.

**Differential Methods**

The differential methods are tools to transform the equation of radiation transfer into a set of partial differential equations which can be formulated in the coordinate system of the fluid flow and combustion transport equations:

\[ \frac{\partial I}{\partial x} + \frac{\partial}{\partial y} \left( \frac{\mu I}{\kappa} \right) + \frac{\partial}{\partial z} \left( \frac{\mu I}{\kappa} \right) = \kappa_s (I_b - I) \]  
(4)

where \( \mu, \eta, \text{ and } \zeta \) are the direction cosines in the \( x-, y-, \text{ and } z \)-direction of a Cartesian coordinate system. After discretising this equation with the finite-volume method fast iterative sparse matrix solvers can be used to compute the equations, and the code for the differential radiation models can effectively be vectorised without significant increase of memory.

**Moment method**

The moment method is based on a Milne-Eddington approximation to the equation of radiation transfer in which a Taylor series expansion, truncated after the terms of first order, is used for the intensity:

\[ I(r, \Omega) = I_0(\Omega) + \mu I_x(\Omega) + \eta I_y(\Omega) + \zeta I_z(\Omega) \]  
(5)

Substituting equation (5) into the radiation transfer equation a partial differential equation can be found (Ozisik, 1973):

\[ \frac{\partial}{\partial x} \left( \frac{1}{\kappa z} \frac{\partial I}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{\kappa y} \frac{\partial I}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{1}{\kappa z} \frac{\partial I}{\partial z} \right) = 3 \kappa_s (I_0 - I_b) \]  
(6)

The source term of the energy conservation can be computed from the intensity distribution as follows:

\[ S_{\text{radiation}} = k_s V \left( \int 1 d \Omega - 4 \sigma T^4 \right) = 4 k_s V \left( \pi I_0 - \sigma T^4 \right) \]  
(7)

The conservation of radiation energy applied to a wall gives:

\[ Q_r = Q_e + Q_w \]  
(8)

in which \( Q_r \) is the net radiative flux towards and normally to the surface, \( Q_e \) is the flux emitted by the wall and \( Q_w \) is the flux absorbed by the wall. The quantities \( Q_r, Q_e, \text{ and } Q_w \) are given by:

\[ Q_r = \int \frac{I(s) \Omega}{4 \pi} d \Omega \]  
(9)

\[ Q_e = \epsilon W \sigma T^4 \]  
(10)

\[ Q_w = \alpha_w \int I(\Omega) \frac{\Omega}{4 \pi} d \Omega \]  
(11)

A partial differential equation of first order for the intensity \( I_0 \) is yielded substituting equations (9)-(11) into equation (8) and integrating over the solid angle:

\[ \epsilon I_0 \pm \frac{2(1-\epsilon) \partial I_0}{3 \kappa_s \sigma T^4} = \epsilon \sigma T^4 \]  
(12)

The signs (-) and (+) correspond to the positive and negative coordinate directions respectively.

An identical set of equations for intensity, boundary conditions and radiation source-term found for the moment method can be derived by the spherical harmonics method applying the P1-approximation and the Marshak boundary conditions (Koch, 1992).

**Flux model**

The flux model used is based on a Schuster-Schwarzschild approximation and was proposed by De Marco and Lockwood (1975). Similar to the moment method the intensity is represented by a truncated Taylor series expansion which is done for the intensity:

\[ I(r, \Omega) = \mu A_x(\Omega) + \mu B_y(\Omega) + \mu C_z(\Omega) + \eta A_y(\Omega) + \eta B_z(\Omega) + \zeta B_x(\Omega) \]  
(13)

The coefficients \( A_i, B_i \) are related to the intensities in the directions of coordinate axes,

\[ A_i = \frac{I_{i\text{pos}} - I_{i\text{neg}}}{2} \]  
(14)

in which \( I_{i\text{pos}} \) and \( I_{i\text{neg}} \) represent the intensities in the positive and negative directions of the subscripted coordinate. The intensity distribution equation (13) is substituted into the transfer equation (4) and some mathematical operations have to be done (De Marco and Lockwood, 1975) to yield three coupled second order equations for the quantities \( C_x, C_y, \text{ and } C_z \). The equation for \( C_x \) reads:

\[ \frac{d}{dx} \left( k_s C_x \right) = 3 k_s C_x - k_s (\sigma T^4 + C_y + C_z) \]  
(15)

\[ C_y = \frac{\pi}{2} \left( B_x + \frac{1}{2} B_y \right) \]  
(16)

\[ C_z = \frac{\pi}{2} \left( B_x + \frac{1}{2} B_z \right) \]  
(17)

Each heat flux \( C_i \) is dependent on the intensities in all six directions and therefore this model is named a coupled flux model. In simpler flux models this coupling is not guaranteed leading to non-physical coupling of heat flux and coordinate directions.

