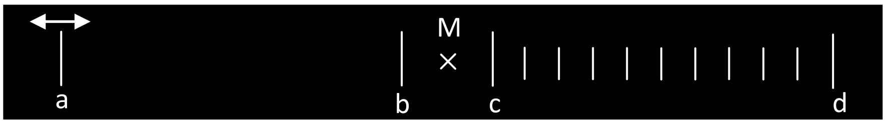
On the intensity maximum of the Oppel-Kundt illusion



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1. Illusion triggered by a gradually filled space

In the Oppel-Kundt illusion, a linear space of horizontal extent is gradually filled with vertical strokes. It turns out that the apparent length of the space first increases with the number of filling elements, but then, after a maximum, decreases again. (Oppel, 1854; Kundt, 1863; Spiegel, 1937; Surkis, 2007; 2008). In the following, the illusion is interpreted as a consequence of the size constancy effect. An algebraic expression is derived and fitted to the experimental results.

The experiments performed by Spiegel (1937) were carried out in the dark. Slits were cut into a piece of black cardboard and illuminated from the back (Fig. 1). The distance *cd* between the long slits was fixed in length. It was filled successively with up to x=47 short vertical slits. In the basic experiments they were equally spaced. The position of slit *a* was variable. The subjects compared the two distances (*cd* vs. *ab*) and indicated as soon as they judged them to be equal.

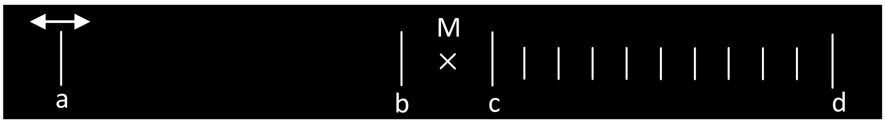

Fig. 1 The distance between the long slits on the left (*ab*) appears to be shorter than on the right (*cd*). Arrangement of slits in the first four experiments. In the fifth experiment the filled space was on the left.

