

Social Networks in a Frictional Labour Market

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1 Introduction

This dissertation consists of three essays investigating the role of social networks in a labour market with search and matching frictions. There is strong empirical evidence that 30 - 60% of new employees find jobs through their social contacts¹. Therefore, this topic has recently gained a lot of attention as it extends the famous search and matching framework of Mortensen and Pissarides to account for social networks making it even more realistic. In particular, there is an ongoing discussion in both theoretical and empirical literature about the effect of referrals on wages which is called a "referral puzzle" (Pelizzari (2010))². Thus, first of all, this thesis aims to shed light on this issue and is even able to explain both effects - positive and negative in one model (Chapter 3). In addition, the second objective of the thesis is to analyze the implications of social networks on the equilibrium welfare and market efficiency.

The impact of social contacts is considered here in the sense of an opportunity of word-of-mouth information transmission about job offers (social capital). Employees hear about vacancies and refer to them their unemployed friends. This increases unemployed workers' outside options (reservation wage) in the search theory framework thus influencing equilibrium wages, unemployment rates and the social welfare level in the decentralized economy. In particular, with full information, this dissertation shows that the presence of social networks in the model with workers heterogeneous in productivity may lead to the equilibrium inefficiency different to that found in the literature which doesn't account for social networks (Chapter 2). On the contrary, the asymmetric information of firms with respect to the workers' social capital may neutralize the standard search externality and raise welfare which is different from the classical view (Chapter 4).

It is easy to see the evolution of the dissertation exposition. The first two chapters consider the workers' productivity heterogeneity and assume its connection to the social capital. On the contrary, Chapter 4 abstracts from this since workers here are assumed to be equally productive and differ only in the amount of the social capital thus focusing purely on the effects of the latter. In addition, in the Chapter 2 there are only two worker types: high and low ability, and only professional contacts, while Chapter 3 allows for arbitrarily many worker types with different productivity and adds family contacts as well. Chapter 4 accounts for all social contacts' types useful for job search without any restrictions.

Chapter 2 investigates the social welfare in a model with two types of workers differing in their productivity (high and low ability). The main assumption is a positive correlation between the ability of a worker and her number of professional contacts who can give a reference for the job. It is justified by empirical findings of Hensvik and Skans (2013) who support the original idea of Montgomery (1991) and report that incumbent workers with a high test score

¹For example, Staiger (1990), Granovetter (1995), Pistaferri (1999), Kugler (2003), Pelizzari (2010), Bentolila et. al. (2010), Delattre and Sabatier (2007) for different countries.

²For example, Staiger (1990), Simon and Warner (1992) and Granovetter (1995) report that referrals are associated with wage premiums in the United States. The hypothesis of wage premiums is also supported by Margolis and Simonnet (2003) and Goos and Salomons (2007) for France and the United Kingdom. In contrast, Bentolila et al. (2010) report wage penalties in the United States and the European Union. This result is supported by Delattre and Sabatier (2007), Pistaferri (1999) as well as Addison and Portugal (2002) for France, Italy and Portugal respectively. Simon and Warner (1992), Montgomery (1991,1992, 1994), Mortensen and Vishwanath (1994), Kugler (2003), Ioannides and Soetevent (2006) and Galenianos (2014) provide theoretical models supporting wage premiums due to referrals while Bentolila et al. (2010) and Ponzio and Scoppa (2010) argue in favor of wage penalties. Tumen (2013) and Zaharieva (2015) explain both effects depending on parameters.

are more likely to be linked to the new hires than low ability employees. For simplicity, high ability workers are assumed to be linked in a network and have the same exogenous number of professional contacts. In contrast, the number of professional contacts of low ability workers is normalized to zero. There are two labour markets in the economy: regular and referral. As wages are determined through the Nash bargaining, high-ability workers are better paid than low ability workers. On the one hand, high ability workers are more productive which leads to higher wages. On the other hand, their reservation wages are high due to the additional possibility of finding jobs through the network of contacts.

High ability workers are more productive but they also bargain a higher wage. Which of these two effects is dominating for profits depends on the productivity difference between workers and the number of social contacts. If the productivity (wage) effect is dominating then the expected profit of firms in the regular market is increasing (decreasing) in the proportion of high ability workers. For realistic parameter values, the effect of higher wages is dominating already for a small number of social contacts and therefore high ability workers impose a negative externality on low ability workers. This effect generates an equilibrium wage dispersion which is inefficiently large. The optimal policy in this case is associated with increasing (decreasing) the reservation wage of low (high) ability workers through redistributive taxes and subsidies and reducing the equilibrium inequality in wages. This finding questions the traditional view that social contacts increase efficiency by mitigating the problem of adverse selection (see Montgomery (1991)). It is also different from the literature on heterogeneous worker groups where wages are generally compressed when two types of workers are simultaneously searching for jobs in the same labour market (see Blazquez and Jansen (2008)). In addition, firms hiring through referrals do not impose a negative search externality on other firms which is the case in the regular market. Therefore, employment subsidies in the referral market should be imposed.

Chapter 3 of the thesis explains an empirically observed U-shape referral hiring pattern, namely that referrals are mostly used by workers in the tails of the skill distribution, whereas all other workers in the middle are more likely to use a formal channel of job search³. There are arbitrarily many worker types different in their productivity and three job search channels in the model: costly formal applications and two costless informal channels - through family and professional networks. Every worker has the same small probability of hearing about job openings through her family members, while the model is also robust to the endogenous job-finding rate through family. However, the job-finding rate through the network of professional contacts is productivity-specific. Every worker has a fixed number of professional contacts but a strong degree of network homophily along the productivity dimension is assumed.

Workers choose the search intensity through the formal channel while firms choose the advertising intensity of open positions to their incumbent employees. The result of the model is a strong self-selection of workers on productivity across the three channels: low (high) productivity types rely more on family (professional) contacts while middle productivity workers search rather formally. Moreover, as a wage bargained is a function of the worker's productivity, combining family and professional referrals into one informal channel may generate a spurious result of wage premiums (penalties) if high (low) productivity workers are dominating in the

³It was found in empirical studies by Brown et al. (2012) for the US, Corak and Piraino (2011) for Canada, Boxman et al. (1991) for the Netherlands

empirical data. Thus, the novelty of the model is that it is able to explain both average effects of referrals on wages: positive and negative, and the U-shape referral hiring pattern in a unified framework.

The mechanism generating the U-shape is following. Low productivity workers expect low wages thus it is not optimal for them to exert costly search effort. At the same time hiring these workers is not profitable for firms, so that firms prefer to direct their search towards more productive worker groups. Hence low productivity workers rely on family referrals as a method of last resort. Average workers mostly use the formal channel: their expected wages are sufficiently high and motivating to exert search effort but their outside opportunities in terms of professional referrals are not yet too good. Finally, high productivity workers naturally tend to find their jobs by means of professional referrals.

Chapter 4 of the dissertation presents a model in which firms are uncertain about the job seekers' number of friends, who can help them in the job search (or their outside options). All workers have the same productivity and differ only in the social capital. For simplicity, there are only two worker types in the model: low and high type. A firm offers a take-it-or-leave-it wage contract to a worker after checking the worker's profile and the non-fictitious number of social contacts in the Social Network System (SNS) in the Internet (for example, Facebook, LinkedIn or Xing) for the worker to accept. This number serves as a noisy signal of the social capital for firms. Indeed, "nearly 39 percent of firms use SNS to research job candidates, up from 37 percent in 2012" (CareerBuilder.com (2013)). Firms pay attention, whether the job seeker has great communication skills and whether other people posted great references about the candidate.

The model generates a positive relationship between the number of contacts in the Social Network System and the wage offered by firms in the equilibrium. Thus, there will be a wage dispersion between the equally productive workers with different number of contacts in the Internet, which extends the classical result on wage dispersion with respect to the signal in the literature on uncertainty about the workers productivity (for instance, Spence (1973)).

In addition, the comparative statics w.r.t. the firms' uncertainty level increase was conducted. Moreover, the equilibrium outcomes were compared numerically with the two extreme cases: the case of perfect information, when workers' social capital is observed perfectly, and the case of a full information asymmetry, when firms don't have any reliable signal to make inferences about workers' outside options. The overall social welfare turns out to be increasing with the increase in the firms' uncertainty level.

One of the reasons for this is that firms anticipate that expected profits from an open vacancy will decrease due to more mismatched wages offered and open less vacancies thus decreasing their overall cost and leading to the welfare increase. So the information asymmetry turns out to be welfare improving as firms, by chance, will employ less workers which they would not like to employ. In the standard search theory with perfect information (for example, Pissarides (2000)), the social welfare is maximized when the workers' bargaining power is equal to the elasticity of the job-filling rate. This result is known as the Hosios condition. Otherwise, when their bargaining power is too low (high), firms will open too many (few) vacancies due to low (high) wages leading to more inefficiency. Since in the present model the wage is offered only by firms, the workers' wages are relatively low. This gives an intuition why the social welfare in the perfect

information case is not the largest since the Hosios condition is not satisfied.

It is interesting to compare this finding to the conclusion of Montgomery (1991) that social contacts use leads to a higher level of social welfare due to a lower mismatch between firms and workers as referrals reveal the quality of the match. In the present paper, a higher level of mismatched wages offered contributes to the increase in the welfare.

2 Optimal policy and the role of social contacts in a search model with heterogeneous workers

2.1 Introduction

The purpose of this paper is to investigate social welfare and optimal policy in a search model with heterogeneous workers. In our model workers differ with respect to their productivity (high and low ability workers) and the structure of social networks, in particular, there is a positive correlation between ability and the number of professional contacts. In this setting, when both types of workers are mixed in the regular labour market, the decentralized equilibrium is inefficient as high ability workers congest the market for workers with low abilities. Moreover, this inefficiency is increasing in the number of social contacts and is associated with a larger wage gap between the two groups of workers. This finding questions the traditional view that social contacts increase efficiency by mitigating the problem of adverse selection (see Montgomery (1991)). It is also different from the literature on heterogeneous worker groups where wages are generally compressed when two types of workers are simultaneously searching for jobs in the same labour market (see Blazquez and Jansen (2008)).

There is strong empirical evidence that 30 – 60% of new hires find jobs through personal contacts (see for example Staiger (1990), Granovetter (1995), Pistaferri (1999), Kugler (2003), Pelizzari (2010), Bentolila et. al. (2010) for different countries). In addition, Hensvik and Skans (2013) report that incumbent workers with a high test score are more likely to be linked to the new hires than low ability employees. In particular, in their data firms rely on referrals from high-ability workers in order to attract applicants with higher unobserved ability. To incorporate these empirical findings into the model we assume that high ability workers are linked in a network and have the same exogenous number of professional contacts who can give a reference for the job. In contrast, low ability workers do not have professional contacts and are restricted to search for jobs in the regular labour market. Therefore, there is a tight connection in the model between the productivity of the worker and the amount of social capital.

The choice of search methods by firms is endogenous. When entering the labour market, firms decide between a high cost vacancy in the regular job market and a low cost informal job opening in the referral market. The pool of job applicants in the regular labour market is mixed as both types of unemployed workers apply for the publicly advertised positions. On the contrary, the pool of applicants in the referral market is limited to unemployed workers with high ability as only these workers are connected in a network. This assumption is in line with the original idea of Montgomery (1991) that workers hired through social networks are on average more productive than job applicants hired through the formal channel of search.

To keep the model tractable we assume that the worker type is immediately observed by the firm upon the match. Thus there is no asymmetric information in the model and wages are negotiated ex-post between the firm and the applicant by means of the individual Nash-bargaining. Depending on the parameters, there are two types of equilibria. If the number of social contacts is low it is not optimal for firms to rely solely on referrals as the probability of hiring in the referral market is relatively low. In this situation there exists a unique equilibrium without referrals where both types of workers are mixed in the regular labour market. In contrast, if the number of social contacts is sufficiently large, then some firms prefer to use referrals in

the hiring process, so both search channels are active in the equilibrium. In the numerical part of the paper we estimate the net welfare gain of referrals at 1.2%.

High-ability workers are better paid than low ability workers. On the one hand, high ability workers are more productive which leads to higher wages. On the other hand, their reservation wages are high due to the additional possibility of finding jobs through the network of contacts. In this setting, the model predicts that a larger number of social contacts puts an upward pressure on wages of high ability workers and reduces the equilibrium unemployment of these workers. Low ability workers are negatively affected: their wages fall and the unemployment rate is increased. This implies that a more intensive use of referrals is associated with an increased wage dispersion between the two groups of workers. Thus a more important role of social networks in the modern society may provide an additional explanation for the increased income inequality in the United States in the recent decade. Some indirect support for this argument can also be found in Dawid and Gemkow (2013). These authors find that an increase in network density leads to a polarization of firms and a concentration of more productive workers at firms with high productivities (and wages) thereby enlarging the wage dispersion.

Next our model predicts that the decision of firms to use referrals may be inefficient from a social perspective. The job-filling rate in the referral market does not depend on the number of other informal vacancies in this market. It is rather that the hiring probability depends on the architecture of the social network. So firms hiring through referrals do not impose a negative search externality on other firms which is the case in the regular market. From a social perspective this means that vacancies in the referral market should be created up to the point where the expected cost of an open position is equal to the expected surplus of a filled job. In contrast, in the decentralized equilibrium firms start using referrals at the point where the expected cost is equal to the expected profit. This means that the optimal threshold number of contacts which is necessary for firms to use referrals is lower than the minimum number of contacts in the decentralized economy. In the paper we show that this inefficiency may be mitigated by means of employment subsidies in the referral market. In reality such subsidies can take the form of referral bonuses which are reimbursed by the state.

