

Calculation of electric charge density based on a numerical approximation method using triangular functions

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Abstract

This article focuses on the calculation of electric charge density induced on a conducting incomplete spherical surface. The analysis is based on an existing numerical method that has previously been proposed for solving some electromagnetic scattering problems. The method uses vector forms of triangular functions to reduce the solution of an integral equation into solving a system of algebraic equations. Some plots are given to better illustrate the numerical results.

Keywords: Charge density; Numerical method; Triangular functions.

1 Introduction

Many problems in Electromagnetics can be modelled by Fredholm integral equations of the first kind [1–6]. These equations are in general ill-posed. That is, small changes to the problem's data can make very large changes to the answers obtained [7, 8]. Hence, obtaining the numerical solutions is difficult.

Several regularization methods have been proposed to overcome the ill-posedness [9, 10]. In recent years, some numerical methods based on different basis functions have been illustrated. These methods often use a projection method and transform a first kind integral equation to a linear system of algebraic equations. This system usually has large condition number, therefore a suitable approach such as the Conjugate Gradient (CG) method should be considered [11, 12].

An effective numerical direct method for solution of first kind Fredholm integral equations has been proposed by Hatamzadeh-Varmazyar et al. in [3] for analysis of some electromagnetic scattering problems. In this article, we apply their approach in determination of electric charge density. This method uses vector forms of triangular functions (TFs) and reduces the solution of a first kind Fredholm integral equation into solving a linear system of algebraic equations. The organization of this article is as follows. Two sets of TFs together with some properties are reviewed in section 2. Section 3 reviews a special representation of TFs vector forms and some of its properties which are the main basis for introducing the method. Section 4 reviews the numerical direct method proposed and formulated in [3] for solving Fredholm integral equation of the first kind. An integral equation model for charge density induced on a conducting incomplete spherical surface is posed in section 5 and solved by the numerical direct method. Some plots are given to better illustrate the numerical results. Finally, conclusions will be given in section 6.

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2 Triangular functions

2.1 Definition

Let us consider two m -sets of TFs over the interval $[0, H)$ as in [13]

$$\begin{aligned}
 T1_i(t) &= \begin{cases} 1 - \frac{t-ih}{h}, & ih \leq t < (i+1)h, \\ 0, & \text{otherwise,} \end{cases} \\
 T2_i(t) &= \begin{cases} \frac{t-ih}{h}, & ih \leq t < (i+1)h, \\ 0, & \text{otherwise,} \end{cases}
 \end{aligned} \tag{2.1}$$

where $i = 0, 1, \dots, m-1$, with a positive integer value for m . Also, consider $h = H/m$, and $T1_i$ as the i th left-handed TF and $T2_i$ as the i th right-handed TF.

From the definition of TFs, it is clear that they are disjoint, orthogonal, and complete [3, 13]. Therefore, we can write

$$\int_0^H T1_i(t)T1_j(t) dt = \int_0^H T2_i(t)T2_j(t) dt = \begin{cases} \frac{h}{3}, & i = j, \\ 0, & i \neq j. \end{cases} \tag{2.2}$$

Also,

$$\varphi_i(t) = T1_i(t) + T2_i(t), \quad i = 0, 1, \dots, m-1, \tag{2.3}$$

where φ_i is the i th BPF defined as

$$\varphi_i(t) = \begin{cases} 1, & ih \leq t < (i+1)h, \\ 0, & \text{otherwise.} \end{cases} \tag{2.4}$$

2.2 Vector forms

Consider the first m terms of left-handed TFs and the first m terms of right-handed TFs and write them concisely as m -vectors [3]

$$\begin{aligned}
 \mathbf{T1}(t) &= [T1_0(t), T1_1(t), \dots, T1_{m-1}(t)]^T, \\
 \mathbf{T2}(t) &= [T2_0(t), T2_1(t), \dots, T2_{m-1}(t)]^T,
 \end{aligned} \tag{2.5}$$

where, superscript T indicates transposition, and $\mathbf{T1}$ and $\mathbf{T2}$ are called left-handed triangular functions (LHTF) vector and right-handed triangular functions (RHTF) vector, respectively.