The source-term for the energy conservation may be written as:

\[ S_{\text{radiation}} = 4 k_s V \left( \frac{1}{2} \sum_{i=x} C_i - \sigma T^4 \right) \]  
(19)

The boundary conditions are determined in the same way as for the moment method and can be formulated for the \( x \)-direction as follows:

\[ \epsilon C_x = \frac{2(1-\epsilon) \partial C_x}{3 k_s \sigma T^4} = \epsilon \sigma T^4 \]  
(20)

**Discrete ordinates method**

In the discrete ordinates method the radiative transport equation is solved for a set of \( m \) discrete directions. Each direction \( \omega_m \) is associated with a solid angle in which the intensity is assumed to be constant. All solid angles are non-overlapping and spanning the total angle range of \( 4 \pi \). The integrals over the direction are replaced by numerical quadrature weights \( w_m \) summed over each ordinate. Thus the radiative transport equation can be approximated by a set of \( m \) equations:

\[ \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \left( \frac{\mu I}{\kappa} \right) + \frac{\partial}{\partial z} \left( \frac{\mu I}{\kappa} \right) = \kappa_s (I_b - I_m) \]  
(21)
The differential equation (21) is solved for a control volume $P$ by integrating over the volume,

$$\sum_{f=1}^{6} I_{m,f}(\xi_m \cdot \vec{q}) F_f = k_a(I_{b,P} - I_{m,P})V$$  \hspace{1cm} (22)$$

where $I_{m,f}$ are the intensities on the surfaces $F_f$ of the control volume. In order to reduce the number of unknowns in equation (22) the intensity at the cell-center $I_{m,P}$ can be expressed as an interpolation between the intensities at two opposite faces upstream and downstream:

$$I_{m,P} = \omega I_{m,n_u} + (1 - \omega) I_{m,u}$$ \hspace{1cm} (23)$$

where $\omega$ is a grid dependent weight factor. When the upstream intensities $I_{m,n_u}$ are assumed to be known the cell-centre intensities can be calculated recursively as a function of the upstream values. The intensities at the downstream faces $I_{m,d}$ are computed with equation (23).

The quadrature weights $w_{m}$ are determined by an $S_4$-approximation, in which the number of ordinates is calculated by $m=n(n+2)$. Each direction $s_m$ is fixed by its direction cosines $I_{b,n_u}$. Table I presents a set of positive direction cosines together with the corresponding weights which are summarised for the $S_4$-approximation (Fiveland, 1988):

<table>
<thead>
<tr>
<th>Approximation</th>
<th>$\mu$</th>
<th>$\eta$</th>
<th>$\xi$</th>
<th>$w$</th>
</tr>
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<tr>
<td>$S_4$</td>
<td>0.2958758</td>
<td>0.2958758</td>
<td>0.9082483</td>
<td>0.5235987</td>
</tr>
<tr>
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<td>0.2958758</td>
<td>0.5235987</td>
</tr>
</tbody>
</table>

Table I: Direction cosines and weights factors of the first quadrant

The source-term of the energy conservation equation can be computed from the difference of the intensities in all directions multiplied by the weights $w_{m}$ at the cell centres and the emissivity:

$$S_{\text{radiation}} = k_a V \left( \sum_{m=1}^{n(n+2)} I_{m,w_m} \Delta s_m - 4\sigma T^4 \right)$$ \hspace{1cm} (24)$$

By analogy to the moment method and the flux model, the intensities at the boundaries result from the conservation of energy at the wall:

$$I_m = \frac{\sigma T^4}{\pi} + \frac{(1 - \varepsilon)}{\pi} \sum_{m=1}^{n(n+2)} w_m I_m \Delta s_m$$ \hspace{1cm} (25)$$