Finally, we identify a pooling inefficiency in the regular labour market. High ability workers are more productive but they also bargain a higher wage. Which of these two effects is dominating for profits strongly depends on the productivity difference between the two types of workers and the number of social contacts. If the productivity (wage) effect is dominating then the expected profit of firms in the regular market is increasing (decreasing) in the proportion of high ability workers. In a numerical example we show that the effect of higher wages is dominating already for a small number of social contacts and therefore high ability workers impose a negative externality on low ability workers. This effect generates an equilibrium wage dispersion which is inefficiently large. The optimal policy in this case is associated with increasing (decreasing) the reservation wage of low (high) ability workers and reducing the equilibrium inequality in wages. Moreover, we show that this pooling inefficiency is an artefact of referrals and does not exist in the labour market without social contacts. Once the optimal policy is implemented the net welfare gain of referrals rises to 1.8%.

2.1.1 Related literature

This paper is closely related to the literature on heterogeneous workers and social networks. Albrecht and Vroman (2002) is the first study analysing an economy with skill differences across workers and varying skill requirements of firms. Gautier (2002) extends their framework to allow for on-the-job search and Blazquez and Jansen (2008) analyse welfare in a model economy of Albrecht and Vroman (2002). Our comparative static result is similar to the one reported in Gautier (2002), namely that mixing two types of workers in the same labour market may congest the market for low ability workers and their unemployment may increase with a higher proportion of high ability workers. However, the channel of influence is different in our model. In Gautier (2002) high-skilled workers employed in simple jobs continue searching on-the-job and so the expected profit of firms in this market is reduced due to the lower job duration of high-skilled workers. In contrast, in our model the average job duration is the same for both groups of workers and the negative effect on profits is produced due to better outside opportunities of high-skilled workers. This effect is absent in Gautier (2002) as outside opportunities do not affect wages in his model.

From the perspective of welfare our paper is close to Blazquez and Jansen (2008) and Igarashi (2013). The first two authors find that wage bargaining when agents are matched at random compresses the wage distribution relative to workers' shadow values and doesn't lead to the efficient outcome. This means that low (high) ability workers receive more (less) in the decentralized equilibrium than in the socially efficient allocation. We show that this situation is a special case of our model when productivity differences are large and the number of social contacts is small. However, in the opposite case when the number of social contacts is large and output differences are small the direction of inefficiency is reversed and the gap in wages is inefficiently large. This is a situation which is not considered by Blazquez and Jansen (2008).

Igarashi (2013) considers a search model with two groups of workers – networked and non-networked workers – which is similar to our study. However, the primary research question is different between the two papers. While Igarashi (2013) investigates the question whether restricting the network search will have a positive or a negative effect on the welfare of non-networked workers, having an additional channel of search is always beneficial in our model for the aggregate social welfare. Therefore, the primary task of our paper is on designing the optimal budget transfers to compensate the group of agents who is adversely affected by the presence of networks. Besides that, our model has two dimensions of heterogeneity (productivity and networks) and the network size is explicitly included as a parameter of the model, whereas in the model of Igarashi (2013) all workers are identical with respect to their productivity and every networked worker knows all other workers of this group.

From the perspective of social networks our study is most closely related to Cahuc and Fontaine (2009) and Zaharieva (2013). Cahuc and Fontaine (2009) restrict workers to choose between the two job search methods, so in their model, decentralized decisions by workers and firms to use networks can suffer from a coordination failure. On the contrary, the choice of search methods is not limited in our model and thus both search channels are simultaneously used by high ability workers to find a job. Zaharieva (2013) considers a matching model with family networks and wage posting and examines welfare in this model. Wage posting and directed search lead to the ex-ante separation of unemployed workers in the regular labour market.

Consequently the decentralized equilibrium with family contacts is constrained efficient.

Early economic studies on social contacts include Montgomery (1991, 1992, 1994) and Mortensen and Vishwanath (1994). The focus of Montgomery (1991) is on the effect of asymmetric information on wage inequality in the presence of the "inbreeding bias", implying clustering of workers with respect to their ability type. As a result the equilibrium is characterized by the positive correlation between ability and wages. Mortensen and Vishwanath (1994) consider the population of workers differing with respect to the probability of receiving job offers through personal contacts, they show that wages paid in jobs obtained through personal contacts are more likely to be higher than wage offers obtained through a direct application. This conclusion is questioned in the recent empirical literature, and moreover, "both the models of Montgomery (1991) and Mortensen and Vishwanath (1994) ignore what may be the most important role for network: to increase the job offer arrival rate." (p. 7, Margolis and Simonnet (2003)).

Recent theoretical studies emphasizing the positive effect of referrals on wages include Kugler (2003) and Galenianos (2014). Specifically, Kugler (2003) finds that the benefit of using referrals for firms is that they lower monitoring costs, because workers can exert peer pressure on co-workers. As a result, firms relying on referrals find it cheaper to elicit effort by paying efficiency wages than firms using formal hiring methods. Galenianos (2014) extends the original idea of Montgomery (1991) and shows a positive link between the intensity of referrals and the job finding rate. Other studies investigating the link between referrals and the job-finding rate are Calvo-Armengol and Jackson (2004, 2007) as well as Fontaine (2004, 2007, 2008). A larger overview of this literature can be found in Ioannides and Datcher Loury (2004).

The plan of the paper is as follows. Section 2.2 explains notation and the general economic environment. Section 2.3 deals with the existence of the decentralized equilibrium. Section 2.4 contains welfare analysis of the decentralized equilibrium. Section 2.5 illustrates our theoretical results by means of a numerical example, while section 2.6 concludes the paper.

2.2 Labour market modeling framework

The labour market is characterized by the following properties. There is a unit mass of infinitely lived risk neutral workers and an endogenous number of firms, both workers and firms discount the future at rate r . Workers are ex-ante heterogeneous with respect to their ability and social capital. Let μ denote the fraction of low ability workers, once employed these workers produce the flow output y_0 . The fraction of high ability workers is $1-\mu$, these workers are more productive and generate the flow output $y_1 \geq y_0$ when employed. Output variables y_0 and y_1 are known to workers and are immediately observable by firms upon the match. So there is no problem of asymmetric information in the model.

From the perspective of social capital, high ability workers have an equal number of professional contacts $l > 0$. The network size of low ability workers is normalized to zero. Despite simplifying the model, this assumption preserves the idea that worker's ability and the number of professional contacts are positively correlated. By professional contacts we mean connected employees who are willing to refer a given unemployed worker to the potential employer. Workers can be either employed and producing output or unemployed and searching for a job. Let u_1 and u_0 denote the total numbers of unemployed workers with high and low ability, so that $u \equiv u_0 + u_1$. Unemployed workers enjoy the flow value of leisure ζ , but also search for jobs which

is associated with a search cost h in the regular market. Workers do not incur search costs if they find a job by recommendation in the referral market. In order to simplify the notation we set $z = \zeta - h$ which is a net value of leisure for workers.

Every firm entering the labour market can choose between a public vacancy in the regular submarket at cost c and a job opening in the referral submarket at cost c_2 . Following the literature (for example, Cahuc and Fontaine (2009) and Zaharieva (2013)) we assume that referral openings allow firms to save on the advertising costs so that $c_2 \leq c$. Let v and v_2 be the numbers of vacancies in the two submarkets respectively. Job information in the referral submarket is exclusively transmitted by employees, therefore, due to the absence of professional contacts, workers with low ability are restricted to search in the regular job market. On the contrary, high ability workers can simultaneously search in both submarkets. The matching function in the regular job market is then given by $m(u, v)$, and the market tightness is $\theta = v/u$. This matching function is assumed to be increasing in both arguments, unemployment and vacancies, concave, and exhibiting constant returns to scale. Therefore, the job finding rate $\lambda(\theta)$ and the vacancy filling rate $q(\theta)$ in the regular job market are given by:

$$q(\theta) = m(u, v)/v = m\theta^{-\eta} \quad \lambda(\theta) = \theta q(\theta) = m\theta^{1-\eta}$$

where $0 < \eta < 1$ is the elasticity of the job filling rate with respect to the market tightness.

In the referral market firms with open positions contact high ability employees at an exogenous rate a per unit time (see Cahuc and Fontaine (2009)). Every employee who was contacted by the firm transmits vacancy information to exactly one randomly chosen unemployed social contact out of a pool of l contacts. Here we assume that job information is only transmitted to the direct social links, so the job offer is lost if all l contacts are employed. The matching function in the referral job market is therefore equal to $m_1(u_1, v_2) = av_2[1 - (1 - \frac{u_1}{1-\mu})^l]$. The term in brackets is the probability to meet an employee with at least one unemployed social contact (as $(1 - \frac{u_1}{1-\mu})^l$ is the probability that all l contacts are employed). Therefore this matching function can be understood as the number of vacancies in the referral job market sent to the employees with at least one unemployed contact at rate a . The job finding rate λ_2 and the vacancy filling rate q_2 in the referral job market are given by:

$$q_2 = m_1(u_1, v_2)/v_2 = a[1 - (1 - \frac{u_1}{1-\mu})^l] \quad \lambda_2 = m_1(u_1, v_2)/u_1 = av_2[1 - (1 - \frac{u_1}{1-\mu})^l]/u_1$$

The job-filling rate q_2 doesn't depend on the total number of vacancies v_2 which means that new job openings in the referral market don't change the hiring probability of other firms. A more general representation of the matching process between employees and open vacancies in the referral market would be $m_1(u_1, v_2) = a(1 - \mu - u_1)^\psi v_2^{1-\xi}[1 - (1 - \frac{u_1}{1-\mu})^l]$ where $1 - \mu - u_1$ is a total number of high ability employees. Thus our specification corresponds to the case $\psi = \xi = 0$. Even though $\psi = 0$ is a simplifying assumption¹, $\xi = 0$ is a fundamental property of

¹In a companion paper Stupnytska and Zaharieva (2015) we consider a more general case $\psi > 0$. We find that this extension doesn't change the qualitative properties of the model, with the exception that it may give rise to multiple equilibria. Note that the extended matching function with $\psi > 0$ is equal to zero for $u_1 = 0$ and $u_1 = 1 - \mu$. It means that a larger number of unemployed agents raises the probability that a randomly contacted employee will recommend his/her contact for the job ($[1 - (1 - \frac{u_1}{1-\mu})^l]$ is increasing). But when the number of unemployed workers is increasing further, then there are fewer employees who can give a recommendation

the referral process. If a firm in the referral market is in contact with some employee then the probability of filling a vacancy depends on the number of contacts in the network of this employee and on the probability of having at least one unemployed worker in the network. However, it does not depend on the number of other vacancies in the referral market as these vacancies are not observable by unemployed workers. This is different in the regular market with random matching: vacancy information is public and observable in this submarket and therefore a new posted vacancy will reduce the probability of getting an application for competing vacancies (for a given fixed total number of unemployed workers). This search externality in the regular market is a well documented empirical fact, see Petrongolo and Pissarides (2001) for a survey.

Finally, note that firms have to make a decision whether to post a regular vacancy or fill the opening through the informal channel of search. Moreover, when making this decision firms face a trade off between the quality of the applicant pool and the total number of job applicants. Since both types of workers are mixed in the regular job market the average productivity of an applicant in this market is low but the total number of searching workers is large. On the contrary, the total number of applicants having access to the referral market is low but the average productivity is high since only high ability workers are recommended through this channel. Finally, any job can be destroyed for exogenous reasons with a Poisson destruction rate δ . Upon a separation the worker becomes unemployed and the firm may open a new job.

2.3 The decentralized equilibrium

2.3.1 Bellman equations

Let U_i , $i = 0, 1$ denote the present values of being unemployed and, similarly, W_i , $i = 0, 1, 2$ – the present values of being employed. The subindex 0 refers to low ability workers. The subindex 1 refers to high ability workers obtaining jobs in the regular market, while the subindex 2 stands for the present values of workers finding jobs in the referral market. The structure of the labour market is illustrated in figure 2.1. In addition, let variables τ_0 and τ_1 denote the flow values of transfers that unemployed workers receive from the public budget. The present values U_0 and U_1 for the unemployed can be written as:

$$rU_0 = z + \tau_0 + \lambda(\theta)(W_0 - U_0) \quad rU_1 = z + \tau_1 + \lambda(\theta)(W_1 - U_1) + \lambda_2(W_2 - U_1) \quad (2.1)$$

where the latter equation incorporates the fact that high ability workers can simultaneously search for jobs in both submarkets. The present values W_i for the employed are given by:

$$rW_0 = w_0 - \delta(W_0 - U_0) \quad rW_i = w_i - \delta(W_i - U_1), \quad i = 1, 2 \quad (2.2)$$

Next consider firms and let J_i , $i = 0, 1, 2$ denote the present values of profits. Bellman equations for filled jobs are then given by:

$$rJ_i = y_i - w_i - \delta J_i, \quad i = 0, 1 \quad rJ_2 = y_1 - w_2 - \delta J_2 \quad (2.3)$$

($a(1 - \mu - u_1)^\psi v_2^{1-\xi}$ is decreasing). Thus we can prove that in this extended economy there may be two equilibria for the same parameter values; in the first one, the unemployment rate is low, in the second one, the unemployment rate is high, but firms obtain the same profits and are indifferent between the two equilibria.