The following properties of the product of the two TFs vectors are presented in [14]:

$$\begin{aligned}
 \mathbf{T1}(t)\mathbf{T1}^T(t) &\simeq \begin{pmatrix} T1_0(t) & 0 & \dots & 0 \\ 0 & T1_1(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & T1_{m-1}(t) \end{pmatrix}, \\
 \mathbf{T2}(t)\mathbf{T2}^T(t) &\simeq \begin{pmatrix} T2_0(t) & 0 & \dots & 0 \\ 0 & T2_1(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & T2_{m-1}(t) \end{pmatrix},
 \end{aligned} \tag{2.6}$$

and

$$\begin{aligned}
 \mathbf{T1}(t)\mathbf{T2}^T(t) &\simeq \mathbf{0}, \\
 \mathbf{T2}(t)\mathbf{T1}^T(t) &\simeq \mathbf{0},
 \end{aligned} \tag{2.7}$$

where $\mathbf{0}$ is the zero $m \times m$ matrix. Also,

$$\begin{aligned} \int_0^H \mathbf{T1}(t)\mathbf{T1}^T(t) dt &= \int_0^H \mathbf{T2}(t)\mathbf{T2}^T(t) dt \simeq \frac{h}{3}I, \\ \int_0^H \mathbf{T1}(t)\mathbf{T2}^T(t) dt &= \int_0^H \mathbf{T2}(t)\mathbf{T1}^T(t) dt \simeq \frac{h}{6}I, \end{aligned} \quad (2.8)$$

in which I is $m \times m$ identity matrix.

2.3 TFs expansion

The expansion of a function $f(t)$ over $[0, H]$ with respect to TFs, may be compactly written as [3]

$$\begin{aligned} f(t) &\simeq \sum_{i=0}^{m-1} c_i T1_i(t) + \sum_{i=0}^{m-1} d_i T2_i(t) \\ &= \mathbf{c}^T \mathbf{T1}(t) + \mathbf{d}^T \mathbf{T2}(t), \end{aligned} \quad (2.9)$$

where we may put

$$\begin{aligned} c_i &= f(ih), \\ d_i &= f((i+1)h), \quad i = 0, 1, \dots, m-1. \end{aligned} \quad (2.10)$$

3 Special representation of TFs vector forms and other properties

3.1 Definition and expansion

Let $\mathbf{T}(t)$ be a $2m$ -vector defined as follows [3, 15]:

$$\mathbf{T}(t) = \begin{pmatrix} \mathbf{T1}(t) \\ \mathbf{T2}(t) \end{pmatrix}, \quad 0 \leq t < 1, \quad (3.11)$$

where $\mathbf{T1}$ and $\mathbf{T2}$ have been defined in (2.5). Now, the expansion of f with respect to TFs can be written as

$$\begin{aligned} f(t) &\simeq F1^T \mathbf{T1}(t) + F2^T \mathbf{T2}(t) \\ &= F^T \mathbf{T}(t), \end{aligned} \quad (3.12)$$

where $F1$ and $F2$ are TFs coefficients with $F1_i = f(ih)$ and $F2_i = f((i+1)h)$, for $i = 0, 1, \dots, m-1$. Also, $2m$ -vector F is defined as

$$F = \begin{pmatrix} F1 \\ F2 \end{pmatrix}. \quad (3.13)$$

Now, assume that $k(s, t)$ is a function of two variables. It can be expanded with respect to TFs as follows:

$$k(s, t) \simeq \mathbf{T}^T(s) K \mathbf{T}(t), \quad (3.14)$$

where $\mathbf{T}(s)$ and $\mathbf{T}(t)$ are $2m_1$ - and $2m_2$ -dimensional TFs vectors respectively, and K is a $2m_1 \times 2m_2$ TFs coefficient matrix. For convenience, we put $m_1 = m_2 = m$. Hence, matrix K can be written as [3, 15]

