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Research Article

ADVANCED COMPUTATIONAL TECHNIQUES IN ELECTROMAGNETICS

Applying integral equation modeling technique in determination of charge distribution on conducting structures

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Abstract

We use, in this article, the integral equation technique for modeling the problem of determination of charge distribution on conducting structures. Such problems may be modeled by Fredholm integral equations and, in general, have no analytical solution. Hence, an appropriate computational approach should be used for obtaining the approximate solutions. For this purpose, an effective collocation method is used and two conducting structures are analyzed by it. Numerical results are given to illustrate the computational efficiency of the method.

Keywords: Integral equation modeling technique; Charge distribution; Conducting structures.

1 Introduction

Integral equation technique is a well-known approach for modeling of Electromagnetics problems and many of such problems are modeled by Fredholm integral equations of the first kind [1–6]. These equations are in general ill-posed. That is, small changes to the problem's data can make very large changes to the answers obtained [7, 8]. Hence, obtaining the numerical solutions is difficult.

In recent years, some numerical methods based on different basis functions have been illustrated. These methods often use a projection method and transform a first kind integral equation to a linear system of algebraic equations. This system usually has large condition number.

A collocation method for solution of Fredholm integral equations of the first kind has been presented and formulated by Masouri and Hatamzadeh-Varmazyar in [9], essentially for analysis of some electromagnetic scattering problems. In this article, we apply their approach in calculation of electric charge distribution induced on conducting surfaces. The method reduces the solution of a first kind Fredholm integral equation into solving a linear system of algebraic equations by using an appropriate set of basis functions.

The organization of this article is as follows. As two conducting surfaces, a finite-width strip and an incomplete cylindrical surface are surveyed in section 2, and the integral equation models associated with the electric charge distribution induced on them are formulated. For solution of these models, we use the collocation method proposed in [9], therefore a review on this method is performed in section 3. The numerical results for the surface charge densities are given in section 4. Finally, conclusions will be given in section 5.

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2 Integral equation modeling for analysis of electric charge distribution on conducting surfaces

Here, we survey the integral equation modeling for analysis of charge density induced on two conducting surfaces; a finite-width strip, and an incomplete cylindrical surface, both of infinite length.

2.1 Finite-width strip

In Fig. 1 there is a conducting strip of width a containing electric charge that is very long in $\pm z$ direction. It can be imagined that the strip consists of line charges stacked next to one another and parallel to z-axis. The potential ϕ at a filed point $\boldsymbol{\rho}$ produced by a line charge (electric filed source) of density q_L is given by [2]

$$\mathrm{d}\phi(\boldsymbol{\rho}) = \frac{q_L}{2\pi\varepsilon_0}\ln\left(\frac{1}{|\boldsymbol{\rho}-\boldsymbol{\rho}'|}\right), \tag{2.1}$$

where $\varepsilon_0 \simeq 8.854e-12$ (F/m) is free space permittivity, $\boldsymbol{\rho}$ is the position vector of observation (field) point, $\boldsymbol{\rho}'$ is the position vector of source point.

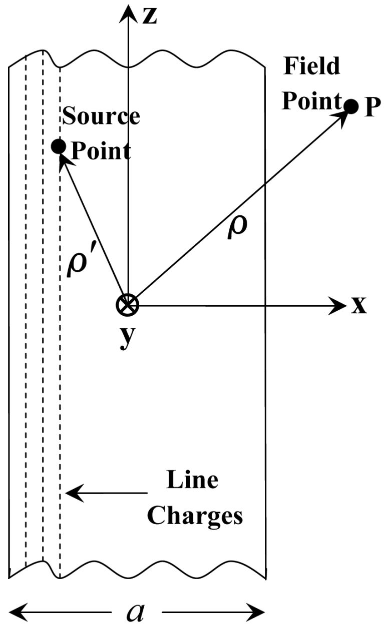

Figure 1: A conducting strip of width a containing electric charge.