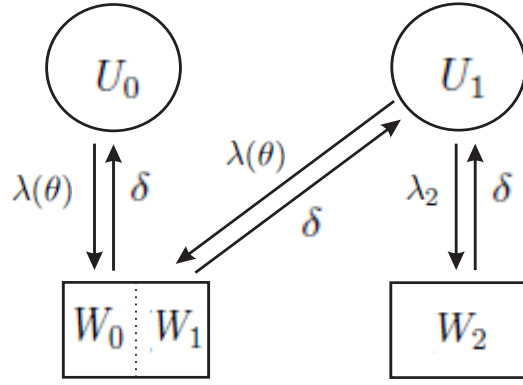


Figure 2.1: Labour market flows

Further, we describe firms with open vacancies. In the regular labour market, let $\gamma = u_0/u$ denote the probability of meeting a low ability unemployed worker, so that $1 - \gamma = u_1/u$ is the probability of meeting a high ability unemployed worker. Besides, let s denote the flow values of transfers that firms in the referral market obtain from the public budget. For example, these transfers can cover the traveling expenses of job applicants and the costs of accommodation at the place of the job interview. In the next section we consider the optimal policy of the social planner, so the vector of policy instruments $\{\tau_0, \tau_1, s\}$ will allow the social planner to affect wages and the job creation. The present values of open vacancies V and V_2 in the regular and the referral market respectively can then be written as:

$$rV = -c + q(\theta)(\gamma J_0 + (1 - \gamma)J_1 - V) \quad rV_2 = -c_2 + s + q_2(J_2 - V_2) \quad (2.4)$$

where the term $\gamma J_0 + (1 - \gamma)J_1$ is the expected present value of firm profits in the regular labour market. In the following we investigate the economy in the steady state. Hence, the equilibrium unemployment for both types of workers reads:

$$u_0 \lambda(\theta) = \delta(\mu - u_0) \quad u_1(\lambda(\theta) + \lambda_2) = \delta(1 - \mu - u_1) \quad (2.5)$$

Each of these equations implies that the inflow of workers into unemployment (on the right-hand side) is equal to the outflow of workers from this state (on the left-hand side). It is easy to see therefore that u_0 decreases in θ and that workers with low ability face a higher equilibrium unemployment rate: $u_0/\mu > u_1/(1 - \mu)$.

The steady state conditions (2.5) allow us to express the equilibrium probability of being in contact with a low ability worker in the following way:

$$\gamma \equiv \frac{u_0}{u_0 + u_1} = \frac{\mu(\delta + \lambda(\theta) + \lambda_2)}{\delta + \lambda(\theta) + \mu\lambda_2} = \frac{\delta\mu}{\delta\mu + (\delta + \lambda(\theta))u_1}$$

This means that $\gamma(\theta, u_1)$ is decreasing in both θ and u_1 . Intuitively, a higher market tightness θ reduces the equilibrium unemployment of low ability workers u_0 , so the probability that a randomly chosen applicant is of low ability is decreasing in θ . Similarly, more high-skilled unemployed workers u_1 reduce the chances of meeting a low ability unemployed worker. Finally, note that $\gamma > \mu$ in the presence of social contacts, while $\gamma = \mu$ otherwise. Networks reduce unemployment of high ability workers, so in the equilibrium with social contacts firms are less

likely to meet these workers in the regular market: $(1 - \gamma) < (1 - \mu)$. Next section investigates existence and uniqueness of the decentralized equilibrium with social contacts.

2.3.2 Wage determination and the free-entry conditions

This section investigates the labour market without policy instruments ($\tau_0 = \tau_1 = s = 0$). Both the efficient resource allocation and the optimal policy are later addressed in section 2.4.

The equilibrium wages are determined by means of Nash bargaining. When bargaining over w_0 unemployed low ability workers act to maximize the total job rent $W_0 - U_0$ which is an increasing function of w_0 . Similarly, unemployed high ability workers act to maximize the rent $W_i - U_1$, where the subindex i takes values 1 or 2 depending on the type of search channel. Firms are maximizing the surplus value J_i , $i = 0, 1, 2$ so the rent sharing conditions become:

$$J_0 = \frac{(1 - \beta)}{\beta}(W_0 - U_0) \quad J_i = \frac{(1 - \beta)}{\beta}(W_i - U_1), \quad i = 1, 2 \quad (2.6)$$

where we impose the free-entry conditions $V = V_2 = 0$, therefore in the equilibrium firms are indifferent between a formal vacancy in the regular market and an informal vacancy through referrals. The corresponding wage equations are given by:

$$w_0 = \beta y_0 + (1 - \beta)rU_0 \quad w_1 = w_2 = \beta y_1 + (1 - \beta)rU_1 \quad (2.7)$$

Denote $S_0 = J_0 + W_0 - U_0$ the total job surplus in a match between a firm and a low ability worker, similarly let $S = J_i + W_i - U_1$, $i = 1, 2$ the total job surplus in a match between a firm and a high ability worker. Note that S is independent of the search channel, so that $J_1 = J_2$ and $W_1 = W_2$. This is because bargaining is an ex-post wage setting mechanism so the sunk costs of open vacancies are not directly reflected in wages. Surplus values S_0 and S are given by the following system of equations:

$$S_0(\theta) = \frac{y_0 - z}{r + \delta + \beta\lambda(\theta)} \quad S(u_1) = \frac{y_1 - z}{r + \delta + \beta\delta(1 - \mu - u_1)/u_1}$$

where in the last expression we make use of the steady-state condition $\lambda(\theta) + \lambda_2 = \delta(1 - \mu - u_1)/u_1$. Intuitively, a higher market tightness θ improves the outside opportunities of low ability unemployed workers $rU_0 = z + \beta\lambda(\theta)S_0$, so the total job surplus $S_0(\theta)$ is decreasing in θ . At the same time, in the equilibrium a higher number of unemployed high ability workers u_1 can only be attributed to a lower job-finding rate $\lambda(\theta) + \lambda_2$. In this latter case the reservation wage of high ability workers is also lower $rU_1 = z + \beta(\lambda(\theta) + \lambda_2)S$, hence the total job surplus $S(u_1)$ is increasing in u_1 .

The free-entry conditions in each of the two submarkets are then given by:

$$\frac{c_2}{q_2(u_1)} = (1 - \beta)S(u_1) \quad \frac{c}{q(\theta)} = (1 - \beta)[\gamma(\theta, u_1)S_0(\theta) + (1 - \gamma(\theta, u_1))S(u_1)]$$

Both of these equations suggest that the expected cost of an open vacancy in the equilibrium should be equal to the expected present value of profits. Consider first the referral market. Expression $c_2/q_2(u_1)$ is decreasing in u_1 since more high ability unemployed workers make it easier for firms to fill informal vacancies. At the same time a higher unemployment level u_1

worsens the bargaining position of workers. This leads to lower wages w_2 and higher profits $J_2 = (1 - \beta)S(u_1)$ in the referral market. Consequently the free-entry condition in the referral market defines a unique equilibrium value of u_1 if $c_2/a < (1 - \beta)(y_1 - z)/(r + \delta)$. In the following we assume that this condition is satisfied. Now consider the regular labour market. The right hand side of the corresponding free-entry condition is an expected firm profit from an open vacancy in the regular job market. Indeed, bargaining implies that firms obtain a fraction $1 - \beta$ of the total surplus and with probability $\gamma(\theta, u_1)$ the firm is in contact with a low ability worker.

Define the equilibrium with social contacts in the following way:

Definition 2.1. *Search equilibrium with social contacts is a vector of variables $\{U_0, U_1, W_i, J_i, V, V_2, w_i, \theta, u_1\}$, $i = 0, 1, 2$ satisfying the asset value equations for workers (2.1) and (2.2), for firms (2.3) and (2.4), the rent-sharing equations (2.6) as well as the free-entry conditions $V = 0$ and $V_2 = 0$.*

Further note that existence of the equilibrium with social contacts implies

$$v_2 \geq 0 \quad \Leftrightarrow \quad \lambda_2 \geq 0 \quad \Leftrightarrow \quad \lambda(\theta) \leq \delta(1 - \mu - u_1)/u_1 \quad \text{for a given } u_1$$

which imposes an upper bound on the equilibrium market tightness θ . This means that existence of the equilibrium with referrals is not always guaranteed. Our results concerning this question are summarized in proposition 2.1.

Proposition 2.1. *Define the upper bound $\bar{\theta}(u_1)$ implicitly from equation $\lambda(\bar{\theta}(u_1)) \equiv \delta(1 - \mu - u_1)/u_1$. Then there exists an equilibrium with referrals if the following condition is satisfied²:*

$$\textbf{Condition A:} \quad \frac{c}{q(\bar{\theta}(u_1))} > (1 - \beta) \left(\frac{\mu y_0 + (1 - \mu)y_1 - z}{r + \delta + \beta \lambda(\bar{\theta}(u_1))} \right)$$

where u_1 is determined from the job creation condition in the referral market (JC₂):

$$\frac{c_2}{a \left[1 - \left(1 - \frac{u_1}{1 - \mu} \right)^l \right]} = \frac{(1 - \beta)(y_1 - z)}{r + \delta + \beta(1 - \mu - u_1)\delta/u_1}$$

and $\theta \leq \bar{\theta}(u_1)$ is given by the job creation condition in the regular market (JC):

$$\frac{c}{q(\theta)} = (1 - \beta) \left(\frac{\gamma(\theta, u_1)(y_0 - z)}{r + \delta + \beta \lambda(\theta)} + \frac{(1 - \gamma(\theta, u_1))(y_1 - z)}{r + \delta + \beta(1 - \mu - u_1)\delta/u_1} \right)$$

Moreover, wage dispersion $\Delta w = w_2 - w_0 = w_1 - w_0$ is decreasing in θ and u_1 .

Proof: Appendix 2.8.1.

Suppose condition A is satisfied for some $l > 0$, which means there exists an equilibrium with referrals. A higher number of social contacts makes information transmission more efficient in the referral market. Therefore the equilibrium unemployment of high ability workers is unambiguously decreasing in the number of contacts. Moreover in the limiting case $l \rightarrow \infty$ the job-filling rate in the referral market approaches its upper bound $q_2 \rightarrow a$, hence u_1 asymptotically converges to its minimum value $c_2\beta\delta(1 - \mu)/[a(1 - \beta)(y_1 - z) - c_2(r + \delta(1 - \beta))]$.

² $\lambda(\bar{\theta}(u_1)) \equiv \delta(1 - \mu - u_1)/u_1$ implies that $\bar{\theta}(u_1) = \frac{1}{m} \left[\delta \frac{(1 - \mu - u_1)}{u_1} \right]^{\frac{1}{1 - \eta}}$

In the opposite case a lower number of social contacts raises the equilibrium unemployment of high ability workers u_1 . So there is a negative impact on the upper bound of the equilibrium market tightness $\bar{\theta}(u_1)$. With respect to condition A this means that the difference between the left hand side and the right hand side is diminishing with a lower number of social contacts l (see figure 2.2). Therefore there exists a threshold value $l_0 > 0$ such that condition A is satisfied with a strict equality. This automatically implies that the equilibrium with referrals does not exist for $l \leq l_0$. These results are summarized in corollary 2.1:

Corollary 2.1. *For $l < l_0$ there exists a unique search equilibrium without referrals, where the market tightness θ^* is given by:*

$$\frac{c}{q(\theta^*)} = (1 - \beta) \left(\frac{\mu y_0 + (1 - \mu)y_1 - z}{r + \delta + \beta\lambda(\theta^*)} \right)$$

The threshold number of social contacts l_0 can be obtained from $\theta^* = \bar{\theta}(u_1(l_0))$ and is given by:

$$l_0 = \frac{\ln \left(a(1 - \beta)(y_1 - z) - c_2(r + \delta + \beta\lambda(\theta^*)) \right) - \ln a(1 - \beta)(y_1 - z)}{\ln \lambda(\theta^*) - \ln (\delta + \lambda(\theta^*))}$$

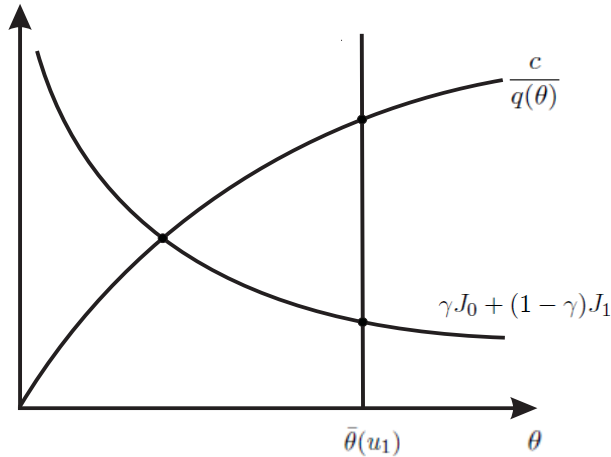


Figure 2.2: Existence of the decentralized equilibrium

Intuitively, if the number of social contacts is low $l \leq l_0$ it is not profitable for firms to rely solely on referrals. This means that social contacts are not valuable and wage dispersion is purely attributed to differences in the productivity: $\Delta w = \beta(y_1 - y_0)$. Moreover, the equilibrium unemployment is the same for both types of workers: $u_0/\mu = u_1/(1 - \mu) = \delta/(\delta + \lambda(\theta^*))$.