$$K = \begin{pmatrix} (K11)_{m \times m} & (K12)_{m \times m} \\ (K21)_{m \times m} & (K22)_{m \times m} \end{pmatrix}, \quad (3.15)$$

where $K11$, $K12$, $K21$, and $K22$ can be computed by sampling the function k at points s_i and t_j such that $s_i = ih$ and $t_j = jh$, for $i, j = 0, 1, \dots, m$. Therefore,

$$\begin{aligned}
 K11 &= \begin{pmatrix} k(s_0, t_0) & k(s_0, t_1) & \dots & k(s_0, t_{m-1}) \\ k(s_1, t_0) & k(s_1, t_1) & \dots & k(s_1, t_{m-1}) \\ \vdots & \vdots & \ddots & \vdots \\ k(s_{m-1}, t_0) & k(s_{m-1}, t_1) & \dots & k(s_{m-1}, t_{m-1}) \end{pmatrix}, \\
 K12 &= \begin{pmatrix} k(s_0, t_1) & k(s_0, t_2) & \dots & k(s_0, t_m) \\ k(s_1, t_1) & k(s_1, t_2) & \dots & k(s_1, t_m) \\ \vdots & \vdots & \ddots & \vdots \\ k(s_{m-1}, t_1) & k(s_{m-1}, t_2) & \dots & k(s_{m-1}, t_m) \end{pmatrix}, \\
 K21 &= \begin{pmatrix} k(s_1, t_0) & k(s_1, t_1) & \dots & k(s_1, t_{m-1}) \\ k(s_2, t_0) & k(s_2, t_1) & \dots & k(s_2, t_{m-1}) \\ \vdots & \vdots & \ddots & \vdots \\ k(s_m, t_0) & k(s_m, t_1) & \dots & k(s_m, t_{m-1}) \end{pmatrix}, \\
 K22 &= \begin{pmatrix} k(s_1, t_1) & k(s_1, t_2) & \dots & k(s_1, t_m) \\ k(s_2, t_1) & k(s_2, t_2) & \dots & k(s_2, t_m) \\ \vdots & \vdots & \ddots & \vdots \\ k(s_m, t_1) & k(s_m, t_2) & \dots & k(s_m, t_m) \end{pmatrix}.
 \end{aligned} \tag{3.16}$$

3.2 Product properties

Let X be a $2m$ -vector which can be written as $X^T = (X1^T \ X2^T)$ such that $X1$ and $X2$ are m -vectors. Now, it can be concluded from Eqs. (2.6) and (2.7) that [3, 15]

$$\begin{aligned}
 \mathbf{T}(t)\mathbf{T}^T(t)X &= \begin{pmatrix} \mathbf{T1}(t) \\ \mathbf{T2}(t) \end{pmatrix} \begin{pmatrix} \mathbf{T1}^T(t) & \mathbf{T2}^T(t) \end{pmatrix} \begin{pmatrix} X1 \\ X2 \end{pmatrix} \\
 &\simeq \begin{pmatrix} \text{diag}(\mathbf{T1}(t)) & \mathbf{0}_{m \times m} \\ \mathbf{0}_{m \times m} & \text{diag}(\mathbf{T2}(t)) \end{pmatrix} \begin{pmatrix} X1 \\ X2 \end{pmatrix} \\
 &= \text{diag}(\mathbf{T}(t))X \\
 &= \text{diag}(X)\mathbf{T}(t).
 \end{aligned} \tag{3.17}$$

Therefore,

$$\mathbf{T}(t)\mathbf{T}^T(t)X \simeq \tilde{X}\mathbf{T}(t), \tag{3.18}$$

where $\tilde{X} = \text{diag}(X)$ is a $2m \times 2m$ diagonal matrix.