Finite volume method

By analogy to the discrete ordinates method the radiative transport equation is solved for a set of $m$ discrete directions $s_m$ and the intensities are assumed to be constant over one solid angle $\Delta s_m$. The intensities at the cell centres are calculated exclusively as a function of their upstream neighbouring cells. However, except for this analogy, there are two characteristic differences between these methods. One difference is that for the finite volume method not the numerical quadrature is used to compute the integral differences between these methods. One difference is that for the determination of the intensity at the control volume can be expressed as a function of its upstream values. The determination of point $uf$ is illustrated in Figure 1:

![Figure 1: Geometry of the finite volume method](image)

After the intensities are determined the source-term of the energy conservation equation can be computed at a cell centre from the difference of the intensities and the emissivity:

$$S_{\text{radiation}} = k_a V \left( \int_\Omega I_{m,f} \, d\Omega - 4\sigma T^4 \right)$$ \hspace{1cm} (29)$$

The boundary conditions for the intensities are calculated by the incoming radiant flux:

$$I_m = \frac{\sigma T^4}{\pi} + \frac{(1 - \varepsilon)}{\pi} \sum_{m=1}^{n(n+2)} Q_m$$ \hspace{1cm} (30)$$

Ray-Tracing Models

For ray-tracing radiation models one-dimensional equations along a multitude of individual rays through the combustion chamber are solved. The formulation of the equations is made straight, arbitrarily oriented rays, which are not inevitably aligned with the coordinate system of the fluid flow:

$$\frac{dl}{ds} = k_a(I_{b,P} - I)$$ \hspace{1cm} (31)$$

Determining all traces of the rays and cells located along its way affords either a lot of computational time or a lot of memory. In addition, the vectorisation and parallelisation of the algorithms of ray-tracing radiation models is limited due to the different lengths of the individual rays.

Semi-stochastic Monte Carlo model

In the pure version of the Monte Carlo model the coordinates of the starting points as well as the directions of the rays are generated by random numbers in the whole computational domain. In contrast, a fixed number of rays are started in the geometrical center of the cells using a fixed angular distribution in this semi-stochastic Monte Carlo model (Richter and Heap, 1981). This can be done if the cells are small enough to assume that radiation is emitted almost isotropically over the solid angle. Each ray is traced from its starting point until it leaves the computational domain or its extinction. The absorbed energy per cell is computed summing over all rays according to:

$$Q_{abs} = \sum_{rays}(Q_\text{abs} - Q_{n+1}) = \sum_{rays} Q_{abs}(1 - e^{-k_A\Delta s})$$ \hspace{1cm} (32)$$

The source-term of the energy conservation equation of one cell is the difference between absorbed and emitted energy:

$$S_{\text{radiation}} = Q_{abs} - 4\sigma k_A T^4 V$$ \hspace{1cm} (33)$$

The reflection of rays at the wall in a direction computed by random numbers follows the usual Monte Carlo approach for an ideal diffusive reflector. From the incident energy flux of the ray the wall absorbs the amount of energy

$$Q_{abs, w} = \varepsilon Q_n$$ \hspace{1cm} (34)$$

and the rest of the energy is reflected according to:

$$Q_{n+1} = (1 - \varepsilon)Q_n$$ \hspace{1cm} (35)$$
The discrete transfer model (Lockwood and Shah, 1976) is built on the concept of solving the radiation transfer equation for representative rays in the combustor and to this extent it is related to the Monte Carlo model. But in contrast to the Monte Carlo model the directions of the rays are specified and not randomly chosen. The rays are started only on the boundary surface and are solved only along paths between two boundary walls and are not individually reflected at walls and traced to extinction. An equation for the intensity results from the integration of the radiation transfer equation along the distance in the direction of the ray looked at:

\[ I_{n+1} = I_n e^{-k \Delta x} + I_k (1 - e^{-k \Delta x}) \]  

(36)

The intensity balance per cell and ray is given in the following equation:

\[ I_{\text{netto}} = I_n - I_{n+1} = I_k (1 - e^{-k \Delta x}) + I_k (1 - e^{-k \Delta x}) \]  

(37)

The radiation source-term is the sum of all intensity changes caused by all rays passing through a cell.

\[ S_{\text{radiation}} = \sum_{\text{rays}} (I_{\text{netto}} \Delta \Omega F) \]  

(38)

For all rays starting only at the walls of the computational domain an intensity at the boundary is defined

\[ I_0 = (1 - \varepsilon) \sum_{\text{rays}} \left( I_k \xi < 0 \frac{\Delta \Omega}{\pi} \right) + \varepsilon \frac{\sigma}{\pi} T^4 \]  

(39)

and therefore they are dependent on the intensities towards the boundary surface. This explicit treatment of the intensities at walls leads to a slightly slower convergence in comparison to the Monte Carlo model.