2.3.3 Participation decisions

In the previous section, we have investigated the decision of firms to use referrals as a hiring channel. In this section we consider the decision of workers to apply for regular vacancies, which is a costly search channel for both types. Recall from before that $z = \zeta - h$, where h is the cost of searching for regular vacancies. Intuitively, one would expect that a high search cost may lead to situations when some workers don't search in the regular market. Consider first

the decision of unemployed high ability workers. For given variables $\lambda(\theta)$ and λ_2 , their present value of unemployment is given by:

$$rU_1 = \zeta - h + (\lambda(\theta) + \lambda_2)\beta\left(\frac{y_1 - \zeta + h}{r + \delta + \beta(\lambda(\theta) + \lambda_2)}\right) \quad \text{versus} \quad \zeta + \lambda_2\beta\left(\frac{y_1 - \zeta}{r + \delta + \beta\lambda_2}\right)$$

where the last term is a hypothetical present value of unemployment if high ability workers stopped searching in the regular market. Comparing these two values reveals the threshold search cost h_1 for a given vector $\{\lambda(\theta), \lambda_2\}$:

$$h_1 = \frac{\lambda(\theta)\beta(y_1 - \zeta)}{r + \delta + \beta\lambda_2} \quad (2.8)$$

Thus, for high ability workers searching in the regular market is gainful only if the cost is not too high, i.e. $h < h_1$. Next consider the decision of low ability workers. Their present value of unemployment is then:

$$rU_0 = \zeta - h + \lambda(\theta)\beta\left(\frac{y_0 - \zeta + h}{r + \delta + \beta\lambda(\theta)}\right) \quad \text{versus} \quad \zeta$$

Comparing ζ with rU_0 allows us to find the threshold search cost of low ability workers h_0 :

$$h_0 = \frac{\lambda(\theta)\beta(y_0 - \zeta)}{r + \delta} \quad (2.9)$$

These results imply the following. If productivity differences are small, for example, $y_1 = y_0$, then $h_1 < h_0$. This means that high ability workers would stop searching for jobs in the regular market if $h_1 < h < h_0$. Intuitively, these workers have better outside opportunities, which is associated with a lower rent from finding jobs. Thus high ability workers would stop searching already at moderate levels of search costs, whereas low ability workers would continue searching even if $h_1 < h < h_0$. Low ability workers have similar productivity in this case but their outside opportunities are worse, which is associated with a higher job rent. And a higher expected job rent makes low ability workers more likely to pay the search cost in the regular market. This is the equilibrium with a full segregation of workers, where only low ability workers search in the regular market, whereas high ability workers rely exclusively on referrals. In order to find the first threshold value h_1 , consider an equilibrium where it holds that $h = h_1$. In this situation high ability workers become indifferent between continuing or stopping the search in the regular market. The market tightness θ is then given by:

$$\frac{c}{q(\theta)} = (1 - \beta) \left[\gamma(\theta, u_1) \frac{(y_0 - \zeta + h_1)}{r + \delta + \beta\lambda(\theta)} + (1 - \gamma(\theta, u_1)) \frac{(y_1 - \zeta + h_1)}{r + \delta + \beta\delta(1 - \mu - u_1)/u_1} \right]$$

where the unemployment rate u_1 and the cost threshold h_1 are given by:

$$\frac{c_2}{q_2(u_1)} = \frac{(1 - \beta)(y_1 - \zeta + h_1)}{r + \delta + \beta\delta(1 - \mu - u_1)/u_1} \quad \text{and} \quad h_1 = \frac{\lambda(\theta)\beta(y_1 - \zeta)}{r + \delta + \beta(\delta(1 - \mu - u_1)/u_1 - \lambda(\theta))}$$

For any h larger than h_1 , it's not optimal for high ability workers to incur the search cost in the regular market, which gives rise to the segregated equilibrium. Next consider the cost h_0 for which even low ability workers stop searching in the regular way. For the case $h = h_0$, the

market tightness in the segregated equilibrium is given by:

$$\frac{c}{q(\theta)} = (1 - \beta) \frac{(y_0 - \zeta + h_0)}{r + \delta + \beta\lambda(\theta)} \quad \text{where} \quad h_0 = \frac{\lambda(\theta)\beta(y_0 - \zeta)}{r + \delta}$$

In the numerical example below, we explicitly derive the two threshold values h_0 and h_1 and analyze their implications for the existence of the segregated equilibrium. Finally, note that h_0 doesn't depend on the productivity y_1 , whereas h_1 is increasing in this productivity. Thus hypothetically there is also a possibility that y_1 is so high that h_1 becomes larger than h_0 . If this condition is satisfied, then there exists an equilibrium where high ability workers are incurring the cost h_1 and sending their applications in both markets, whereas low ability workers can not cover the cost and stop searching altogether. However, numerically this case arises only for unrealistically high values of y_1 and, therefore, it is only of minor relevance.

2.4 Social optimum

This section investigates efficiency properties of the decentralized equilibrium. Consider the problem of a social planner, whose objective is to maximize the present discounted value of output minus the costs of job creation:

$$\max_{\theta, v_2} \int_0^{\infty} e^{-rt} ((1 - \mu - u_1)y_1 + (\mu - u_0)y_0 + (z - c\theta)(u_0 + u_1) - c_2v_2) dt \quad (2.10)$$

In addition, the social planner is subject to the same matching constraints as firms and workers, therefore the dynamics of unemployment is described by the following differential equations $\dot{u}_0 = \delta(\mu - u_0) - \lambda(\theta)u_0$ and $\dot{u}_1 = \delta(1 - \mu - u_1) - (\lambda(\theta) + \lambda_2)u_1$. The next proposition presents solution of the planner's optimization problem.

Proposition 2.2. *Consider a social planner choosing the market tightness θ in the regular market and the number of vacancies v_2 in the referral market. Let $\phi = (\partial m_1(u_1, v_2)/\partial u_1) \cdot (u_1/m_1(u_1, v_2))$ be the elasticity of the matching function $m_1(u_1, v_2)$. Then the optimal job creation is:*

$$\frac{c}{q(\theta)} = (1 - \eta)(\gamma k_0 + (1 - \gamma)k_1) \quad \text{and} \quad \frac{c_2}{q_2} = k_1 \quad (2.11)$$

where the costate variables k_0 and k_1 ($\Delta k = k_0 - k_1$) are obtained as:

$$k_0 = \frac{y_0 - z - \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma)}{r + \delta + \eta\lambda(\theta)} \quad k_1 = \frac{y_1 - z + \lambda(\theta)\Delta k(1 - \eta)\gamma + (\eta - \phi)\lambda_2 k_1}{r + \delta + \eta\lambda(\theta) + \eta\lambda_2}$$

Proof: Appendix 2.8.2.

Costate variables k_0 and k_1 can be interpreted as shadow prices or marginal gains associated with a unit decrease in unemployment u_0 and u_1 respectively. Thus k_0 and k_1 are the present values of net output in the socially optimal allocation (social surplus from the job). These variables should be compared to S_0 and S which are the private surplus values of workers and firms in the decentralized economy. Comparing k_0 with S_0 for low ability workers and k_1 with S for high ability workers reveals that the decentralized equilibrium is not constrained efficient. Consider first the situation when $k_0 > k_1$ which means that high ability workers create a lower

job surplus than low ability workers. This situation is possible since high ability workers have an additional possibility of employment in the referral market. Hence their outside opportunities are better and their reservation wages are higher. Therefore, if $k_0 > k_1$ every additional high ability worker searching in the regular market reduces the expected profits of firms. To see this let $\bar{J} = (1 - \eta)(\gamma k_0 + (1 - \gamma)k_1)$ be the expected firm profit at the optimum, so that $\partial \bar{J} / \partial (1 - \gamma) = -(1 - \eta)(k_0 - k_1) < 0$. This implies that high ability workers impose a negative externality on low ability workers in the regular labour market.

Next consider the opposite case when $k_0 < k_1$ which means that high ability workers are significantly more productive and create a higher surplus than low ability workers. Then the external effect is reversed. Every additional high ability worker searching in the regular market increases the expected profits of firms and so high ability workers impose a positive externality on low ability workers. From proposition 2.2 the surplus difference Δk can be expressed as follows:

$$\Delta k = \frac{y_0 - y_1 + \phi c_2 \theta_2}{r + \delta + \lambda(\theta)} \quad \text{where} \quad \theta_2 = \frac{v_2}{u_1}$$

See appendix 2.8.2 for a more detailed derivation. Δk is positive if $y_1 - y_0 < c_2 \phi \theta_2$ and negative otherwise. Intuitively, a lower difference in productivities and a larger number of social contacts (which increase the market tightness θ_2) make the first case more likely. In contrast, a large productivity difference and a low number of social contacts contribute to the occurrence of the second case. In addition, the above equation implies that $\Delta k > 0$ if productivity differences between workers are negligibly small, that is $y_0 = y_1$. In this latter case high ability workers unambiguously impose a negative externality on low ability workers in the regular labour market.

Finally, consider the labour market without contacts, so that $v_2 = 0$. For the traditional Hosios value of the bargaining power ($\beta = \eta$) it is then true that: $\gamma k_0 + (1 - \gamma)k_1 = \gamma S_0 + (1 - \gamma)S$, so the externality is neutralized and the market tightness θ coincides with the optimal choice of the social planner. If $v_2 = 0$ it follows that $\Delta k < 0$ so more productive high ability workers unambiguously impose a positive externality on agents with low ability. But at the same time low ability workers produce less output which explains a negative external effect on high ability workers. In the equilibrium without networks these two external effects are automatically neutralized and the fact that the two worker groups are pooled in the same submarket does not create an inefficiency. Hence the inefficiency from pooling is an artefact of referrals. In a more general framework with referrals the two external effects are not internalized and so there is a strong need for the optimal redistribution policy. This policy is described in proposition 2.3:

Proposition 2.3. *Let the Hosios condition be satisfied, so that $\beta = \eta = \phi$. For $l > l_0$ the equilibrium with social contacts is constrained inefficient but there exists a policy scheme $\{\tau_0^*, \tau_1^*, s^*\}$ that can restore the optimal allocation:*

$$\tau_0^* = \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma) \quad \tau_1^* = -\lambda(\theta)\Delta k(1 - \eta)\gamma \quad s^* = \eta c_2$$

where endogenous variables θ , Δk and γ are evaluated at the socially optimal allocation described in proposition 2.2. In addition, the two transfers τ_0^* and τ_1^* are purely redistributive as it holds that $u_0 \tau_0^* + u_1 \tau_1^* = 0$.

Proof: Appendix 2.8.3.

First, observe that firms in the referral market do not impose a negative externality on other firms because q_2 does not depend on v_2 . This is a fundamental property of the model because a new job opening in the referral market does not change the hiring probability of other firms. But if other vacancies are not affected, the optimal job creation in this market is obtained at the point where the total surplus of the job k_1 is equal to the expected cost c_2/q_2 . In contrast, in the decentralized economy firms capture a fraction $(1 - \beta)$ of the total surplus S , so the job creation is distorted downwards. The optimal policy then includes paying employment subsidies s to firms in the referral market. One immediate implication of this policy should be a lower unemployment of high ability workers u_1 and a higher job-finding rate λ_2 . As described above, the situation is different in the regular market. These firms impose a standard search externality on other firms which is neutralized for $\beta = \eta$. A more general case when this condition is not satisfied is analyzed later in the section.

Further consider an economy with the optimal employment subsidy s^* . Proposition 2.3 describes a system of Pigouvian taxes τ_0^* and τ_1^* . When $\Delta k > 0$ high ability workers impose a negative congestion externality on low ability workers in the regular market, so the optimal policy implies a negative value of $\tau_1^* < 0$. These transfers are supposed to reduce the reservation wage of high ability workers and increase the expected profit \bar{J} . In a similar way, low ability workers create more profits and impose a positive congestion externality on high ability workers. So the optimal policy implies a positive value of $\tau_0^* > 0$, these transfers are supposed to increase the reservation wage of low ability workers. Finally, note that this policy should increase the wage w_0 and decrease both wages w_1 and w_2 , so a lower equilibrium wage dispersion is a positive side effect of this policy.

For $\Delta k < 0$, proposition 2.3 implies $\tau_0^* < 0$ and $\tau_1^* > 0$. In this case the bargained wage of low ability workers is too high and the wage of high ability workers is too low. Wages are then compressed in the decentralized equilibrium and the planner needs to raise the reservation wage of high ability workers and reduce the reservation wage of low ability workers. These predictions coincide with the results of Blazquez and Jansen (2008), however they do not describe the possibility of the reverse policy when $\Delta k > 0$. Identifying and characterising this latter case is the primary contribution of this paper.