Now, let B be a $2m \times 2m$ matrix as

$$B = \begin{pmatrix} (B11)_{m \times m} & (B12)_{m \times m} \\ (B21)_{m \times m} & (B22)_{m \times m} \end{pmatrix}. \tag{3.19}$$

Hence, it can be similarly concluded from Eqs. (2.6) and (2.7) that

$$\begin{aligned}
 \mathbf{T}^T(t)B\mathbf{T}(t) &= \begin{pmatrix} \mathbf{T1}^T(t) & \mathbf{T2}^T(t) \end{pmatrix} \begin{pmatrix} B11 & B12 \\ B21 & B22 \end{pmatrix} \begin{pmatrix} \mathbf{T1}(t) \\ \mathbf{T2}(t) \end{pmatrix} \\
 &\simeq \mathbf{T1}^T(t)B11\mathbf{T1}(t) + \mathbf{T2}^T(t)B22\mathbf{T2}(t) \\
 &\simeq \hat{B}11^T\mathbf{T1}(t) + \hat{B}22^T\mathbf{T2}(t),
 \end{aligned} \tag{3.20}$$

where $\hat{B}11$ and $\hat{B}22$ are m -vectors with elements equal to the diagonal entries of matrices $B11$ and $B22$, respectively. Therefore,

$$\mathbf{T}^T(t)B\mathbf{T}(t) \simeq \hat{B}^T\mathbf{T}(t), \tag{3.21}$$

in which \hat{B} is a $2m$ -vector with elements equal to the diagonal entries of matrix B . Also, it is immediately concluded from Eqs. (2.8) that [3, 15]

$$\begin{aligned} \int_0^H \mathbf{T}(t)\mathbf{T}^T(t) dt &= \int_0^H \begin{pmatrix} \mathbf{T1}(t) \\ \mathbf{T2}(t) \end{pmatrix} (\mathbf{T1}^T(t) \quad \mathbf{T2}^T(t)) dt \\ &= \int_0^H \begin{pmatrix} \mathbf{T1}(t)\mathbf{T1}^T(t) & \mathbf{T1}(t)\mathbf{T2}^T(t) \\ \mathbf{T2}(t)\mathbf{T1}^T(t) & \mathbf{T2}(t)\mathbf{T2}^T(t) \end{pmatrix} dt \\ &\simeq \begin{pmatrix} \frac{h}{3}\mathbf{I}_{m \times m} & \frac{h}{6}\mathbf{I}_{m \times m} \\ \frac{h}{6}\mathbf{I}_{m \times m} & \frac{h}{3}\mathbf{I}_{m \times m} \end{pmatrix}. \end{aligned} \tag{3.22}$$

Therefore,

$$\int_0^H \mathbf{T}(t)\mathbf{T}^T(t) dt \simeq D, \tag{3.23}$$

where D is the following $2m \times 2m$ matrix:

$$D = \frac{h}{3} \begin{pmatrix} 1 & 0 & \dots & 0 & 1/2 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & 1/2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 1/2 \\ 1/2 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ 0 & 1/2 & \dots & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1/2 & 0 & 0 & \dots & 1 \end{pmatrix}. \tag{3.24}$$

4 Numerical direct method for solution of first kind Fredholm integral equation

By using the results obtained in section 3, an effective numerical direct method for solution of first kind Fredholm integral equation has been proposed and formulated in [3]. Here, we review this method. Also, we assume that TFs are defined over an arbitrary interval $[a, b)$. It is clear that all the mentioned properties and relations can be easily generalized over this interval.

Consider the Fredholm integral equation of the first kind of the form

$$\int_a^b k(s,t)x(t) dt = f(s), \quad a \leq s < b, \tag{4.25}$$

where the functions k and f are known but x is the unknown function to be determined. Moreover, $k \in \mathcal{L}^2([a, b) \times [a, b))$ and $f \in \mathcal{L}^2([a, b))$, in which \mathcal{L}^2 is the space of square integrable functions.

Approximating the functions k , f , and x with respect to TFs, using Eqs. (3.12) and (3.14), gives [3]

$$\begin{aligned} x(s) &\simeq X^T \mathbf{T}(s), \\ f(s) &\simeq F^T \mathbf{T}(s), \\ k(s,t) &\simeq \mathbf{T}^T(s)K\mathbf{T}(t), \end{aligned} \tag{4.26}$$

where $2m$ -vectors F and X , and $2m \times 2m$ matrix K are TFs coefficients of f , x , and k , respectively. Note that X is the unknown vector and should be obtained.