FLUID FLOW AND COAL COMBUSTION

The radiation heat transfer models are embedded in a method for predicting steady, three-dimensional, turbulent reacting flows in coal-fired utility boilers. Submodels for fluid flow, turbulence and combustion are included in the program named AIOLOS alongside the heat radiation. For this purpose equations for the conservation of mass and momentum are solved. Coupling between velocity and pressure is achieved by the SIMPLEx method (Van Doormaal and Raithby, 1984). The turbulence closure is done by a standard k-\varepsilon model. Coal combustion is described by a four-step reaction scheme. The model covers two heterogeneous reactions for coal pyrolysis and char combustion and two gas-phase reactions for the oxidation of volatile matter (hydrocarbons and carbon monoxide). The gaseous reactions are assumed to be mixing-limited, which is modelled using an Eddy-Dissipation Concept (Magnussen and Hjertager, 1976). The procedure is based on a fully conservative, structured finite-volume formulation employing Cartesian vector and tensor components. The discretisation can be carried out on control volumes defined by a Cartesian grid using a non-staggered variable arrangement (Rhie and Chow, 1983). A more detailed description of the simulation program AIOLOS can be found in Schnell et al. (1995).

RADIATION HEAT TRANSFER IN AN IDEALISED SQUARED COMBUSTION CHAMBER

The geometry of the combustion chamber and the boundary conditions approximate a strongly idealised, horizontally fired test facility. The combustion chamber has a squared cross-section, cooled side walls and uncooled rear and front walls. A burner is located in the centre of the front wall and therefore an axisymmetric temperature distribution can be assumed. The computations are based on a given constant temperature distribution. No conservation equations for mass, momentum and energy are solved and therefore the influence of a model other than the radiation model can be excluded. The emissivity of all walls is set to \( e = 1 \), which means that they behave like a black body. In Figure 2 a 90° sector of the combustion chamber cross section is pictured showing the grid used and the areas where the computational results are opposed to the analytic solution along the positive z-coordinate direction. The geometry of this test case and the boundary conditions are described in detail by Selçuk (1985).
Irrespective of the moment method and the discrete transfer model the computed source-terms of most of the models agree quite well with the reference. The moment method and the discrete transfer model show the greatest differences for the average deviation as well as for the maximum deviation (Table II). Furthermore, the source-terms of the Monte Carlo model and the finite volume method differ from the reference directly in front of the front wall.

<table>
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<tr>
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<td>average deviation</td>
<td>1.3</td>
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<td>1.5</td>
<td>0.8</td>
<td>2.2</td>
<td>8.2</td>
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<td>maximum deviation</td>
<td>7.8</td>
<td>9.9</td>
<td>10.5</td>
<td>4.4</td>
<td>5.5</td>
<td>14.4</td>
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Table II: Deviation of the computed source-terms from the reference

With regard to the wall heat fluxes the differences between the reference and the solution computed by the radiation models are more visible. Especially in the front of the combustion chamber, where the highest gradients due to high temperatures appear, all models have difficulties in predicting the test conditions.

Least of all the moment method can predict the maximum of the heat flux profile. One of the reasons is the description of the intensity distribution by only one equation, so that inhomogeneity of the intensity distribution is not registered. The flux model also fails to predict the wall heat fluxes more exactly although it should be able to reproduce the inhomogeneity. The main problem of both models is the formulation of the boundary conditions for the intensities only perpendicular to the wall surfaces.

The ray tracing models, namely the Monte Carlo model and the discrete transfer model, also show distinct differences from the reference. This is a result of the ray effect, which is a principal weakness of the pre-determined distribution of the solid angles. Because information can only be spread along the traces of rays and only a fixed number of directions is defined, cells which are not affected by a particular ray can not get any information from it. This effect can cause an irregular heat flux and source-term distribution in the computational domain. For the Monte Carlo model further investigation is done later in this chapter.