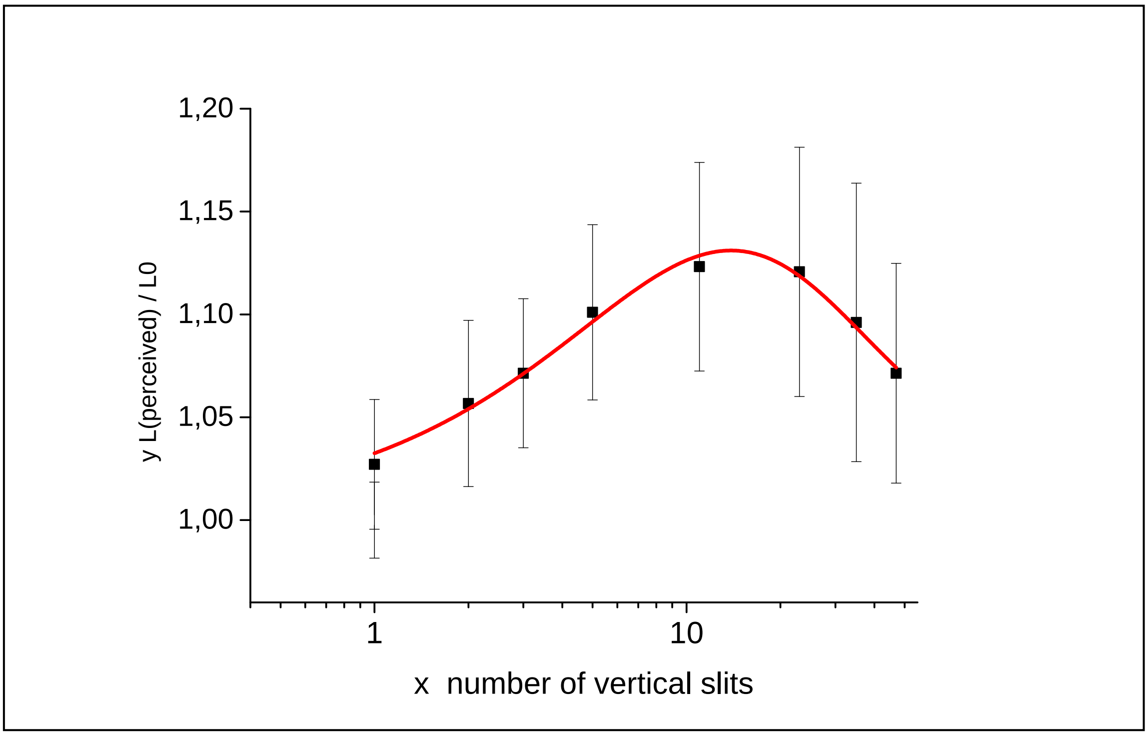

Fig. 2. Perceived length of a distance of horizontal extent (L0=400 mm) as a function of the number of slits. The increase at low x is interpreted as due to the attempt of the visual system to improve resolution while the decrease at high x indicates that the filling structure is gradually regarded as a uniform pattern. Distance of observation was 2.7 m.

In the first 5 experiments *cd* measured 400 mm, while the distance of observation was varied between 1.65m and 5.90m. The length of *cd* was always overestimated. Fig. 2 gives an example. The striking feature is the maximum between x=11 and x=23. Because the undivided distance appears always shorter, the author claims kind of a force which pushes the long slits limiting the empty space towards each other while the shorter slits in between are believed to exert some kind of resistance. This, so the author, works as long as the slits can be clearly separated from each other. In case a large number of slits combines to a continuous bright ribbon, there will be no resistance any more. A similar arguments holds, when the slits get pretty dense, giving the impression of a uniform texture.

Here, it is assumed that increasing structural density (in the sense of the number of geometric features per unit length) leads to increasing perceptual magnification. This is based on the conceptual model of size constancy: Due to its limited channel capacity, the visual system can process a certain amount of information per unit time only. If it were a technical electronic device one would say it could handle a certain number of pixels per second only. As a consequence, only a limited section of the retinal image is processed and projected on to kind of an internal "visual memory screen", finally resulting in the perceived image. The memory screen just illustrates the assumption that the visual system always employs its full data processing and storage capacity, irrespective whether the pictorial information is retrieved from a large or a small retinal section. In order to resolve fine structure, the size of this angle of attention (or conspicuity range) has to be narrowed down, leading to enlargement of the perceived details. Several authors come to the conclusion that the neural mechanism of top-down modulation serves as a common framework for selective attention processes (see for example Beck & Kastner, 2009). An overview is given by Gazzaley & Nobre (2012).

Eq (1) is the function fitted to the experimental values.

$$y = 1 + A \cdot x^{[n \cdot \exp(-B \cdot x)]} \quad (1)$$

y gives the perceived length of the target relative to its true length (400 mm in the experiments 1 to 5). The 1 stands for the perceived length of the empty target. Concerning the illusion, it is assumed that, to first order of approximation, its intensity first increases in proportion to the number of filling elements, x. (In this case the exponent of x, the expression in the square bracket, is assumed to be equal to 1.) n is the size constancy parameter, originally introduced to describe the moon illusion (Kreiner 2004, 2009); n= 0 means that the perceived image is in proportion to the size of the retinal image of the target while n=1 indicates that the size of the apparent image is determined by the illusion only. The exponential decay function takes into account that the size constancy parameter n may vary with the density of strokes. In its extreme, a continuous row of elements will merge into a bright line without any structure. High resolution would be idle then. For this reason, n is assumed to decrease gradually with x.

The results given in Fig.3 and Table 1 indicate that the effect increases roughly with the apparent distance of observation: With binoculars (2fold, experiment 4) the target appears at half the distance (285 instead of 570 cm). Correspondingly, the illusion is quite comparable to experiments 1 and 5 (270 cm).