Substituting (4.26) into (4.25) gives [3]

$$\int_a^b \mathbf{T}^T(s)K\mathbf{T}(t)X^T\mathbf{T}(t) dt = F^T \mathbf{T}(s), \tag{4.27}$$

or

$$\mathbf{T}^T(s)K \int_a^b \mathbf{T}(t)X^T\mathbf{T}(t) dt = F^T \mathbf{T}(s). \tag{4.28}$$

Note that $X^T \mathbf{T}(t) = \mathbf{T}^T(t)X$. Hence, from (4.28) we obtain [3]

$$\mathbf{T}^T(s)K \int_a^b \mathbf{T}(t)\mathbf{T}^T(t)X dt = F^T \mathbf{T}(s), \quad (4.29)$$

or

$$\mathbf{T}^T(s)K \left(\int_a^b \mathbf{T}(t)\mathbf{T}^T(t) dt \right) X = F^T \mathbf{T}(s). \quad (4.30)$$

Using (3.23) gives

$$\mathbf{T}^T(s)KDX = F^T \mathbf{T}(s). \quad (4.31)$$

Considering $F^T \mathbf{T}(s) = \mathbf{T}^T(s)F$ we obtain [3]

$$\mathbf{T}^T(s)KDX = \mathbf{T}^T(s)F. \quad (4.32)$$

Therefore,

$$(KD)X = F. \quad (4.33)$$

Equation (4.33) is a linear system of $2m$ algebraic equations for the $2m$ unknowns $X_{10}, X_{11}, \dots, X_{1m-1}, X_{20}, X_{21}, \dots, X_{2m-1}$, components of $X^T = (X_{10}^T \ X_{20}^T)$. Hence, an approximate solution $x(s) \simeq X^T \mathbf{T}(s)$, or $x(s) \simeq X_{10}^T \mathbf{T}_1(s) + X_{20}^T \mathbf{T}_2(s)$ can be computed for Eq. (4.25) without applying any projection method [3].

5 Analysis of charge density induced on a conducting incomplete spherical surface by the numerical direct method

5.1 Modeling the problem

In Fig. 1, there is a conducting incomplete spherical surface of radius a containing a surface electric charge that produces a constant potential at its surface. According to the symmetry of the problem (as shown in Fig. 1), it is

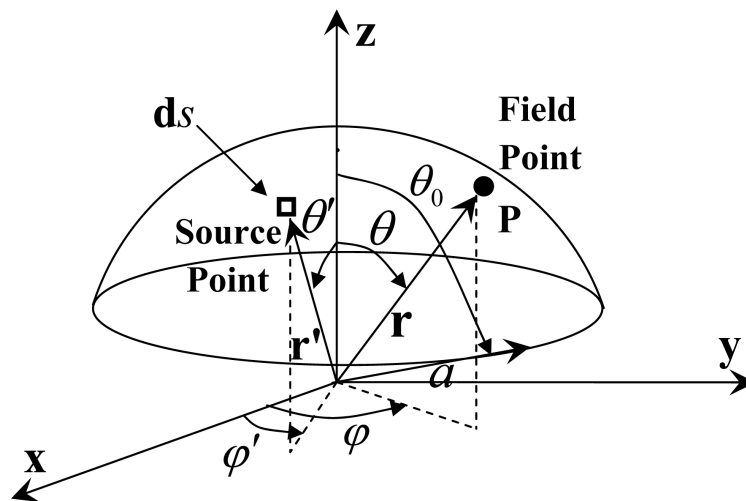


Figure 1: A conducting incomplete spherical surface of radius a containing electric charge.

clear that the charge distribution on the surface of the sphere is independent of variable ϕ of the spherical coordinates system. However, it depends on θ . Consider a differential surface element ds at an arbitrary source point on the surface of the sphere. For element ds , the resulting electric potential at the field point P (that is moved to the surface of sphere) is

$$d\phi = \frac{q_s(\theta')}{4\pi\epsilon_0|\mathbf{r}-\mathbf{r}'|} ds, \quad (5.34)$$

where ϕ is the electric potential; q_s is the surface charge density; $\epsilon_0 \simeq 8.854e - 12$ (F/m) is free space permittivity; and \mathbf{r}' and \mathbf{r} are respectively the position vectors of source point and observation point (field point) on the surface of sphere.