The ray effect was first mentioned by Lathrop (1968) in connection with the discrete ordinates method. However, the sensitivity of the discrete ordinates method seems to be weaker than for the ray tracing models using the same number of rays.

For this test case and a restriction to 24 directions of rays, the solutions of the finite volume and discrete ordinates methods lead to the best agreement with the exact solution. Maximum and average deviation of the six models are summarised in Table III.

<table>
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<td>3.6</td>
<td>2.3</td>
<td>1.2</td>
<td>6.3</td>
<td>4.7</td>
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<tr>
<td>maximum deviation</td>
<td>9.4</td>
<td>6.0</td>
<td>3.7</td>
<td>3.8</td>
<td>34.7</td>
<td>20.8</td>
</tr>
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</table>

Table III: Deviation of the computed wall heat fluxes from the reference

Besides the exactness of the radiation models, their computational effort with respect to time and memory has to be considered especially if a coupled simulation of fluid flow, combustion and radiation heat transfer is intended. The memory requirement is shown in Table IV in which the additional number of arrays with the dimension of the number of cells is specified. For the finite volume and discrete ordinates methods, the values are referred to the $S_4$-approximation.

<table>
<thead>
<tr>
<th></th>
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<td>additional arrays</td>
<td>4</td>
<td>6</td>
<td>45</td>
<td>33</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Table IV: Memory requirement of the radiation models

These additional requirements of memory for the radiation models have to be seen in relation to the approximately 80 arrays for a computation of fluid flow and combustion, which is quite high especially for the finite volume and discrete ordinates methods. Due to the dropping prices of memory the available memory on computers will no longer be the limiting factor for the application of a model.

More important than the memory requirement is the computational time of the radiation models depending on the time per iteration and the number of iterations until a converged solution is achieved. A comparison of the performance of the different models has been worked out on a DEC alpha 3000/400 scalar computer and a CRAY C94 vector computer (Table V). The solution is assumed to be converged when no further change of the averaged source-term appears.

<table>
<thead>
<tr>
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<th>iterations until convergence</th>
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<th>CRAY C94</th>
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<td>CPU-seconds per iteration</td>
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<td>CPU-sec. until convergence</td>
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<td>3.18</td>
<td>11.0</td>
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</table>

Table V: Computational time of the radiation models

The different solution behaviour of the radiation models is shown by the number of iterations, which is needed to get a converged solution. For the ray tracing models a converged solution is yielded after only one iteration if the temperature distribution is given and the emissivity is set to $\varepsilon=1$. The flux model and the moment method need a very high number of iterations and therefore the computational effort is high although the numerical effort per iteration is low. Both models can be fully vectorised which explains the reduction of the computational time for vector computers in comparison to the scalar computer.

The discrete ordinates and finite volume methods can not be fully vectorised because the solution algorithm is partly recursive. So the speed up on vector computers for these models is not as good as for the flux model and the moment method depending on their degree of vectorisation.

The degree of vectorisation for the Monte Carlo model and the discrete transfer model is inferior. Thus the fast vector processors of the C94 computer can not be used effectively by these methods. However, the ray tracing models can be vectorised if the drawback of an extreme increase of memory is accepted (Brown and Martin, 1984). The computational time for the Monte Carlo model is by far the highest of all models although there is no reflection of the rays at the wall ($\varepsilon=1$).

The radiation boundary conditions are a combination of emission, which depends on the wall temperature, and the reflection of the incident radiation. The emission is constant for a given wall temperature and is in contrast to the reflected part, not dependent on the iteration process. Besides the physical interpretation, the emissivity $\varepsilon$ can also be seen as a weight factor between the fixed and variable part of the boundary conditions.

The influence of the boundary conditions onto the convergence behaviour has been investigated varying the emissivity coefficient between the values $\varepsilon=0$ and $\varepsilon=1$ (Table VI). These two extreme values for $\varepsilon$ stand for boundary conditions completely depending on the iteration process and constant boundary conditions respectively.