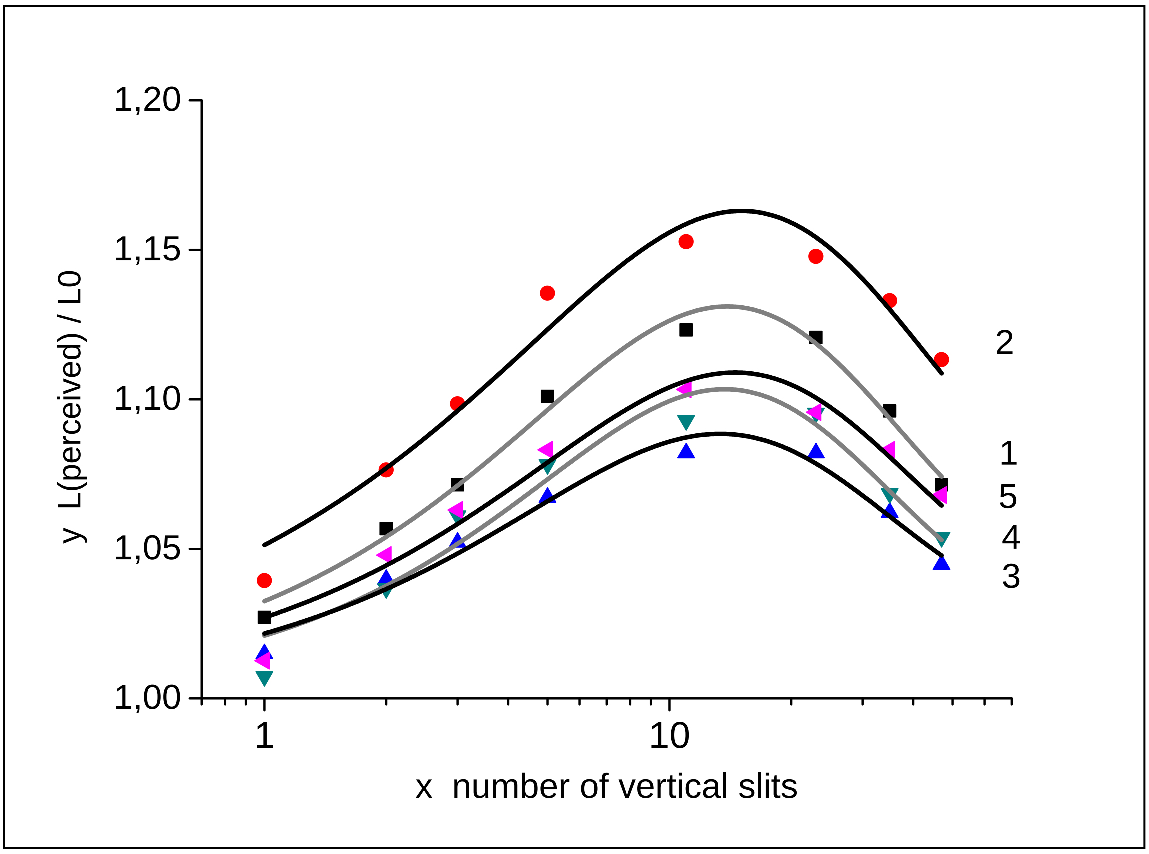

Fig. 3 Apparent length of the target (distance *cd*) as a function of the number x of vertical slits. Data and parameters obtained from a fit are listed in Table 1. L0 means the perceived length of the empty target.

Table 1. The first five experiments. Parameters obtained from fit. Target length is 400 mm.

Experiment Distance	1 270cm	2 590 cm	3 165 cm	4 590 cm	5 270 cm
Exp. Cond. Parameter				Binocular 2x	Filled space on the left
A	0.0325(22)	0.0513(48)	0.0217(24)	0.0210(39)	0.0270(38)
n	0.775(47)	0.617(65)	0.798(76)	0.894(125)	0.757(96)
B	0.0274(13)	0.0245(21)	0.0289(22)	0.0279(30)	0.0257(25)

The question is whether the illusion is triggered by the number of slits or rather by their structural density. For this purpose, targets of different length were tested, of 300, 200, and

100 mm in addition to the 400 mm target already investigated. It was found that the optimum number of slits was approximately in proportion to the length of the target. From this, one can draw the conclusion that it is not the number of the filling elements presented within the length of the target causing the intensity of the illusion, but rather the structural density. In Fig. 4 the fitted functions are plotted. The (theoretical) position of the maxima was obtained from Eq (2).