According to Fig. 1 we have

$$|\mathbf{r} - \mathbf{r}'| = a\sqrt{2[1 - (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi'))]}, \quad (5.35)$$

and

$$ds = a^2 \sin \theta' d\theta' d\varphi'. \quad (5.36)$$

Then the total potential is

$$\phi = \int_0^{2\pi} \int_0^{\theta_0} \frac{a}{4\pi\epsilon_0} \frac{q_s(\theta') \sin \theta' d\theta' d\varphi'}{\sqrt{2[1 - (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\varphi - \varphi'))]}}. \quad (5.37)$$

The potential is constant throughout the conducting surface, therefore the observation point (P) can be moved to $\varphi = 0$. So, Eq. (5.37) can be rewritten as

$$\phi(\theta) = \int_0^{2\pi} \int_0^{\theta_0} \frac{a}{4\pi\epsilon_0} \frac{q_s(\theta') \sin \theta' d\theta' d\varphi'}{\sqrt{2[1 - (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \varphi')]}}, \quad (5.38)$$

where $\phi(\theta)$ has a constant value. Now, assume that the potential produced by the surface charge is chosen 1 (V). Then, Eq. (5.38) becomes

$$\int_0^{2\pi} \int_0^{\theta_0} \frac{a}{4\pi\epsilon_0} \frac{q_s(\theta') \sin \theta' d\theta' d\varphi'}{\sqrt{2[1 - (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \varphi')]}} = 1. \quad (5.39)$$

Equation (5.39) is a first kind Fredholm integral equation of the following form

$$\int_0^{\theta_0} k(\theta, \theta') x(\theta') d\theta' = f(\theta), \quad (5.40)$$

in which

$$k(\theta, \theta') = \int_0^{2\pi} \frac{a}{4\pi\epsilon_0} \frac{\sin \theta' d\varphi'}{\sqrt{2[1 - (\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos \varphi')]}}, \quad (5.41)$$

$x(\theta') = q_s(\theta')$ is the unknown function (surface charge density) to be determined, and $f(\theta) = 1$.

5.2 Numerical results

Now, we apply the numerical direct method formulated in section 4 in solution of integral equation (5.39) to determine the surface charge density q_s . The results for $a = 1$ (m) are given in Figs. 2–4 respectively for $\theta_0 = \frac{\pi}{4}$, $\frac{\pi}{2}$, and $\frac{3\pi}{4}$ (rad).

For $\theta_0 = \pi$ (rad) we have a complete sphere and the surface charge density q_s should have a constant value, according the Gauss's law, throughout the surface of the sphere. To calculate the exact value of q_s in this case, we can write

$$\phi = \frac{Q}{4\pi\epsilon_0 a}, \quad (5.42)$$

where Q is the total charge on the sphere. Considering $Q = q_s(4\pi a^2)$ and substituting $\phi = 1$ (V) and $a = 1$ (m) we obtain

$$\begin{aligned} q_s &= \epsilon_0 \\ &\simeq 8.854e - 12 \text{ (C/m}^2\text{)}, \end{aligned} \quad (5.43)$$

for any arbitrary θ .

The results obtained by the numerical direct method for the case of $\theta_0 = \pi$ (rad) and $a = 1$ (m) are shown in Fig. 5 which are in good agreement with the exact constant value of $q_s \simeq 8.854e - 12$ (C/m²) given by (5.43).