<table>
<thead>
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<th>$\varepsilon$</th>
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<td>2700</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>2100</td>
<td>1700</td>
</tr>
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</table>

Table VI: Convergence behaviour as a function of the wall emissivity

As expected the number of iterations for all models except the Monte Carlo model is increased to yield a converged solution when the emissivity of the walls is decreased because the boundary conditions become more explicit. For the Monte Carlo model the solution is independent from the emissivity coefficient yielded after one iteration. This behaviour results from the individual reflection of each ray, so emission and absorption can be decoupled in the algorithm. The length of each ray is extended for $\varepsilon=1$ because the rays are reflected at the walls and therefore the computational time per iteration is increased. For the other models the time per iteration is maintained constant.
The more favourable convergence behaviour of the discrete transfer model in comparison to the finite volume and discrete ordinates methods is the result of the one-dimensional solution of the radiation transfer equation along individual rays. Both models mentioned last, having about the same rate of convergence, are based on a three-dimensional balance over the cell faces of the control volumes and therefore the influence of three directions has to be taken into consideration, which slightly slows down convergence.

An additional influence on the convergence behaviour of a radiation model is the absorption coefficient $k_a$ of the fluid in the combustion chamber. The influence of the absorption coefficient on the convergence can be demonstrated by the intensity equation of the discrete transfer method.

The intensity $I_{n+1}$ emerging from a cell depends on the incident intensity $I_n$ and the intensity of a black body $I_{b}$ weighted by the factors $e^{-k_a s}$ and $(1 - e^{-k_a s})$, respectively. For a given $\Delta s$ and an infinitely great absorption coefficient, which means an infinite optical thickness, the whole energy incident in the cell is absorbed. Therefore the emerged intensity is dependent on the emission of the cell only, so that the influence of local changes to the surroundings is small. Accordingly the rate of convergence is increased for higher optical thickness. The influence of the variation of the absorption coefficient from 0.035 to 35 m$^{-1}$ is shown in Table VII ($\varepsilon = 0.7$):

<table>
<thead>
<tr>
<th>$k_a$</th>
<th>MC</th>
<th>DT</th>
<th>FV</th>
<th>DO</th>
<th>FS</th>
<th>MM</th>
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<td>0.035</td>
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<td>12</td>
<td>16</td>
<td>&gt;7500</td>
<td>&gt;7500</td>
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<tr>
<td>0.35</td>
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<td>1</td>
<td>3</td>
<td>7</td>
<td>7</td>
<td>97</td>
<td>103</td>
</tr>
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<td>1</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>22</td>
<td>8</td>
</tr>
</tbody>
</table>

Table VII: Convergence behaviour in dependency on the absorption coefficient

It is interesting to note that very high absorption coefficients have a favourable effect on the convergence of the flux model and the moment method. However, for practical applications this is not of great benefit because typical values of $k_a$ in combustion chambers are between 0 and 5.

The convergence of the Monte Carlo model does not depend on the absorption coefficient but the computational time per iteration is decreased for an increased $k_a$ because the distance to extinction of the rays is shortened. The computational effort per iteration for the other models is independent from the absorption coefficient.

Finally some remarks for the computation of this idealised test case using the Monte Carlo model are made, which are also valid for the discrete transfer model with certain reservations. Using a division of the solid angle into 24 segments for the Monte Carlo model, the axial profiles of the wall heat fluxes shown in Figure 4 are not in good correspondence with the reference. For the semi-stochastic Monte Carlo model used, only the angle of reflection to the wall is determined by random numbers. In this test case it is assumed that there is no reflection from the walls, so the stochastic characteristic of this model is totally lost and it is no longer free from the ray effect. The appearance of the ray effect is due to inhomogeneity of the boundary conditions or of the properties of the combustion chamber, which certainly occurs in combustion chambers caused by the high temperature gradients. A remedy for this problem is to use a greater number of rays leading to a finer division of the solid angle.