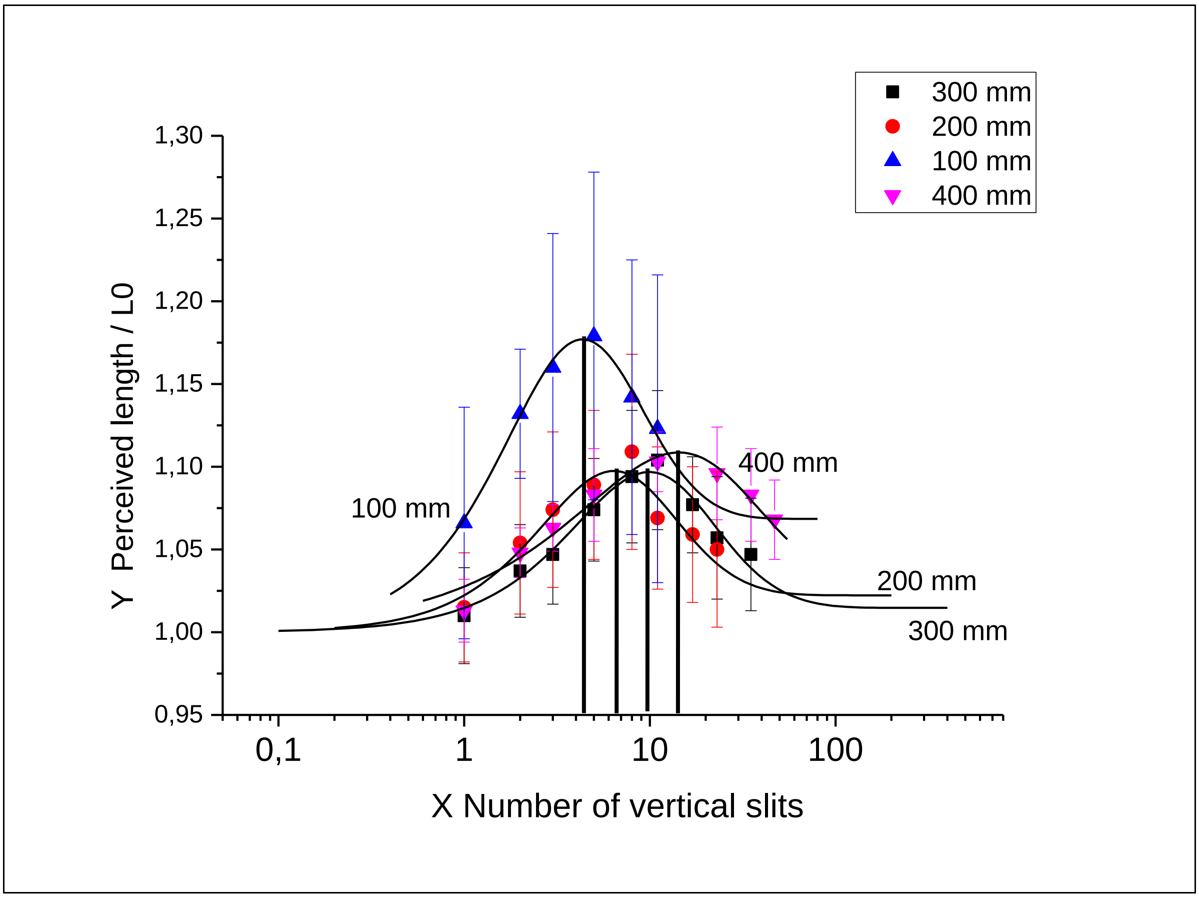

Fig. 4 Experiments performed with different target lengths. Distance of observation was 2.7 m. Eq (1) was fitted to the experimental values. The calculated values of x corresponding to the maxima decrease approximately in proportion to the target length. This shows that the maximum illusion is mainly triggered by a particular optimum structural density of the filling elements, rather than by their number.