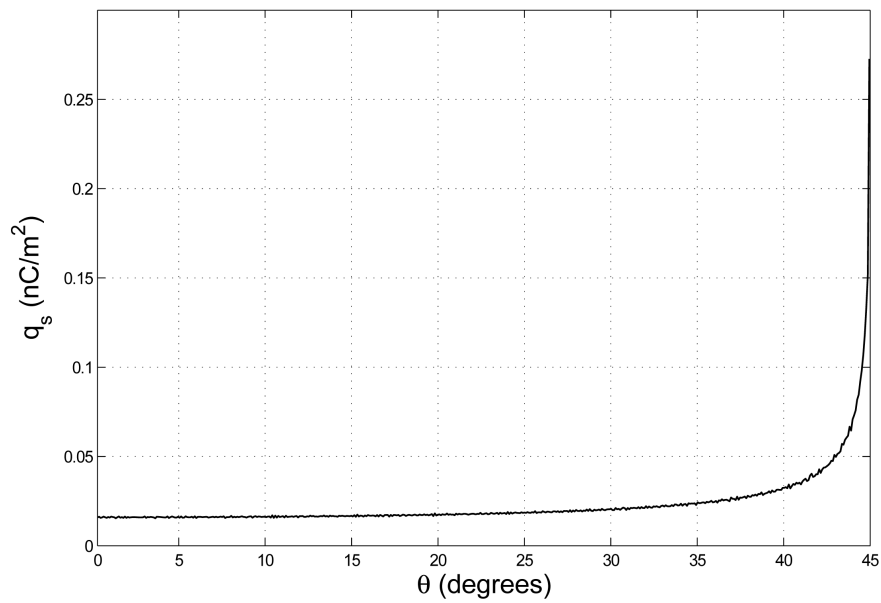


Figure 2: Electric charge density induced on the conducting incomplete spherical surface for $a = 1$ (m) and $\theta_0 = \frac{\pi}{4}$ (rad), obtained by the numerical direct method.

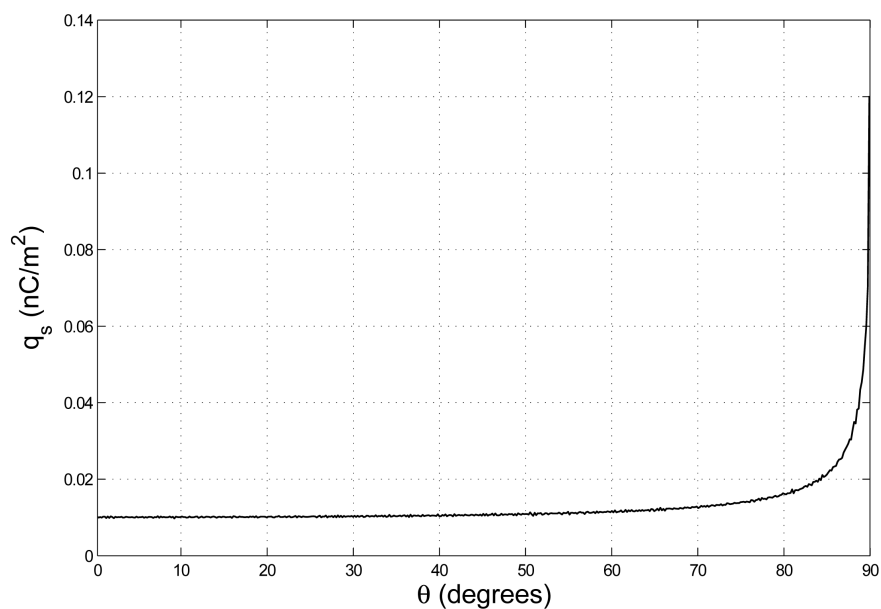


Figure 3: Electric charge density induced on the conducting incomplete spherical surface for $a = 1$ (m) and $\theta_0 = \frac{\pi}{2}$ (rad), obtained by the numerical direct method.