Based on these considerations, the obvious way to try to get a better agreement of the wall heat fluxes computed with the Monte Carlo model and the reference is achieved by increasing the number of rays started in each cell. This was attempted for 8, 24, 56, 96 and 200 rays per radiant volume. The computational effort increases approximately linear with the number of rays. Because already 24 rays lead to very high computing times, a further division of the solid angle for a practical application does not seem to be feasible. The purpose of this investigation was to determine whether or not the semi-stochastic Monte Carlo model used is suitable to provide a reference radiation model for judging the exactness of other models. For the Selçuk test case 24 rays are definitely not enough to ensure that the solution of the model and the analytic solution are in such good agreement that it can be considered as a reference solution. The findings of the investigations using a higher number of rays are shown in Figures 5 and 6 and Tables VIII and IX. It is obvious that the agreement between the exact reference and the numerical computations is improved by increasing the number of rays. However, in order to achieve both very small average and maximum deviation, a very fine subdivision of the solid angle is necessary.
In the following, a coupled computation of fluid flow, combustion and radiation in the furnace of a coal-fired utility boiler is done to judge the presented radiation models under the conditions of a real application. The furnace under investigation has a height of 29 m and a cross section of approximately 8m in square. The power station has a steam output of 306 t/h (Größterlinden, 1995) and the combustion is initiated by 6 burners which are arranged at the front wall on three levels. The burners (Figure 7) are an optimised version of the mixed staging burner concept developed by Steinmüller (Thielen, 1988). The swirled secondary air of the two burners on each level is rotating in opposite directions and the burnout of the coal is ensured by admixture of the tertiary air entering the furnace through four jets arranged around each burner. For the comparison of the radiation models neither the boundary conditions nor the parameters of each model were changed to ensure the comparability of the different models. The emissivity of the walls (\(\varepsilon = 0.65\)), the wall temperature (\(T = 943\) K) and the fluid absorption coefficient \(k_a = 1.1\) m\(^{-1}\) are assumed to be invariant in space. Each simulation takes 4500 iterations until a converged solution is achieved. Therefore the radiation source-terms for the flux model and the moment method have to be computed for each iteration of the Monte Carlo model as a reference the agreement of the differences of the wall heat fluxes for the radiation models are small. In comparison to the Monte Carlo model the deviation is between -0.6% for the flux model and +2.7% for the moment method. The results of all models indicate to be satisfactory in comparison to the reference computed by the Monte Carlo model. Certainly the turbulence and combustion models imply more potential uncertainties than each of the investigated radiation models. The computational time which is needed to achieve a converged solution is summarised in Table XI. The computational time is drastically decreased in comparison to the Monte Carlo model, the temperature profile having significantly higher fluctuations than all other models. Rays are only started at the boundaries of the computational domain and are not reflected at the walls for this model. The number of information paths along which the information is spread out is drastically decreased in comparison to the Monte Carlo model and therefore it has to be concluded that the occurrence of the ray effect is strengthened.

Apart from the temperature distribution in the furnace, the heat fluxes along the walls are of great interest. In Table X the wall heat fluxes computed by the different models are compared. The wall heat fluxes over the rear wall opposite to the burners and the heat fluxes summed over all walls of the furnace are indicated in MW. The convective heat transfer is considered in the simulation but not embodied in the values in Table X. The differences of the wall heat fluxes for the radiation models are small. In comparison to the Monte Carlo model the deviation is between -0.6% for the flux model and +2.7% for the moment method. The results of all models indicate to be satisfactory in comparison to the reference computed by the Monte Carlo model. Certainly the turbulence and combustion models imply more potential uncertainties than each of the investigated radiation models. The computational time which is needed to achieve a converged solution is summarised in Table XI. The computational time is

<table>
<thead>
<tr>
<th></th>
<th>MC</th>
<th>DT</th>
<th>FV</th>
<th>DO</th>
<th>FS</th>
<th>MM</th>
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<td>rear wall [MW]</td>
<td>30.4</td>
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<td>31.5</td>
<td>30.2</td>
<td>30.9</td>
</tr>
<tr>
<td>furnace [MW]</td>
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<td>128.8</td>
<td>129.1</td>
<td>129.6</td>
<td>128.6</td>
<td>132.9</td>
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</table>

Table X: Radiation wall heat flux

<table>
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<th>FV</th>
<th>DO</th>
<th>FS</th>
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</thead>
<tbody>
<tr>
<td>DEC alpha 3000/400</td>
<td>1.0</td>
<td>1.93</td>
<td>1.07</td>
<td>1.24</td>
<td>1.02</td>
<td>1.11</td>
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<td></td>
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<tr>
<td>CRAY C94</td>
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<td>1.05</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
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</tbody>
</table>

Table XI: Computational time of the radiation models related to a simulation of fluid flow and combustion without radiation
given in relation to a computation of fluid flow and combustion without radiation, with the absolute time for the simulation additionally indicated. Values are given for a DEC 3000/4000 scalar computer as well as for a CRAY C94 vector computer. The CRAY C94 computer has four processors whereas only one is used for the computations. With the exception of the ray effect in Monte Carlo model the additional computational effort for the radiative heat transfer models is low in relation to the models for fluid flow and combustion.