Now let's return to the more general case when the Hosios condition is not satisfied ($\beta \neq \eta$). In this situation both sides of the market suffer from the underlying search externality (see Hosios (1990) and Pissarides (2000)). This externality distorts the market in addition to the congestion effects described above and is not specific to the model with heterogeneous workers. By this we mean that the external effects from searching would exist even in a labour market with identical workers. When $\beta < \eta$, the job-filling rate in the regular market $q(\theta)$ is too sensitive to the market tightness and so an additional vacancy in the regular market makes other vacancies significantly worse off. Therefore, both transfers τ_0 and τ_1 should be increased to achieve a lower expected profit. Intuitively, lower profits make vacancies less sensitive to the new entrants and so the negative externality is mitigated. Thus low ability types will unambiguously obtain a positive overall transfer, let it be denoted by $\tilde{\tau}_0$.

In the opposite case when $\beta > \eta$, the job-finding rate in the regular market $\lambda(\theta)$ is too sensitive to the market tightness and so an additional searching worker makes other workers

significantly worse off. Therefore, both transfers τ_0 and τ_1 should be reduced to achieve a lower expected worker rent. Intuitively, a lower expected gain from the job makes unemployed workers less sensitive to the size of the unemployment pool and so the negative externality is again mitigated. Thus high ability types will unambiguously get a negative overall transfer, let it be denoted by $\tilde{\tau}_1$. The following proposition is summarizing our results:

Proposition 2.4. *If the Hosios condition is not satisfied, so that $\beta \neq \eta = \phi$. For $l > l_0$ the equilibrium with social contacts is constrained inefficient but there exists a policy scheme $\{\tilde{\tau}_0, \tilde{\tau}_1, s^* = \eta c_2\}$ that can restore the optimal allocation:*

$$\begin{aligned}\tilde{\tau}_0 &= \lambda(\theta)\Delta k(1-\eta)(1-\gamma) + \frac{(\eta-\beta)}{(1-\beta)}k_0(r+\delta+\lambda(\theta)) \\ \tilde{\tau}_1 &= -\lambda(\theta)\Delta k(1-\eta)\gamma + \frac{(\eta-\beta)}{(1-\beta)}k_1(r+\delta+\lambda(\theta)+\lambda_2)\end{aligned}$$

where endogenous variables $\lambda(\theta)$, λ_2 , k_0 , k_1 and γ are evaluated at the socially optimal allocation described in proposition 2.2.

Proof: Appendix 2.8.4.

Note that both transfers are reduced in the case when $\beta > \eta$ and increased otherwise, which allows the policy maker to balance the negative external effects on workers and firms. However, when standard search externalities are internalized, the optimal policy should still address the asymmetric congestion effects that the two groups of workers impose on each other. And so the vector $\{\tilde{\tau}_0, \tilde{\tau}_1\}$ coincides with $\{\tau_0^*, \tau_1^*\}$ for the case $\beta = \eta$.

To complete this section we also compare the minimum number of contacts l_0 in the decentralized equilibrium and l_0^* in the social optimum. In the equilibrium without referrals the labour market tightness is equal to θ^* (see corollary 2.1) and the corresponding unemployment of high ability workers is $u_1 = \delta(1-\mu)/(\delta+\lambda(\theta^*))$. In this economy opening a vacancy in the referral market is associated with a present value of profits $J_2 = (1-\beta)(y_1-z)/(r+\delta+\beta\lambda(\theta^*))$ and is independent of the number of social contacts l (see figure 2.3).

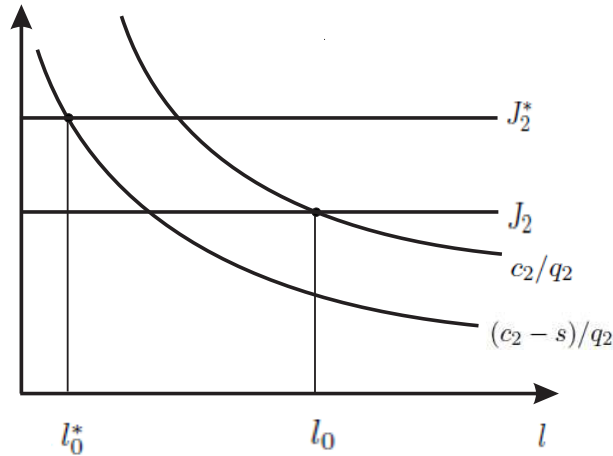


Figure 2.3: Choice of l_0 in the decentralized equilibrium and in the social optimum

In contrast, expected costs from an open referral vacancy are equal to c_2/q_2 which is a decreasing function of l . Intuitively, expected duration of an open vacancy is lower with a

larger number of social links. The threshold value l_0 can then be found as a minimum number of contacts with positive net profits from referral vacancies $J_2 = c_2/q_2$, which is equivalent to $\theta^* = \bar{\theta}(u_1(l_0))$. If the optimal policy is implemented, firms' expected profits are larger as $y_1 - \tau_1^* - z > y_1 - z$, while the expected costs are lower ($c_2 - s < c_2$). Therefore, referral vacancies become attractive for firms at a lower number of social contacts $l_0^* < l_0$. So the decentralized decision of firms not to use referrals may be inefficient from a social perspective.

2.5 Numerical example

This section parameterizes the model to match the average labour market indicators in the OECD countries. Without loss of generality, we normalize the productivity parameter y_0 to 1. The productivity of high ability workers y_1 is taken to be 1.25 for the benchmark case and we also consider the cases $y_1 = 1$ (workers differ only in social capital) and $y_1 = 1.5$. For comparison, Gautier (2002) uses the value of 0.5 for the productivity of the low-skilled workers and 1 for the high-skilled. In Albrecht and Vroman (2002) productivity values of the high-skilled workers are set in the interval from 1.25 to 1.6 which is similar to our range.

We choose a unit period of time to be one quarter and set $r = 0.012$ which corresponds to the annual discount rate of 5%. Further, we follow Shimer (2005) and set the net value of leisure z equal to 0.4. Fontaine (2008) uses the value of 0.15 for the U.S. economy and 0.4 for the French economy. Gautier (2002) and Cahuc and Fontaine (2009) set z equal to 0.2. At the same time, Hall and Milgrom (2008) obtain a larger value of 0.71. Therefore, our choice of z is in the middle range of values in the literature. We also take $\delta = 0.1$ and $\eta = 0.72$ as in Shimer (2005). This choice of δ implies an average employment duration of 2.5 years. Shimer (2005) obtains these estimates from the monthly US transition data for the period 1960-2004. The same value of the separation rate is also used in Pissarides (2009).

The cost of an open vacancy in the regular market c is chosen to be 0.4. Intuitively, this parameter captures the costs of traveling and accommodation of job applicants at the place of the interview. It exactly coincides with the Cahuc and Fontaine's (2009) value of this parameter. Shimer (2005) has chosen the value of 0.213 for the cost of vacancies while Fontaine (2008) uses the value of 0.3. As a starting point we also set $c_2 = 0.4$. Further in section 2.5.2 we deviate from this benchmark constellation and present comparative statics results for lower values of c_2 to capture the fact that referral vacancies are cheaper for firms. Next, the fraction of low-ability workers is set to 60% of the overall population so that $\mu = 0.6$. Albrecht and Vroman (2002) choose a similar value of 0.67 for the proportion of low-skilled workers in their model, while Gautier (2002) uses the value of 0.5 for this parameter.

With respect to the bargaining power, we assume $\beta = \eta = 0.72$ to satisfy the Hosios condition. Moreover, we make a similar assumption in the referral market and set $\phi = \eta$ in the benchmark case where ϕ is the elasticity of the job-finding rate in the referral market. Combining $\phi = \eta$ with $a = 4$ and solving equations (2.11) for l , we find that $l^* = 40$. Therefore, the implied number of professional contacts in a network of high-ability workers is equal to 40. Cahuc and Fontaine (2009) use $l = 50$, while Fontaine (2008) uses $l = 40$ in a benchmark model of his paper. These numbers are in line with the empirical evidence, for example, in their recent study Cingano and Rosolia (2012) find that the median number of professional contacts in Italy is equal to 32. This number is higher in Germany and is equal to 43 according to Glitz (2013).

y_0	y_1	r	δ	z	η	β	ϕ	m	μ	c	c_2	a	l
1	1.25	0.012	0.1	0.4	0.72	0.72	0.72	1.22	0.6	0.4	0.4	4	40

Table 2.1: Values of the model parameters

Finally, we set m to 1.22 which is an efficiency multiplier in the Cobb-Douglas matching function: $\lambda(\theta) = m\theta^\eta$. This parameter yields the following equilibrium unemployment rates: $u_0/\mu = 0.0924$ and $u_1/(1-\mu) = 0.0388$ for the two groups of workers. So the average unemployment rate in the economy is equal to 0.07 which is close to the long-term unemployment rate in the U.S. For comparison, Blazquez and Jansen (2008) set m equal to 1, while Shimer (2005) uses the value of 1.355.

2.5.1 Comparative statics

First, the model shows that it is not profitable for firms to open vacancies in the referral job market when the number of workers' contacts is low enough. Numerically solving the system of equations (JC) , (JC_2) and $\lambda_2 = 0$ we can find the threshold value l_0 after which firms begin to create vacancies in the referral job market. In the benchmark case, l_0 is approximately equal to 5 and it is decreasing in y_1 or a . Hence when the number of contacts is less than 5 it is not profitable for firms to use referrals.

In all our simulations the decentralized equilibrium is unique as can be seen from figure 2.4. The curve (JC) is decreasing for low values of u_1 and then increasing, while (JC_2) is parallel to the θ -axis. It can also be shown that $S_0 > S$ which means that firms obtain higher profits in a match with low ability workers. When u_1 is low and increases, the probability of hiring a low ability worker $\gamma(\theta, u_1)$ falls, the average firm profits decrease and so the market tightness θ is reduced. In contrast, when u_1 is already high and increases further, the fall in $\gamma(\theta, u_1)$ is dominated by the increase in the total surplus value S . Intuitively, a more pronounced unemployment u_1 puts a downward pressure on the reservation wage of high ability workers. This dampens the wage w_1 and leads to a higher profit $J_1 = (1-\beta)S$.

What are the implications of a higher productivity y_1 for $\theta(u_0)$ and u_1 ? The model predicts that both unemployment rates decrease. This result is intuitive as firms expect higher profits and open more vacancies in both job markets (because high ability workers search in both markets). Figure 2.4 (right) illustrates this effect for the benchmark case $\Delta y = 0.25$ and the other two cases when $\Delta y = 0$ and $\Delta y = 0.5$: (JC_2) moves to the left and (JC) to the up-left with the increase in Δy . This result is similar to Gautier (2002) where the author finds that low-skilled workers gain from the increased productivity of high-skilled workers in simple jobs.

What is the impact of the increase in the number of social contacts l on the equilibrium unemployment? First, the model predicts that a larger number of contacts reduces u_1 and raises u_0 . This effect is illustrated in figure 2.4 (left) where the decentralized equilibrium values of u_1 and θ for $l = 5$, $l = 40$ and $l = \infty$ are compared. As only (JC_2) depends on the number of contacts, there is a parallel shift of this line to the left (right) with the increase (decrease) in l . The larger is the number of contacts the smaller is the shift. We can also calculate the asymptotic value of u_1 which is equal to 0.012.

Changes in the unemployment rates u_0 and u_1 with the increase in the number of contacts

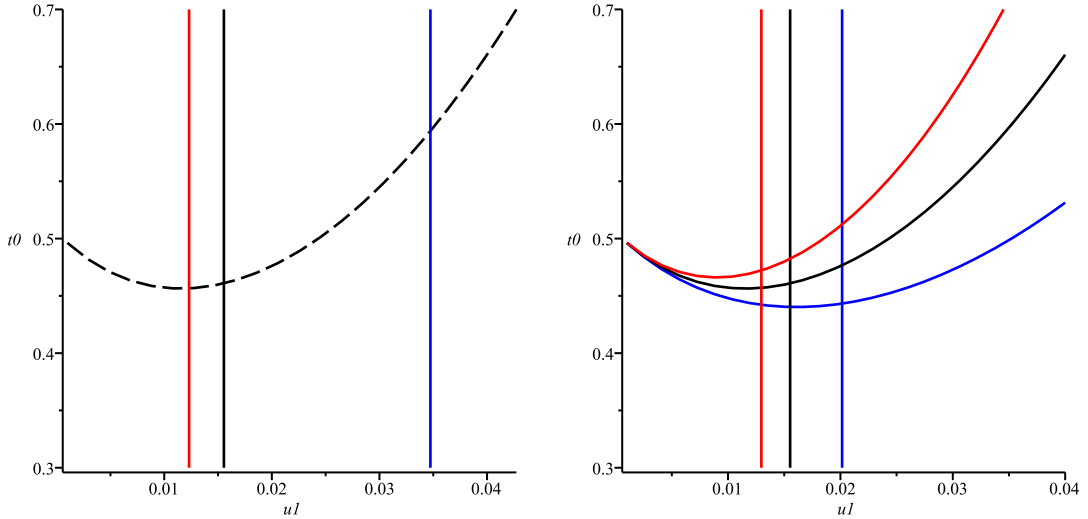


Figure 2.4: Left panel: Job creation curves determining the equilibrium values of θ and u_1 when $l = 5$ (blue), $l = 40$ (black) and $l = \infty$ (red). Right panel: Job creation curves determining the equilibrium values of θ and u_1 when $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red).

are illustrated in figure 2.5. For $l \leq 5$ there exists a unique equilibrium without referrals and so the two unemployment rates coincide. However, if the number of social links is more than 5 firms rely on social contacts to fill their open vacancies. The two unemployment rates are then diverging. On the one hand, as high ability workers are better connected, their equilibrium rate of unemployment is reduced. On the other hand, there is an adverse effect on the equilibrium unemployment of low ability workers which is increasing in l . If high ability workers are better connected, their outside opportunities are improved as finding jobs becomes easier. At the same time, better outside opportunities strengthen the bargaining position of these workers and increase their wages in the regular market. Therefore, firms' profits from regular vacancies and the number of such vacancies are both reduced. Finally, a lower number of vacancies in the regular market worsens the bargaining position of low ability workers and reduces their employment and wages. This latter change is illustrated in figure 2.6.

	u_0	u_1	Δw	l_0
$\Delta y \uparrow$	-	-	+	-
$l \uparrow$	+	-	+	
$a \uparrow$	+/-	-	+	-

Table 2.2: Comparative statics

Our comparative statics results are summarized in table 2.2. Overall, the model predicts that a larger number of social links is associated with a more pronounced wage dispersion between the two groups of workers and a higher unemployment rate of low ability workers. Such predictions are compatible with the observed empirical evidence in the U.S. documenting an increase in the inequality of earnings (see Autor, Katz and Kearney (2008)). This allows us to conclude that a part of this inequality may be generated by a stronger growth and utilization of social networks in the U.S. and other countries in the recent decade³.