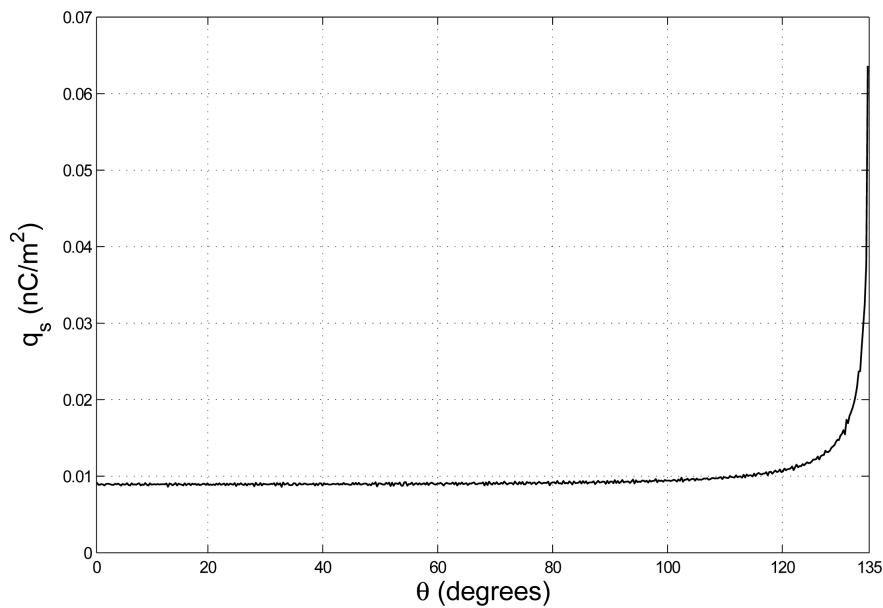


Figure 4: Electric charge density induced on the conducting incomplete spherical surface for $a = 1$ (m) and $\theta_0 = \frac{3\pi}{4}$ (rad), obtained by the numerical direct method.

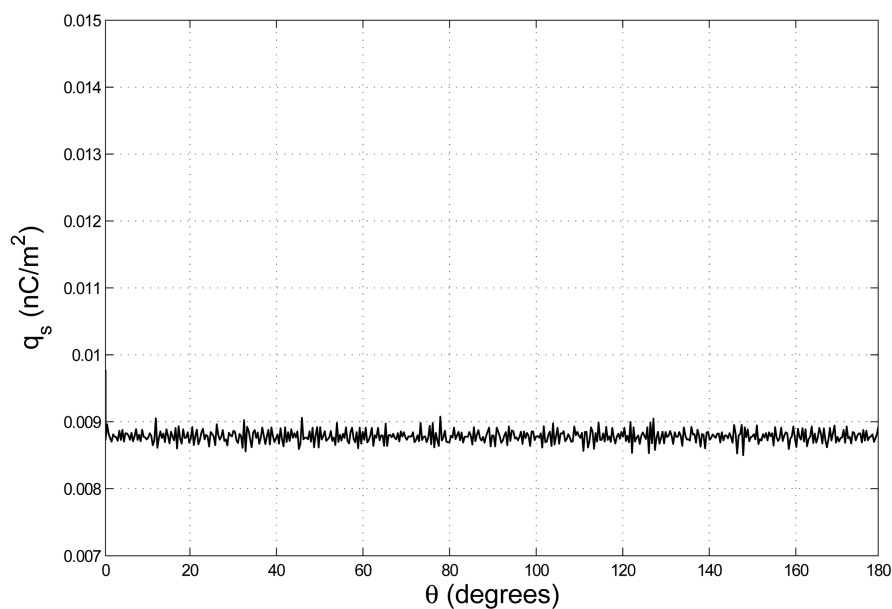


Figure 5: Electric charge density induced on the conducting spherical surface for $a = 1$ (m) and $\theta_0 = \pi$ (rad) (complete sphere), obtained by the numerical direct method, which is in agreement with the exact solution given by the Gauss's law.

6 Conclusion

The problem of calculating the electric charge density induced on a conducting incomplete spherical surface was analyzed by using an effective numerical direct method based on the triangular functions. The approximate results obtained by the method were given for some different values of θ_0 . In the case of $\theta_0 = \pi$ (rad) (complete sphere), we saw that the obtained numerical solution approached to the exact solution given by the Gauss's law. The presented concepts in this article may be generalized to be applied in analysis of some other conducting surfaces.

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