To judge the computational time for the different radiation models on the vector computer, it has to be kept in mind that the values are related to a computation of fluid flow and combustion on the same computer type. The comparison of the computational time needed in comparison to the scalar machine shows that only the degree of vectorisation of the focused radiation model is not as high as for the fluid flow and combustion program modules. This occurs especially for ray-tracking models but also for the discrete ordinates method.

The comparison of the computational time needed for a pure computation of radiation as shown in Table V and the values given in Table XI is very interesting. It shows a stronger increase of the computational time for the Monte Carlo model than for the other models. The extension of the rays which is caused by the reflection on the walls. An additional basic difference of the contents of Table V and Table XI is the unexpected low computational time for the moment method and the flux model in the case of a coupled simulation. One reason is that the convergence is not strongly affected by the absorption coefficient and the reflectivity as shown in Table VI and Table VII. Another reason is that for a coupled computation the convergence of the radiation model does not play the dominant part of the overall system. It is more important to ensure the convergence of the whole system consisting of fluid flow, combustion and radiation and therefore the interactions between all these program modules.

Only 24 rays were used for the simulation of the furnace with the semi-stochastic Monte Carlo model. This is contradictory to the results found for the Selçuk test case in which a division of the solid angle into more than 144 directions is needed to ensure the reference character of the Monte Carlo model. However, computations with different numbers of rays were done for the coal-fired utility boiler and it was found that the difference to a computation with 200 rays is below 1%.

Therefore it is possible to use the results of the Monte Carlo model for judging the exactness of the other radiation models. Contrary to the Selçuk test case one reason for the different number of rays needed is that the angle in which the rays are reflected is computed by random numbers and therefore sufficient spread of information is guaranteed by 24 rays. A second reason is the smaller ratio of length to width of the utility boiler by the factor two in comparison to the Selçuk test case. The ray effect describes the insufficient illumination of the computational domain. A sufficient illumination becomes more difficult the more the ratio of length to width of the geometry is increased. Therefore the computation of the Selçuk test case is affected more seriously by the ray effect than the furnace of the utility boiler.

CONCLUSIONS

Six of the most common numerical radiation models were compared considering their accuracy, numerical stability, memory requirement and need for computational time. The comparison is based on computations of a test case which approximates a strongly idealised, horizontally fired test facility and of a coal-fired utility boiler. The main results are summarised in the following:

- The ray tracing models (Monte Carlo model, discrete transfer model) in the non-vectorised form are not effectively suitable on vector computers. Their vectorisation is coupled with a considerable additional effort of memory and therefore it is no longer possible to run the vectorised type of these models on workstations.
- The Monte Carlo model needs a lot of computational time and therefore it can not be recommended to be used for the simulation of full scale boilers. However, the potential of this model to aim at a very high accuracy allows to use the solution for the validation of other radiation models.
- For the discrete transfer model, rays are only started on the boundaries of the computational domain. For this reason the computational time is reduced in comparison to the Monte Carlo model. Because the illumination of the furnace is not guaranteed, the occurrence of the ray effect can strongly affect the accuracy of the solution. On account of the low memory and computational requirements, this model is of some importance especially on scalar computers.
- The solution of the moment method and the flux model for the real application are in good agreement with the reference computed by the Monte Carlo model. For the Selçuk test case the results are not satisfactory and therefore the application of these models bears uncertainties. The convergence behaviour of both models is very poor but for the reason of the low computational effort and the possibility of full vectorisation the models are superior regarding the computational time.
- The results computed by the finite volume method differ only slightly from the results of the discrete ordinates method but the computational time and the memory effort are drastically increased. The algorithm can be vectorised to a high degree and therefore the model can effectively be used on vector computers.
- The results of the discrete ordinates method yield a good agreement with the reference solutions for both test cases. Drawbacks of this method are the high memory requirement and the possible occurrence of the ray effect which nevertheless seems to be not significantly developed. The algorithm can be vectorised to a high degree and therefore the model can effectively be used on vector computers.

REFERENCES