2. Dots instead of stripes, three intervals instead of two

Surkys (2007, 2008) also investigated the Oppel-Kundt illusion employing dots as filling elements and three intervals instead of two. This is shown in Fig. 5. It turned out that the most pronounced effect (the apparent magnification of the filled section) occurred with experiment 2 (middle row), followed by experiment 1, with only two sections. From a fit of Eq (1) to the results reported by Surkys (2008), the curves shown in Fig. 6 were obtained. In order to calculate the optimum number of filling elements (the x value corresponding to the maximum of a curve) one has to get the first derivative of Eq (1). With x = exp[ln(x)], this expression gives

$$y = 1+A*\exp[\ln x*n*\exp(-B*x)].$$

From its first derivative

$$\frac{dy}{dx} = A*\exp[\ln x*n*\exp(-B*x)]*\{n*[\frac{1}{x}\exp(-B*x)-\ln x*B*\exp(-B*x)]\} = 0 \quad \text{one obtains}$$

$$\frac{1}{x}\exp(-B*x) - \ln x*B*\exp(-B*x) = 0 \quad \text{or} \quad \boxed{\frac{1}{x_{max}} = B*\ln x_{max}} \qquad (2)$$

The calculated x_{max} values are listed in Table 2, together with the number of number of dots producing an illusion closest to the maximum. Fig. 7 gives the maximum perceived lengths of the targets.

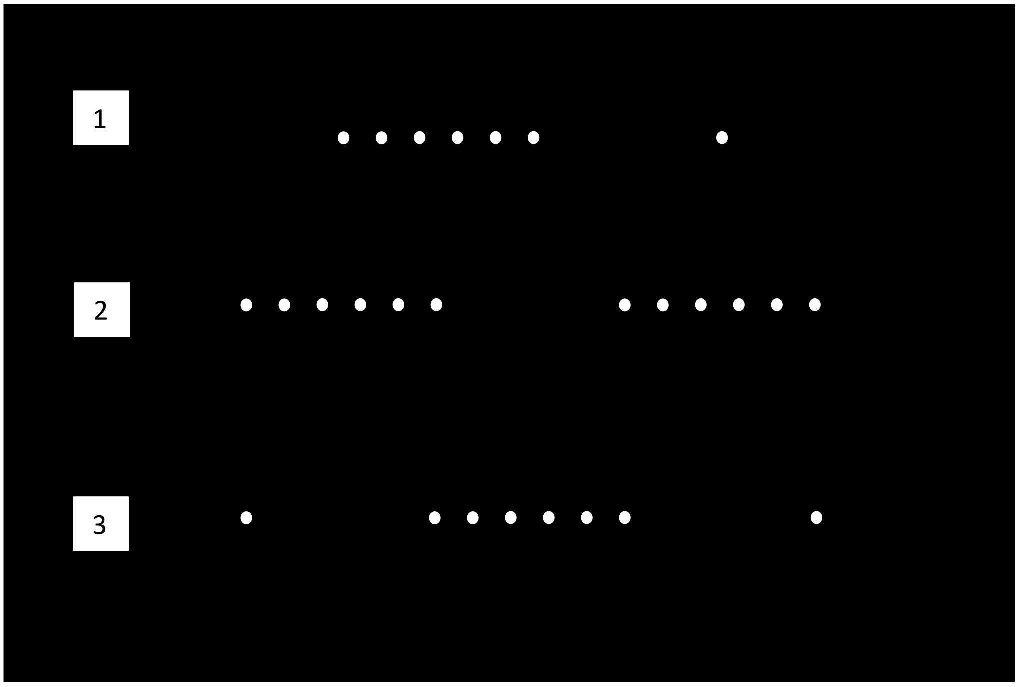

Fig. 5 Stimuli employed by Surkys (2008). The subjects compared the length of filled spatial intervals (rows of dots) with the length the adjacent empty intervals of equal length (50 minutes of arc). The rows of dots always appeared longer, up to 15 minutes arc.

Table 2. Parameters obtained from a fit of Eq (1), together with the calculated values of x and y corresponding to the maxima of the curves shown in Fig. 6.