The surface charge density is constant along z at any x-axis point. It is possible to relate the surface charge on the strip to a line charge by multiplying the surface charge density by $\mathrm{d}x'$ as [2]

$$q_L(x') = q_s(x')\,\mathrm{d}x'. \tag{2.2}$$

Replacing (2.2) into (2.1) and integrating both sides gtives

$$\phi(\boldsymbol{\rho}) = \int_{-a/2}^{a/2}\frac{q_s(x')}{2\pi\varepsilon_0}\ln\left(\frac{1}{|\boldsymbol{\rho}-\boldsymbol{\rho}'|}\right)\mathrm{d}x'. \tag{2.3}$$

Now, the observation point P can be moved to the surface of strip. So, the source and observation points are confined to x-axis. Hence, Eq. (2.3) is reduced to

$$\phi(x) = \int_{-a/2}^{a/2}\frac{q_s(x')}{2\pi\varepsilon_0}\ln\left(\frac{1}{|x-x'|}\right)\mathrm{d}x'. \tag{2.4}$$

Assume that the potential produced by the surface charge of strip is chosen 1 (V). Eq. (2.4) becomes

$$\int_{-a/2}^{a/2} \frac{q_s(x')}{2\pi\varepsilon_0} \ln(\frac{1}{|x-x'|})\,\mathrm{d}x' = 1. \tag{2.5}$$

Equation (2.5) is a Fredholm integral equation of the first kind in terms of the unknown function q_s. Solution of this equation gives the surface charge density on the strip.

For solving (2.5) we will use a numerical method that will be surveyed in section 3. However, we obtain an analytical solution here for a special case. Suppose that the width of the strip is set to $a = 2$. With this assumption, Eq. (2.5) becomes

$$\int_{-1}^{1} \frac{q_s(x')}{2\pi\varepsilon_0} \ln(\frac{1}{|x-x'|})\,\mathrm{d}x' = 1. \tag{2.6}$$

It is possible to choose to observe the potential at any point along x-axis that is desired. Choosing $x = 0$, Eq. (2.6) becomes

$$\int_{-1}^{1} \frac{q_s(x')}{2\pi\varepsilon_0} \ln(\frac{1}{|x'|})\,\mathrm{d}x' = 1. \tag{2.7}$$

This integral has the same form as a known definite integral [2]

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x'^2}} \ln(\frac{1}{|x'|})\,\mathrm{d}x' = \pi \ln 2. \tag{2.8}$$

Normalizing the right side and comparing with Eq. (2.7) we have

$$q_s(x') = \frac{2\varepsilon_0}{(\ln 2)\sqrt{1-x'^2}}. \tag{2.9}$$

Rearranging (2.9), the exact solution of the charge distribution as a function of x is obtained as follows:

$$q_s(x) = \frac{2\varepsilon_0}{(\ln 2)\sqrt{1-x^2}}. \tag{2.10}$$

2.2 Incomplete cylindrical surface

In Fig. 2 there is a conducting incomplete cylindrical surface of radius a containing electric charge that is very long in $\pm z$ direction. According to the figure, it is clear that we can write

$$q_L(\varphi') = q_s(\varphi')(a\,\mathrm{d}\varphi'), \tag{2.11}$$

and

$$|\boldsymbol{\rho} - \boldsymbol{\rho}'| = a\sqrt{2(1-\cos(\varphi-\varphi'))}, \tag{2.12}$$

where $\boldsymbol{\rho}$ and $\boldsymbol{\rho}'$ are the position vectors of observation and source points, respectively. By replacing (2.11) and (2.12) in (2.1) and integrating both sides we have

$$\phi(\varphi) = \int_0^{\varphi_0} q_s(\varphi') \frac{a}{2\pi\varepsilon_0} \ln(\frac{1}{a\sqrt{2(1-\cos(\varphi-\varphi'))}})\,\mathrm{d}\varphi'. \tag{2.13}$$

If the potential produced by the surface charge of cylindrical surface is chosen 1 (V), then Eq. (2.13) becomes

$$\int_0^{\varphi_0} q_s(\varphi') \frac{a}{2\pi\varepsilon_0} \ln(\frac{1}{a\sqrt{2(1-\cos(\varphi-\varphi'))}})\,\mathrm{d}\varphi' = 1. \tag{2.14}$$

Equation (2.14) is a Fredholm integral equation of the first kind in terms of the unknown function q_s. Solution of this equation gives the surface charge density on the incomplete cylindrical surface.

In the next section we will survey a numerical method for solving integral equations (2.5) and (2.14).