³For example, there exist labor market policies trying to encourage the establishment or the improvement of social networks (McClure (2000), OECD (2001)). One of them is Australian Working Together program which

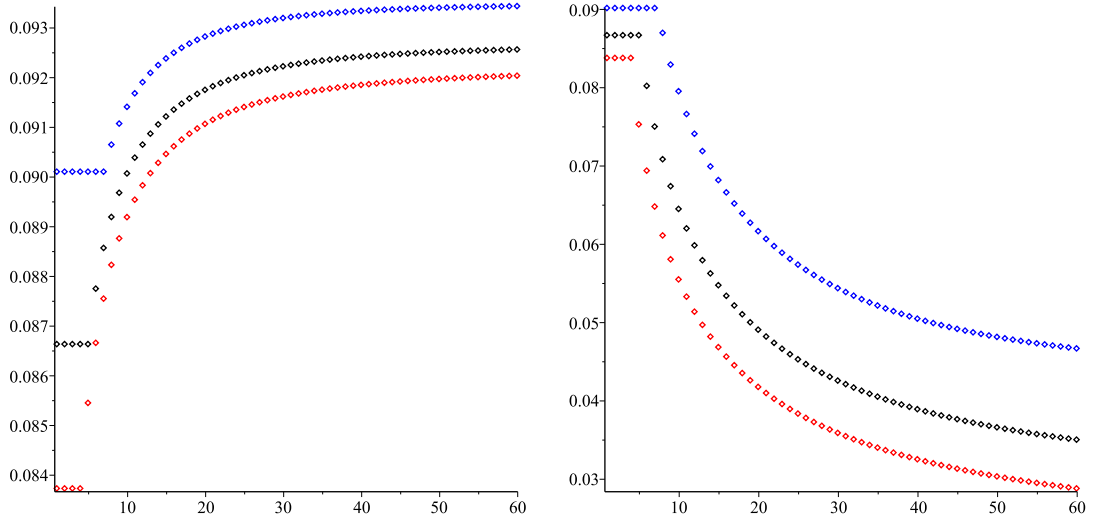


Figure 2.5: Left panel: Change in u_0/μ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red). Right panel: Change in $u_1/(1 - \mu)$ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red)

2.5.2 Search costs and participation decisions

In this section we consider the implications of search costs for workers and firms. First, consider the case $y_1 = y_0 = 1$ and recall that $z = \zeta - h$ denotes the net value of leisure, whereas ζ is the gross value of leisure before subtracting the flow search cost h . For example, if the search cost of workers h is equal to 0.2, then $\zeta = z + h = 0.4 + 0.2 = 0.6$. We consider these parameters as a benchmark cost constellation. From equation (2.8) we find that the threshold cost of searching in the regular market for high ability workers is $h_1 = 0.31$. This means that high ability workers prefer to search in both markets if the search cost h is below $h_1 = 0.31$ and stop searching in the regular market for the search cost higher than 0.31. This is illustrated on the left panel of figure 2.7 where y_1 is measured on horizontal axis. Note that h_1 is an increasing function of the productivity y_1 . For instance, h_1 is increasing from 0.31 to 0.45 for productivity y_1 increasing from 1 to 2 (blue curve). The fact that h_1 is increasing in the productivity of high ability workers is intuitive as higher productivity implies higher wages which makes high ability workers more tolerant to higher costs.

Further, we consider changes in the firms' search cost c_2 in the referral market. Reducing this cost is beneficial for high ability workers as the number of referral vacancies v_2 is rising, leading to lower unemployment u_1 and a higher job-finding rate λ_2 . This is illustrated on the right panel of figure 2.7. At the same time, note from equation (2.8) that a higher referral job-finding rate λ_2 has a negative effect on the threshold search cost of high ability workers h_1 . Thus more frequent referrals reduce the willingness of high ability workers to pay the cost of searching in the regular market. This is illustrated on the left panel of figure 2.7, for instance, for a lower c_2 the threshold value h_1 falls down to 0.24 (in the case $y_1 = y_0 = 1$) and further down to 0.17 for $c_2 = 0.2$ (black curve). This means that our benchmark equilibrium will not exist anymore if posting referral vacancies has a low cost $c_2 = 0.2$ because high ability workers

aims at providing people the incentives to stay together with their communities even if they are economically disadvantaged (OECD (2003)).

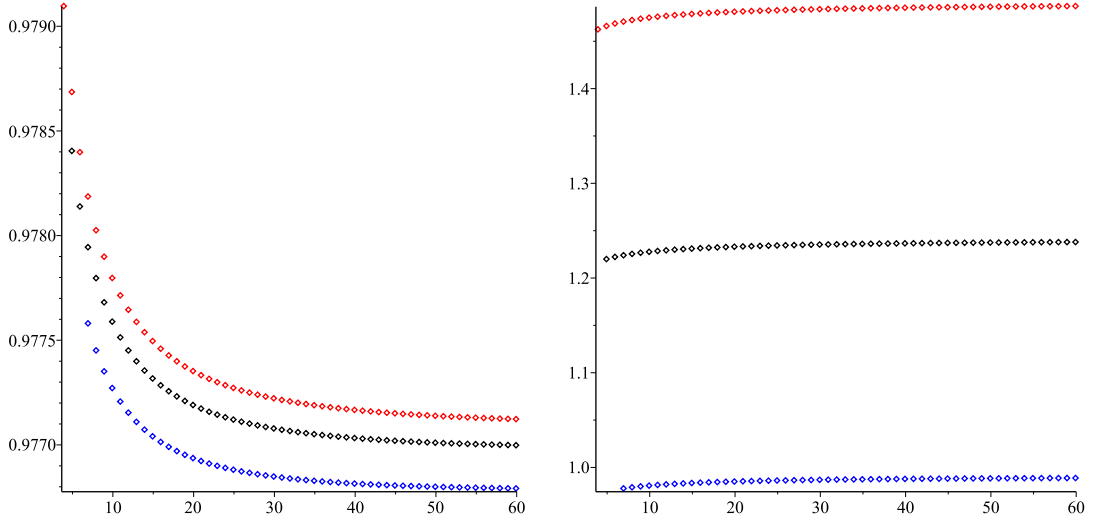


Figure 2.6: Left panel: Change in w_0 with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red). Right panel: Change in $w_1 = w_2$ with the increase in l for $y_1 = 1$ (blue), $y_1 = 1.25$ (black) and $y_1 = 1.5$ (red)

would stop searching in the regular market for any search cost h higher than 0.17.

2.5.3 Optimal policy

This subsection investigates the effect of optimal policy $\{s, \tau_0, \tau_1\}$ on endogenous variables in the labour market. Consider first the case $y_1 = y_0 = 1$ when the productivity of the job is not sensitive to worker's ability. The optimal vector of policy instruments is given by: $s = 0.288$, $\tau_0 = 0.017$, $\tau_1 = -0.160$. For the ease of exposition policy changes are implemented in two steps: only employment subsidies $s = \eta c_2$ in the referral market and the final policy. In addition, we introduce two new welfare variables Ω_0 and Ω_1 for low- and high-ability workers respectively (with social transfers from the budget):

$$\begin{aligned}\Omega_0 &= u_0(z + \tau_0) + (\mu - u_0)y_0 - c\theta u_0 \\ \Omega_1 &= u_1(z + \tau_1) + (1 - \mu - u_1)y_1 - c\theta u_1 - (c_2 - s)v_2\end{aligned}$$

where the first term is the flow income of unemployed workers and the second term is the flow output of employed workers net of the costs of job creation⁴. The net welfare gain of the policy can then be obtained as a gross welfare gain $\Delta\Omega_0 + \Delta\Omega_1$ minus the cost of this policy BC . The total cost BC is a sum of budget expenses:

$$BC = u_0\tau_0 + u_1\tau_1 + sv_2$$

Our results for the first case $y_0 = y_1 = 1$ are presented in table 2.3. Note that high ability workers earn the same wage $w_1 = w_2$ which is independent of the search channel.

⁴More precisely, Ω_0 is a part of total welfare attributed to low ability workers which includes net leisure and transfers of low ability workers $u_0(z + \tau_0)$, wages of employed low ability workers $(\mu - u_0)w_0$ and net firm profits from hiring low ability workers $(\mu - u_0)(y_0 - w_0) - c\theta u_0$. In a similar way, Ω_1 is a part of total welfare attributed to high ability workers and includes net leisure and transfers of high ability workers, wages of employed high ability workers and net firm profits from hiring high ability workers in both markets. So the total welfare is $\Omega_0 + \Omega_1$

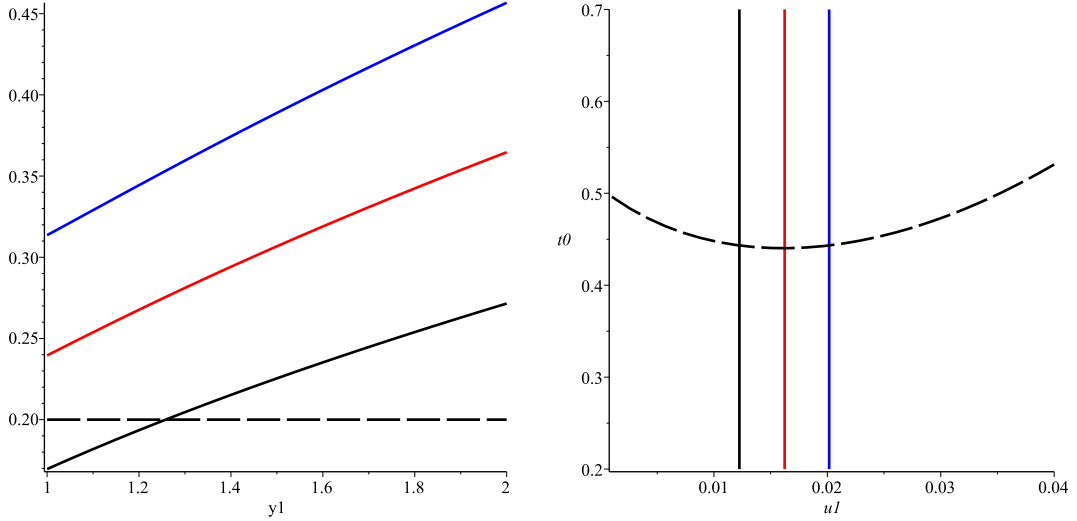


Figure 2.7: Left panel: Search cost threshold h_1 as a function of y_1 when $c_2 = 0.4$ (blue), $c_2 = 0.3$ (red) and $c_2 = 0.2$ (black) relative to the benchmark search cost $h = 0.2$ (black dash). Right panel: Job creation curves determining the equilibrium values of θ and u_1 when $c_2 = 0.4$ (blue), $c_2 = 0.3$ (red) and $c_2 = 0.2$ (black).

Optimal policy	θ	u_0/μ	$u_1/(1-\mu)$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4432	0.0933	0.0504	0.9768	0.9872	0.5565	0.3822
Only subsidy $\{s\}$	0.4527	0.0928	0.0211	0.9769	0.9945	0.5565	0.3919
Final policy $\{s, \tau_0, \tau_1\}$	0.4475	0.0931	0.0182	0.9775	0.9940	0.5574	0.3914

Table 2.3: Optimal policy $s = 0.288, \tau_0 = 0.017, \tau_1 = -0.160$ in a labour market with $y_1 = 1$

As expected, employment subsidies s reduce the equilibrium unemployment of high ability workers $u_1/1 - \mu$ and raise their wages. The unemployment rate of low ability workers u_0/μ is slightly decreased as a consequence of a higher market tightness θ . This is an outcome of a lower competition between agents in the regular labour market (lower $1 - \gamma$). Overall, one can conclude that subsidizing referrals is associated with a large welfare gain for high ability workers (Ω_1 is higher) and no significant welfare changes for low ability workers (Ω_0 is unchanged).