	Exp. 2	1	3
A	0.2169(92)	0.1688(99)	0.0986(99)
n	0.439(63)	0.384(69)	0.469(99)
B	0.148(22)	0.105(18)	0.072(13)
$x(opt)_{calc}$/opt. number of filling elements	4.495 / 4	5.560 / 6	7.090 / 7
$y\ (max)_{calc}$	1,305	1,244	1,171

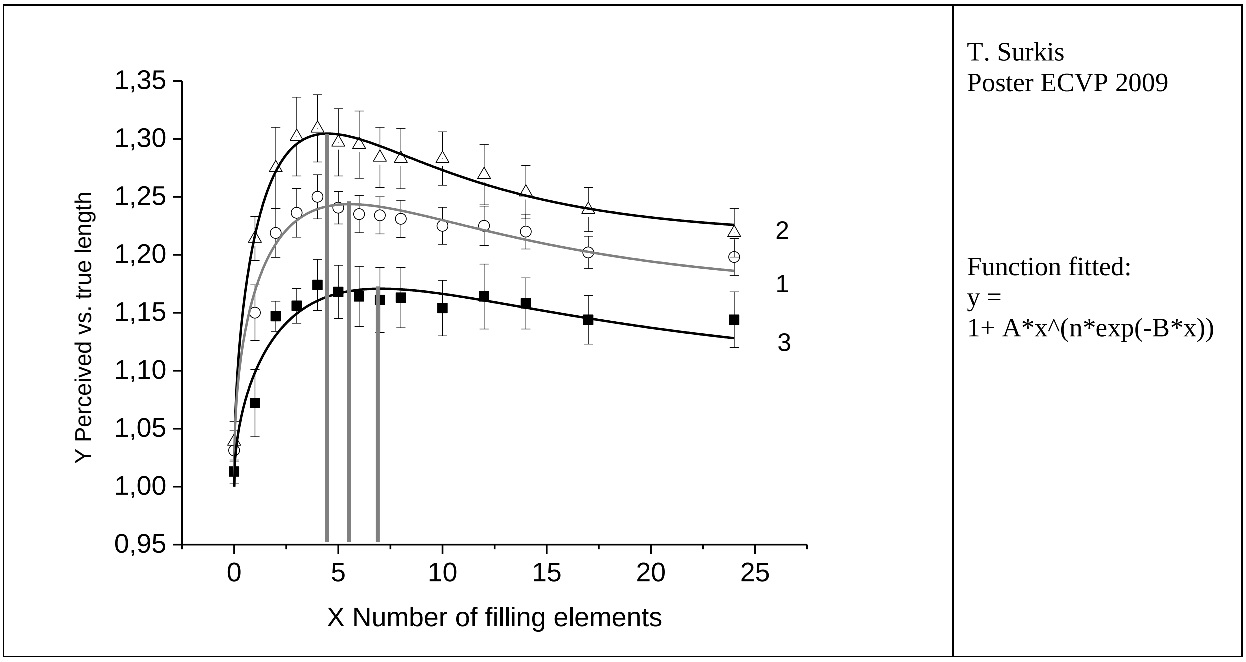

Fig. 6   Results obtained by Surkys (2008). The perceived lengths of the rows of dots are normalized to 50 min arc (reference length). y=1 means: no illusion. Fit of Eq (1). The maxima were determined from Eq (2). Perceived length up to about 30% of true length has been observed.