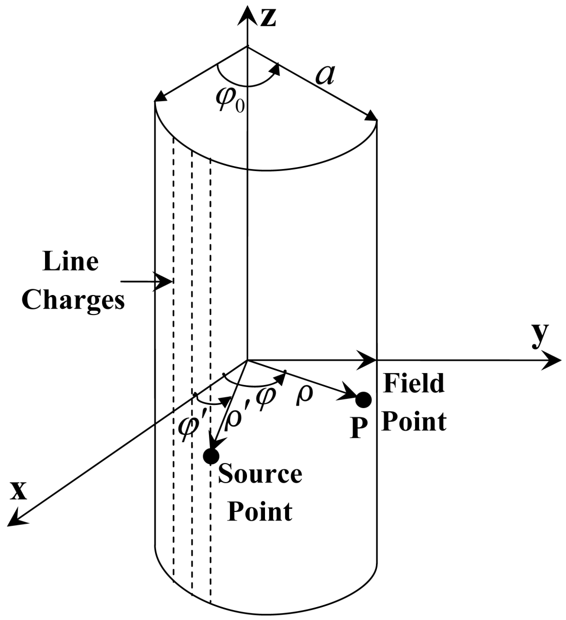

Figure 2: A conducting incomplete cylindrical surface of radius a containing electric charge.

3 Numerical method for solving Fredholm integral equation of the first kind

An efficient collocation method has been proposed and formulated by Masouri and Hatamzadeh-Varmazyar in [9], essentially for analysis of some electromagnetic scattering problems. Here, we review their method and use it for numerically solving integral equations (2.5) and (2.14).

3.1 Basis functions

Let us consider an m-set of truncated cosines for any positive integer m over real interval $[a,b)$ as [9]

$$\mathscr{T}_i(t) = \begin{cases} \cos\left(\gamma(t-a-ih-\frac{h}{2})\right), & a+ih \leqslant t < a+(i+1)h, \\ 0, & \text{otherwise}, \end{cases} \tag{3.15}$$

where $h = \frac{b-a}{m}$, $i = 0,1,\ldots,m-1$, and γ has a real value and may be considered as a regularization factor.
The above definition can obviously make a set of disjoint and orthogonal basis functions. For arbitrary i and j, such that $i = 0,1,\ldots,m-1$ and $j = 0,1,\ldots,m-1$, we have [9]

$$\begin{aligned} \left\langle \mathscr{T}_i, \mathscr{T}_j \right\rangle &= \int_a^b \mathscr{T}_i(t)\mathscr{T}_j(t)dt \\ &= \begin{cases} \frac{h}{2} + \frac{1}{2\gamma}\sin(\gamma h), & i = j, \\ 0, & i \neq j, \end{cases} \end{aligned} \tag{3.16}$$

in which $\langle .,. \rangle$ indicates the inner product.
Moreover, it is clear that function $\mathscr{T}_i$ may be considered as follows:

$$\mathscr{T}_i(t) = \varphi_i(t)\cos\left(\gamma(t-a-ih-\frac{h}{2})\right), \tag{3.17}$$

where φ_i is ith block-pulse function (BPF) defined as

$$\varphi_i(t) = \begin{cases} 1, & a+ih \leqslant t < a+(i+1)h, \\ 0, & \text{otherwise}. \end{cases} \tag{3.18}$$

The disjointness and orthogonality properties of $\mathscr{T}_i$'s can make them very efficient for approximation of functions. The expansion of a function f over $[a,b)$ with respect to $\mathscr{T}_i$, $i=0,1,\ldots,m-1$, may be compactly written as [9]

$$f(t) \simeq \sum_{i=0}^{m-1} f_i \mathscr{T}_i(t), \tag{3.19}$$

where f_i's, the expansion coefficients, may be computed by

$$\begin{aligned} f_i &= \langle f, \mathscr{T}_i \rangle \\ &= \int_a^b f(t)\mathscr{T}_i(t)dt. \end{aligned} \tag{3.20}$$

In the next subsection, we will formulate a numerical method based on these functions for solving the EFIE for the strip problem.

3.2 Formulation of numerical method

Let us consider first kind Fredholm integral equation of the form [9]

$$\int_a^b k(s,t)x(t)dt = f(s), \qquad a \leqslant s < b, \tag{3.21}$$

where the functions k and f are known but x is the unknown function to be determined. Also, $k \in \mathscr{L}^2\big([a,b)\times[a,b)\big)$ and $f \in \mathscr{L}^2\big([a,b)\big)$.
Approximating the unknown function x with respect to the truncated cosines using (3.19) gives

$$x(t) \simeq \sum_{i=0}^{m-1} x_i \mathscr{T}_i(t), \tag{3.22}$$

where x_i's are defined as in (3.20). Substituting (3.22) into (3.21) results in [9]

$$\int_a^b k(s,t)\left(\sum_{i=0}^{m-1} x_i \mathscr{T}_i(t)\right)dt = f(s), \tag{3.23}$$

or

$$\sum_{i=0}^{m-1} x_i \int_a^b k(s,t)\mathscr{T}_i(t)dt = f(s). \tag{3.24}$$

Now, choosing m appropriate points s_j, $j=0,1,\ldots,m-1$, we obtain [9]

$$\sum_{i=0}^{m-1} x_i \int_a^b k(s_j,t)\mathscr{T}_i(t)dt = f(s_j), \qquad j=0,1,\ldots,m-1. \tag{3.25}$$