Table 2.3 further shows that the optimal transfers $\tau_0 = 0.017$ and $\tau_1 = -0.160$ internalize congestion externalities in the regular market. This finding is in contrast to Blazquez and Jansen (2008) as we find that $\Delta k > 0$ for the chosen parameter values. Firm profits are lower in a match with high ability workers and so every additional high ability unemployed imposes a negative externality on workers with low abilities making it more difficult for them to find a job. Therefore, the optimal transfer policy favours low ability workers at the cost of the other group (Ω_0 is higher in the second step, while Ω_1 is lower). The wage of low (high) ability workers becomes higher (lower), so the wage inequality is slightly reduced. The total welfare is increasing with policy from 0.9387 to 0.9488. After subtracting the cost $BC = 0.0042$ the new welfare level is reduced to 0.9446. The net welfare gain of the policy is then calculated as 0.6% of the total welfare. Similar tables for $y_1 = 1.25$ and 1.5 are presented in Appendix 2.8.5 and confirm our predictions.

Next we ask a question whether a welfare gain of 0.6% is economically significant. In order to

answer this question, let us discount the net annual wage in Germany ($2200 \cdot 12.5 = 27500$ EUR)⁵ over 50 years with an annual discount rate of 5%. We get an amount of 502000 EUR which is an average present value of wages per worker. Therefore, a welfare gain of 0.6% is approximately equivalent to the lump sum transfer of 3000 EUR per worker.

Another interesting question is a change in optimal policy if the Hosios condition is not satisfied, i.e. $\beta \neq \eta$. The optimal transfers to unemployed workers are then modified to account for the standard search externalities on other workers and firms. When $\beta < \eta$, equilibrium wages are lower than socially optimal, although there are too many vacancies in the regular labour market. Therefore, both transfers should be increased to improve the bargaining position of workers. Thus low ability types will unambiguously get a positive payoff $\tilde{\tau}_0$. In the opposite case, when $\beta > \eta$, equilibrium wages are higher than socially optimal but the job creation is inefficiently low in the regular market. In this case high ability types will unambiguously get a negative budget payoff $\tilde{\tau}_1$. Table 2.4 below presents the list of optimal policy instruments and the resulting change of the social welfare for high and low values of β . Note that both Ω_0 and Ω_1 have unique maximal values for $\beta \in [0, 1]$.

	$\beta = 0.4$	$\beta = 0.49$	$\beta = 0.61$	$\beta = 0.69$	$\beta = 0.72$	$\beta = 0.8$
$\tilde{\tau}_0$	0.4259	0.3679	0.2363	0.0920	0.0166	-0.2949
$\tilde{\tau}_1$	0.3903	0.3124	0.1356	-0.0583	-0.1596	-0.5783
Ω_0 without policy	0.5496	0.5534	0.5561	0.5566	0.5565	0.5554
Ω_0 with final policy	0.5806	0.5770	0.5697	0.5616	0.5574	0.5400
Ω_1 without policy	0.3847	0.3851	0.3845	0.3831	0.3822	0.3780
Ω_1 with final policy	0.3955	0.3949	0.3936	0.3922	0.3914	0.3884
$\Omega_0 + \Omega_1$ without policy	0.9344	0.9385	0.9405	0.9396	0.9387	0.9334
Net welfare gain (%)	1.1004	0.6548	0.4354	0.5306	0.6327	1.2009

Table 2.4: The values of $\tilde{\tau}_0$ and $\tilde{\tau}_1$, Ω_0 and Ω_1 with and without final policy as well as $\Omega_0 + \Omega_1$ without policy for different values of β in the case of $y_1 = 1$

To perform this comparison, we consider deviations of β around the Hosios value and capture in table 2.4 those outcomes, for which the equilibrium values of Ω_0 , Ω_1 or $\Omega_0 + \Omega_1$ are maximal, as well as the benchmark case $\beta = \eta = 0.72$. In addition, socially optimal values of Ω_0 and Ω_1 are provided. Both $\tilde{\tau}_0$ and $\tilde{\tau}_1$ as well as Ω_0 and Ω_1 at the optimum are decreasing in β . For $y_1 = 1.25$ and $y_1 = 1.5$ the numbers are presented in Appendix 2.8.6.

The equilibrium values of Ω_0 (red) and Ω_1 (blue) for different β -points are presented in figure 2.8 (left panel). These curves, indeed, have a unique maximum in β . As follows from table 2.4, the maximal value of Ω_0 is achieved for $\beta = 0.69$ and is approximately equal to 0.5566. Low ability workers are only searching in the regular submarket, moreover the profits of firms in this submarket are reduced due to the presence of high ability/wage job applicants. Therefore, low ability workers would obtain a higher level of welfare if the bargaining power of all workers was reduced below the Hosios value. This change would increase the equilibrium profits of firms and so low ability workers would gain from a more intensive job creation in the regular market.

The situation is potentially different for high ability workers due to the simultaneous job search in both submarkets. On the one hand, high ability workers gain from the presence of low ability workers in the regular market. So their desired bargaining power in the regular market

⁵Average monthly salary in Germany (Statistisches Bundesamt yearly report, 2013), multiplied by 12.5 months.

should be above the Hosios value. But on the other hand, the total output/welfare in the referral market is maximized for a zero value of the bargaining power. Hence, Ω_1 is maximized for some β which can be above or below the Hosios value depending on which of the two effects dominates. In the numerical example, we find that the maximal value of Ω_1 is achieved for $\beta = 0.49$ and is approximately equal to 0.3851. So the marginal welfare gain from a lower β in the referral market outweighs the welfare loss in the regular market.

The maximal value of $\Omega_0 + \Omega_1$ is equal to 0.9405 and is reached for $\beta = 0.61$, which is in between 0.49 and 0.69 and is smaller than η (black solid curve on the right panel of figure 2.8). This value is lower than $0.9446 = \Delta\Omega_0 + \Delta\Omega_1 - BC + \Omega_0 + \Omega_1$, which is the new welfare level reached after implementing the policy (dashed black line). Thus the decentralized economy is inefficient for any value of β . However, the total welfare gain of the policy varies with β . It is minimized at $\beta = 0.61$ where the two black curves come closely together, but it can be sufficiently large where the two curves are diverging. For example, the total net welfare gain of the optimal policy is equal to 1.1% for $\beta = 0.4$ and 1.2% for $\beta = 0.8$.

Next, let us compare our benchmark economy with social networks versus the case without networks. The red solid curve on the right panel of figure 2.8 illustrates welfare in the decentralized economy without networks, whereas the dashed red line corresponds to the maximum welfare in this restricted economy. As it was already mentioned in the theoretical part, welfare in the economy without networks is maximized for the Hosios value of the bargaining power $\beta = \eta$. We find that the level of welfare at this point is equal to 0.9276, which is lower than the maximal welfare in the economy with referrals: 0.9387 without policy and 0.9446 with policy. The total net gain of referrals can then be estimated as 1.2% before the policy and 1.8% after the policy.

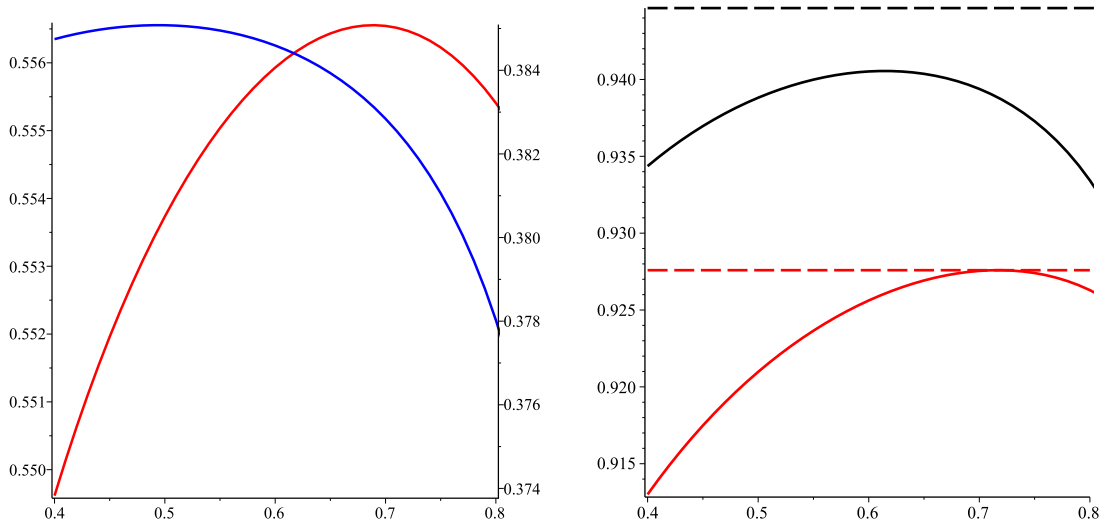


Figure 2.8: Left panel: Ω_0 (red) and Ω_1 (blue) for different values of β . Right panel: $\Omega_0 + \Omega_1$ (solid) and the new welfare level reached after implementing the policy (dashed) with (black) and without (red) allowing for networks for different values of β .

In the final step we investigate the robustness of our results. This can be done by considering the sign of $\Delta k = k_0 - k_1$ with a variation in the key parameters y_1 , l and a . Figure 2.9 confirms our theoretical conclusion from section 2.4 that the case $\Delta k < 0$ is more likely with a higher

difference in productivities and a lower number of social contacts. Every curve on this figure shows the values of Δk for l in the range from 5 (lower line) to 14 (higher line) and for y_1 taking values from 1.05 to 1.95. Moreover $a = 2$ on the left panel of the graph and $a = 4$ (benchmark) on the right panel. It is easy to see that Δk is non-negative when $a = 4$ and $y_1 \leq 1.95$. Although, when $a = 2$, Δk can become negative for low values of l and a high productivity y_1 . This is precisely the case when our result is in line with the finding of Blazquez and Jansen (2008) and the equilibrium exhibits a wage compression in the regular labour market.

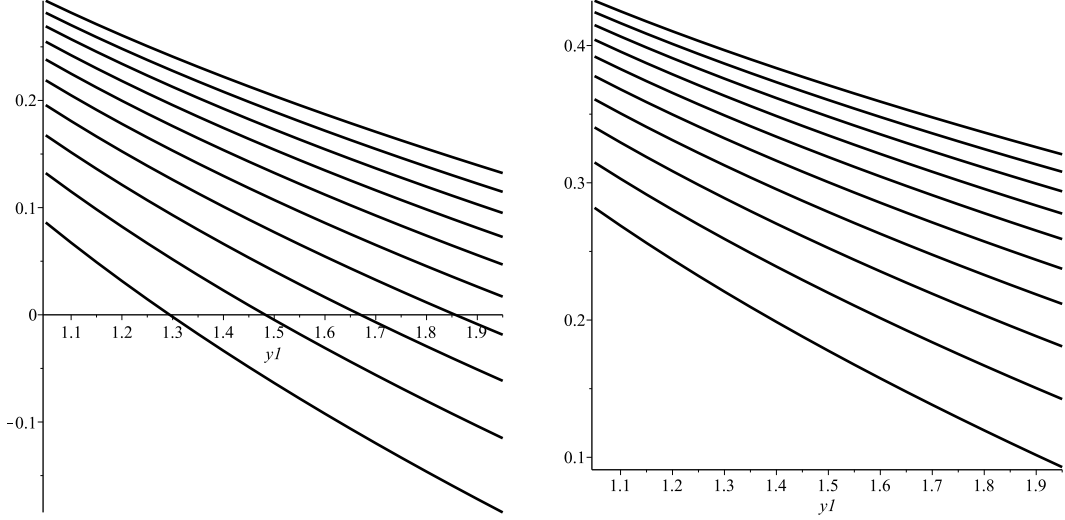


Figure 2.9: Left panel: Values of Δk for $l = [5\dots 14]$, $y_1 = [1.05\dots 1.95]$ and $a = 2$. Right panel: Values of Δk for $l = [5\dots 14]$, $y_1 = [1.05\dots 1.95]$ and $a = 4$.

Figure 2.10 compares the threshold number of social contacts in the decentralized economy l_0 with the optimal planner's choice l_0^* . In particular, it illustrates our result from section 2.4 that l_0^* is always lower than l_0 for every a from 2 to 10. Thus, the decentralized decision not to use referrals may be suboptimal from the social perspective. Cahuc and Fontaine (2009) have already found in their setting that for low values of β formal search methods can be used instead of social networks and this allocation can be inefficient. Our paper extends this result in the sense that it holds for every β and depends on the number of contacts in the networks.