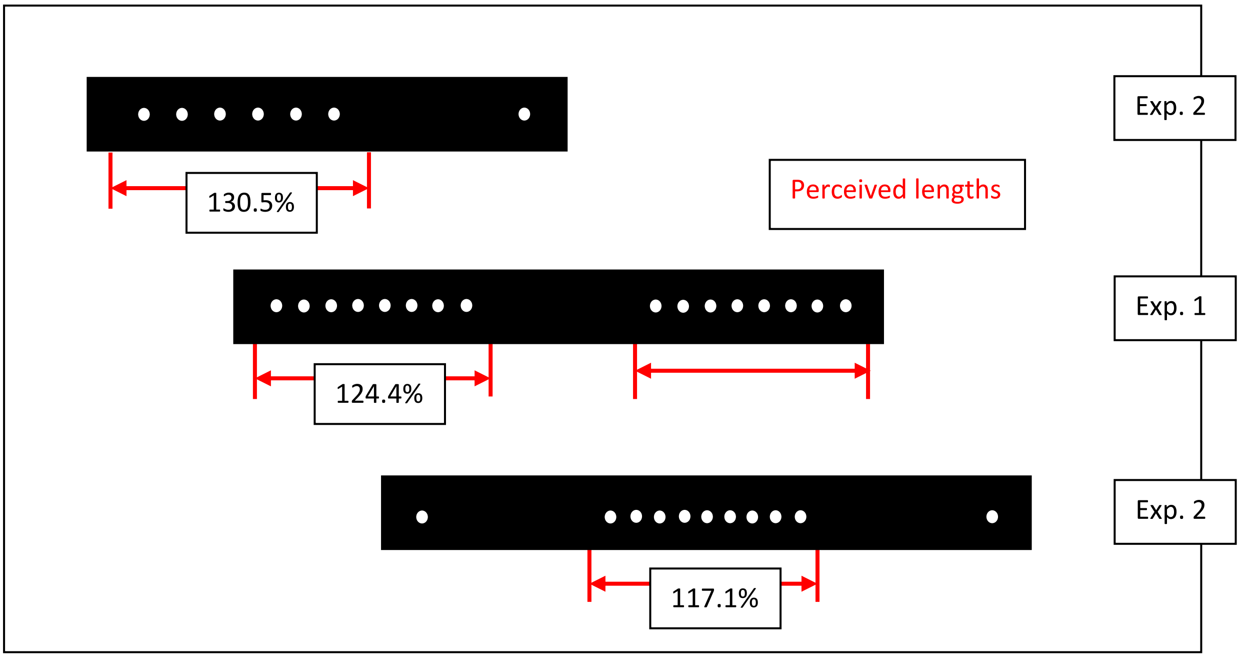

Fig. 7   Perceived lengths of the filled sections. Overestimation ranges from 17.1% to 30.5%. Data in Table 2.

3. More experiments

An experiment which seems to be closely related to the one described has been reported by Giora and Gori (2010). There, subjects estimated the size of a square showing kind of a checkerboard texture. The space frequency of its microstructure was varied. It has been found that the perceived size of the side length as a function of the structural density exhibits a maximum (Fig. 8), the curve being quite similar to the one obtained from the Oppel-Kundt experiments.