(3.25) is a linear system of m algebraic equations in terms of m unknown coefficients x_i. Solution of this systems gives x_i's, and then we obtain an approximate solution $x(s) \simeq \sum_{i=0}^{m-1} x_i \mathscr{T}_i(s)$ for (3.21).

4 Numerical results

Now, we apply the numerical method reviewed in section 3 in solution of integral equations (2.5) and (2.14) to determine the surface charge distribution q_s induced on the conducting finite-width strip and incomplete cylindrical surface. The results for the strip problem are given in Fig. 3 for $a=2$ (m), which are in good agreement with the exact solution. The results for the incomplete cylindrical surface for $a=1$ (m) are shown in Figs. 4–6 respectively for $\varphi_0 = \frac{\pi}{2}$, π, and $\frac{3\pi}{2}$ (rad).

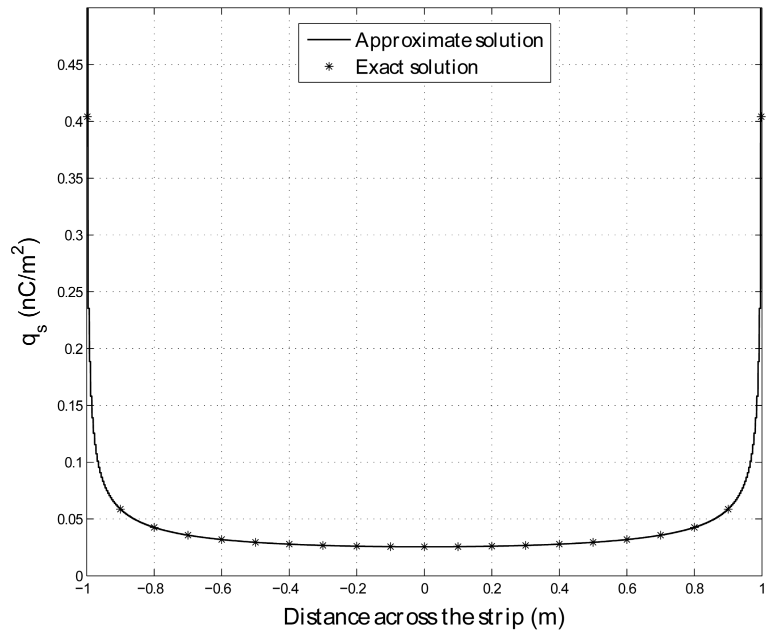

Figure 3: Electric charge distribution induced on the conducting finite-width strip for $a = 2$ (m), obtained by the numerical method together with the exact results.

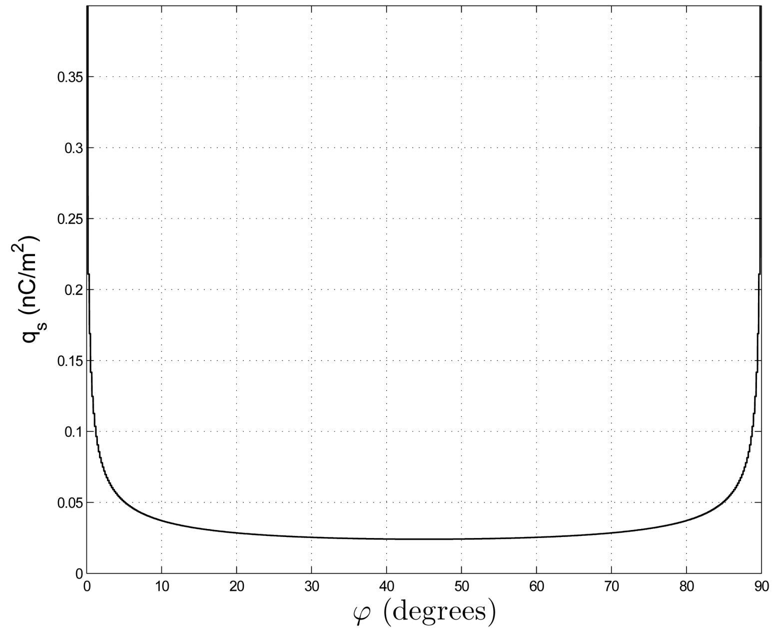

Figure 4: Electric charge distribution induced on the conducting incomplete cylindrical surface for $a = 1$ (m) and $\varphi_0 = \frac{\pi}{2}$ (rad), obtained by the numerical method.