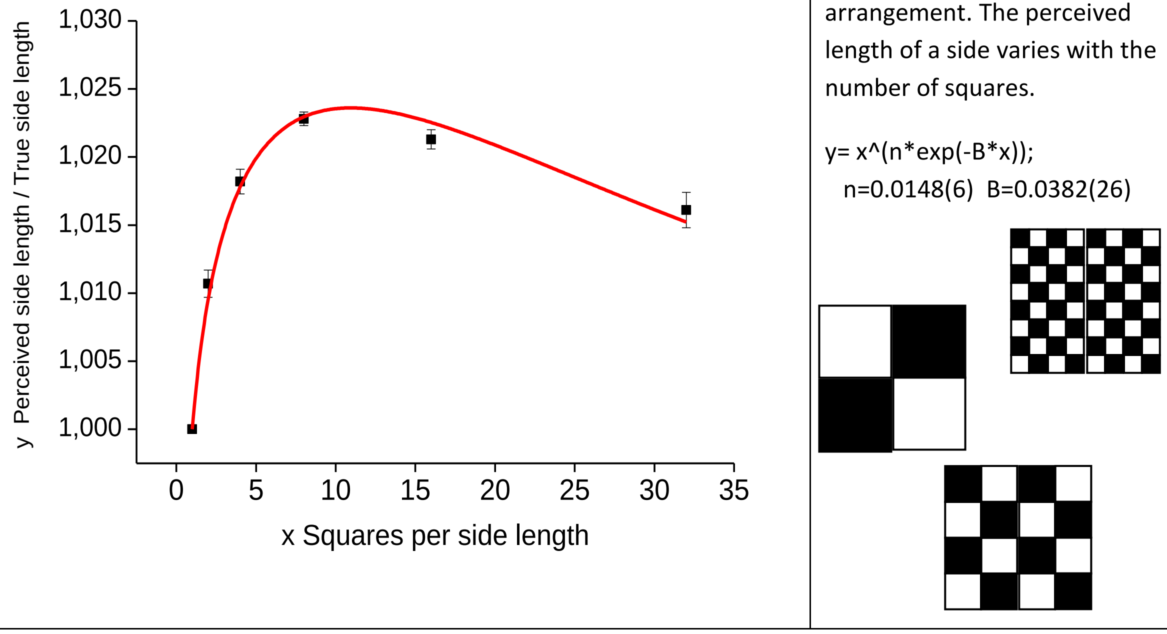

Fig. 8. Checkerboard arrangement. The perceived length of a side varies with the number of squares.

y= x^(n*exp(-B*x));
n=0.0148(6) B=0.0382(26)

In the moon illusion size constancy seems to play an important role as well. Schur (1925) performed experiments in order to investigate the moon illusion in the lab. From their data the following diagram can be plotted (Fig. 9). It gives the perceived size of a bright disk in the

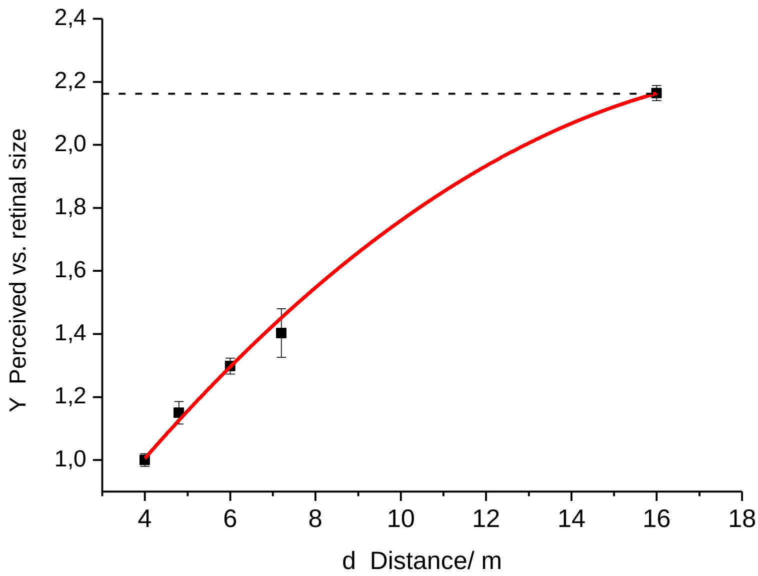

Fig. 9. Perceived size of a bright disk as a function of distance. Between 4m and 16m the apparent size increases by a factor of 2.16 compared to the size one would expect from geometrical optics.

y(x) = (d/d0)^(n);

n= 0.5596(83)

dark as a function of distance, relative to its size at d_0 = 4 m. Between 4m and 16m the apparent size increases by a factor of 2.16 compared to the size one would expect from geometrical optics. As the retinal image gets smaller in diameter, its structural density will increase. However, other than in the Oppel-Kundt phenomenon, there is only one object instead of a number of strokes and the size illusion depends on the distance of observation, d. Therefore, Eq (1) could not be applied. Instead, a power function was employed, with n again being the size constancy parameter.

From experiments performed by Gilinsky (1955) in order to investigate size constancy values around 0.4 have been derived for the parameter n (Kreiner, 2009).

4. Conclusion

The Oppel-Kundt illusion is interpreted as due to a size constancy effect which means that the visual system may concentrate on a narrow section of the retinal image in order to improve resolution which, in turn, leads to perceptional enlargement. An algebraic expression is derived and fitted to results published by several authors. This function is based on the assumption that, at low density of structural elements, the illusion will increase approximately in proportion to their number. However, for high structural density, the intensity of the illusion will reduce again because the filling elements are regarded as a uniform pattern rather than independent geometric entities.

Citations

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