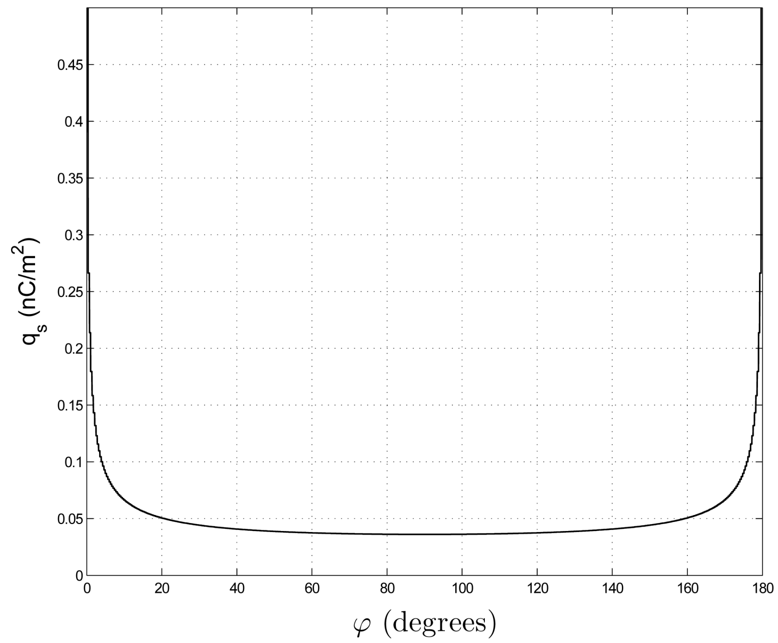

Figure 5: Electric charge distribution induced on the conducting incomplete cylindrical surface for $a = 1$ (m) and $\varphi_0 = \pi$ (rad), obtained by the numerical method.

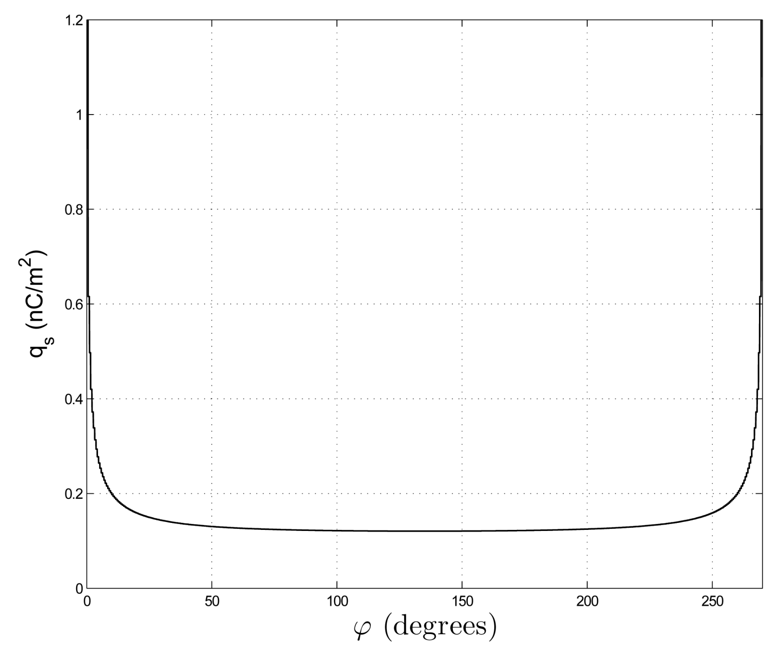

Figure 6: Electric charge distribution induced on the conducting incomplete cylindrical surface for $a = 1$ (m) and $\varphi_0 = \frac{3\pi}{2}$ (rad), obtained by the numerical method.

5 Conclusion

We analyzed in this article two problems of calculating the charge distribution on conducting structures; a finite-width strip and an incomplete cylindrical surface; based on integral equation modeling. We also reviewed an effective numerical method for obtaining an approximate solution for the models because they often have no analytical solution. The numerical results obtained by the method confirmed its computational efficiency in solution of such problems.

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