Figure 2.10 additionally illustrates how the threshold l_0 depends on the cost of referral hiring c_2 . In line with our theoretical results, a lower cost makes referrals a more attractive search channel for firms, so the necessary threshold number of network contacts is reduced for every value of a . Hence there is some complementarity between the size of the network and the flow cost of hiring as $\partial l_0 / \partial c_2 > 0$. If the network size is sufficiently large firms will open referral vacancies even with a high cost c_2 . In contrast, if the network size is small firms will only open referral vacancies if the hiring cost c_2 is also small. This mechanism is similar from the perspective of social planner but it doesn't eliminate the difference between l_0 and l_0^*

2.6 Conclusions

This paper develops a labour market matching model with heterogeneous workers and two channels of job search: formal methods and social networks. Entering firms have an option to post a vacancy in the regular job market or in the referral market, where job information is exclu-

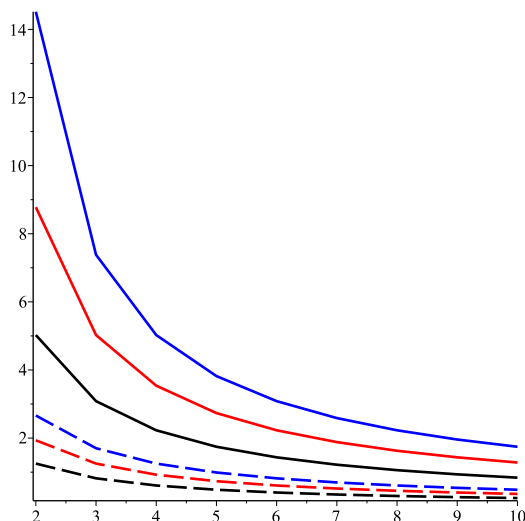


Figure 2.10: Comparison between l_0 (solid) and l_0^* (dashed) for $a = [2...10]$, case $c_2 = 0.4$ (blue), case $c_2 = 0.3$ (red), case $c_2 = 0.2$ (black)

sively transmitted by employees. The paper proves existence of the decentralized equilibrium in this framework and then shows that this equilibrium is inefficient. There are two reasons for the inefficiency. First, firms obtain a fixed fraction of the total job surplus in the referral market which is below the social gain. Therefore, the number of referral vacancies is low and the equilibrium unemployment of high ability workers is inefficiently high. This inefficiency can be corrected by means of employment subsidies in the referral market. Second, high ability workers congest the market for low ability workers. Moreover, this congestion externality is increasing in the number of social contacts. The optimal policy then includes a positive income transfer to low ability workers and a negative transfer to high ability workers. This policy reduces the equilibrium wage dispersion which is different from the result of Blazquez and Jansen (2008) reporting a compressed equilibrium wage distribution in a similar framework without contacts.

Finally, we examine the effect of a larger number of social contacts. High ability workers rely strongly on their networks, thus their unemployment falls and their wages rise with a larger density of the network. In contrast, low ability workers are adversely affected by this change. Their wages fall and the unemployment rate is increased. Overall, the equilibrium wage dispersion is increasing in the number of social contacts.

2.7 Acknowledgements

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2.8 Appendix

Appendix 2.8.1: Proof of Proposition 2.1. The right-hand side of equation (JC_2) is monotonically increasing in u_1 , while the left-hand side of (JC_2) is monotonically decreasing in

u_1 :

$$\lim_{u_1 \rightarrow 1-\mu} \frac{(1-\beta)(y_1-z)}{(r+\delta+\beta\delta\frac{1-\mu-u_1}{u_1})} = (1-\beta)\frac{y_1-z}{r+\delta} \quad \lim_{u_1 \rightarrow 1-\mu} \frac{c_2}{a[1-(\frac{1-\mu-u_1}{1-\mu})^l]} = \frac{c_2}{a}$$

Therefore, (JC_2) will determine a unique value of u_1 when $\frac{c_2}{a} < (1-\beta)\frac{y_1-z}{r+\delta}$.

Condition A: the left-hand side of condition (JC_2) is monotonically increasing in θ :

$$\lim_{\theta \rightarrow 0} \frac{c}{q(\theta)} = 0 \quad \lim_{\theta \rightarrow \bar{\theta}(u_1)} \frac{c}{q(\theta)} = \frac{c}{q(\bar{\theta}(u_1))}$$

The right-hand side of this condition is not necessarily monotonic in θ , however:

$$\lim_{\theta \rightarrow 0} \frac{\gamma(\theta, u_1)(y_0-z)}{r+\delta+\beta\lambda(\theta)} + \frac{(1-\gamma(\theta, u_1))(y_1-z)}{r+\delta+\beta\lambda(\theta)+\beta\lambda_2} = \frac{\mu(y_0-z)}{(\mu+u_1)(r+\delta)} + \frac{u_1(y_1-z)}{(\mu+u_1)(r+\delta+\beta\lambda_2)} > 0$$

where we use that $\gamma(0, u_1) = \mu/(\mu+u_1)$ and

$$\lim_{\theta \rightarrow \bar{\theta}(u_1)} \frac{\gamma(\theta, u_1)(y_0-z)}{r+\delta+\beta\lambda(\theta)} + \frac{(1-\gamma(\theta, u_1))(y_1-z)}{r+\delta+\beta\lambda(\theta)+\beta\lambda_2} = \frac{\mu y_0 + (1-\mu)y_1 - z}{r+\delta+\lambda(\bar{\theta}(u_1))}$$

where $\gamma(\bar{\theta}(u_1)) = \mu$. Then there exists an equilibrium with $\theta(u_1) < \bar{\theta}(u_1)$ if:

$$(1-\beta)\frac{y_0\mu + y_1(1-\mu) - z}{r+\delta+\beta\lambda(\bar{\theta}(u_1))} < \frac{c}{q(\bar{\theta}(u_1))}$$

Wage dispersion Δw is given by:

$$\Delta w = \beta(y_1 - y_0) + (1-\beta)\left(\frac{\beta(y_1-z)(1-\mu-u_1)\delta}{(r+\delta)u_1 + \beta(1-\mu-u_1)\delta} - \frac{\beta(y_0-z)\lambda(\theta)}{r+\delta+\beta\lambda(\theta)}\right)$$

Differentiation of Δw with respect to u_1 and θ gives

$$\frac{\partial \Delta w}{\partial u_1} = -(1-\beta)\frac{\beta(y_1-z)\delta(ru_1 + \delta(1-\mu))}{((r+\delta)u_1 + \beta(1-\mu-u_1)\delta)^2} < 0 \quad \frac{\partial \Delta w}{\partial \lambda(\theta)} = -(1-\beta)\frac{\beta(y_0-z)(r+\delta)}{(r+\delta+\beta\lambda(\theta))^2} > 0$$

Appendix 2.8.2: Proof of Proposition 2.2.

First, we solve the problem following an approach by Pissarides (2000, p. 184). The social planner wants to maximize expression (2.10) subject to the dynamic unemployment equations $\dot{u}_0 = \delta(\mu - u_0) - \lambda(\theta)u_0$ and $\dot{u}_1 = \delta(1 - \mu - u_1) - (\lambda(\theta) + \lambda_2)u_1$. Let k_0 and k_1 be the costate variables corresponding to u_0 and u_1 respectively. The current value Hamiltonian is then:

$$H = (1-\mu-u_1)y_1 + (\mu-u_0)y_0 + (z-c\theta)(u_0+u_1) - c_2v_2 - k_0[\delta(\mu-u_0) - \lambda(\theta)u_0] - k_1[\delta(1-\mu-u_1) - (\lambda(\theta) + \lambda_2)u_1]$$

Note that we use the negative sign in front of k_0 and k_1 to define these two variables as positive-valued functions. The first order conditions are then given by:

$$\frac{\partial H}{\partial \theta} = -c(u_0+u_1) + \lambda'(\theta)(k_0u_0 + k_1u_1) = 0 \quad \Rightarrow \quad c = \lambda'(\theta)(\gamma k_0 + (1-\gamma)k_1)$$

Since we know that $\lambda'(\theta) = q(\theta)(1 - \eta)$ and $q(\theta) = \lambda(\theta)/\theta$, we get:

$$\frac{c}{q(\theta)} = (1 - \eta)(\gamma k_0 + (1 - \gamma)k_1) \Leftrightarrow c\theta = \lambda(\theta)(1 - \eta)(\gamma k_0 + (1 - \gamma)k_1)$$

Next we differentiate with respect to v_2 and u_0 :

$$\begin{aligned} \frac{\partial H}{\partial v_2} &= -c_2 + ak_1[1 - (1 - \frac{u_1}{1 - \mu})^l] = 0 \Rightarrow \frac{c_2}{q_2} = k_1 \\ \frac{\partial H}{\partial u_0} &= -y_0 + (z - c\theta) + k_0(\lambda(\theta) + \delta) = -rk_0 \end{aligned}$$

Note that $c\theta = \lambda(\theta)(1 - \eta)(\gamma k_0 + (1 - \gamma)k_1) = -\lambda(\theta)(1 - \eta)(1 - \gamma)(k_0 - k_1) + \lambda(\theta)(1 - \eta)k_0$, so that $(r + \delta + \lambda(\theta))k_0 = y_0 - z - \lambda(\theta)(1 - \eta)(1 - \gamma)\Delta k + \lambda(1 - \eta)k_0$. This yields:

$$k_0 = \frac{y_0 - z - \lambda(\theta)(1 - \eta)(1 - \gamma)\Delta k}{r + \delta + \eta\lambda(\theta)}$$

Next we differentiate with respect to u_1 :

$$\frac{\partial H}{\partial u_1} = -y_1 + (z - c\theta) + k_1(\lambda(\theta) + \frac{alv_2}{1 - \mu}(1 - u_1/(1 - \mu))^{l-1} + \delta) = -rk_1$$

Again we use that $c\theta = \lambda(\theta)(1 - \eta)(\gamma k_0 + (1 - \gamma)k_1) = \lambda(\theta)(1 - \eta)\gamma(k_0 - k_1) + \lambda(\theta)(1 - \eta)k_1$

$$\begin{aligned} \left[r + \delta + \lambda(\theta) + \frac{alv_2}{1 - \mu}(1 - \frac{u_1}{1 - \mu})^{l-1} \right] k_1 &= y_1 - z + \lambda(\theta)(1 - \eta)\gamma(k_0 - k_1) + \lambda(\theta)(1 - \eta)k_1 \\ k_1 = \frac{y_1 - z + \lambda(\theta)(1 - \eta)\gamma\Delta k}{r + \delta + \eta\lambda(\theta) + \phi\lambda_2} \quad \text{or} \quad k_1 &= \frac{y_1 - z - \phi\lambda_2 k_1 + \lambda(\theta)(1 - \eta)\gamma\Delta k}{r + \delta + \eta\lambda(\theta)} \end{aligned}$$

The difference $\Delta k = k_0 - k_1$ can then be obtained in the following way:

$$\begin{aligned} \Delta k &= \frac{y_0 - z - \lambda(\theta)(1 - \eta)(1 - \gamma)\Delta k}{r + \delta + \eta\lambda(\theta)} - \frac{y_1 - z - \phi\lambda_2 k_1 + \lambda(\theta)(1 - \eta)\gamma\Delta k}{r + \delta + \eta\lambda(\theta)} \\ &= \frac{y_0 - y_1 - \lambda(\theta)(1 - \eta)\Delta k + \phi\lambda_2 k_1}{r + \delta + \eta\lambda(\theta)} \Rightarrow \Delta k = \frac{y_0 - y_1 + \phi c_2 \theta_2}{r + \delta + \lambda(\theta)} \end{aligned}$$

where we make use of the fact that $\lambda_2 k_1 = \lambda_2 c_2 / q_2 = c_2 v_2 / u_1 = c_2 \theta_2$.

The same solution can be obtained in an alternative way which is used by Blazquez and Jansen (2008), who define the Hamiltonian as:

$$\begin{aligned} H &= \frac{\lambda(\theta)(u_0 y_0 + u_1 y_1) + \lambda_2 u_1 y_1}{r + \delta} + (z - c\theta)(u_0 + u_1) - c_2 v_2 + \\ &+ \Lambda_0(\delta(\mu - u_0) - \lambda(\theta)u_0) + \Lambda_1(\delta(1 - \mu - u_1) - \lambda(\theta)u_1 - \lambda_2 u_1) \end{aligned}$$

where Λ_0 , and Λ_1 are costate variables corresponding to u_0 and u_1 respectively. The optimal social planner solution must satisfy:

$$\begin{aligned} \frac{\partial H}{\partial \theta} &= \frac{\lambda'(\theta)(u_0 y_0 + u_1 y_1)}{r + \delta} - c(u_0 + u_1) - \lambda'(\theta)u_0 \Lambda_0 - \lambda'(\theta)u_1 \Lambda_1 = 0 \\ &\Rightarrow \frac{c}{\lambda'(\theta)} = \left[\gamma \frac{y_0 - \Lambda_0(r + \delta)}{r + \delta} + (1 - \gamma) \frac{y_1 - \Lambda_1(r + \delta)}{r + \delta} \right] \end{aligned}$$

where $\gamma = u_0/(u_0 + u_1)$. Further, note that $\lambda'(\theta) = (1 - \eta)q(\theta)$ and define $k_i = [y_i - \Lambda_i(r + \delta)]/(r + \delta)$, $i = 1, 2$. This gives a counterpart of the free-entry condition in the regular market from the perspective of the social planner:

$$\frac{c}{q(\theta)} = (1 - \eta)[\gamma k_0 + (1 - \gamma)k_1] \quad \Rightarrow \quad c\theta = \lambda(\theta)(1 - \eta)[\gamma k_0 + (1 - \gamma)k_1]$$

Differentiating with respect to v_2 we get:

$$\frac{\partial H}{\partial v_2} = a[1 - (1 - \frac{u_1}{1 - \mu})^l] \frac{y_1}{r + \delta} - c_2 - a[1 - (1 - \frac{u_1}{1 - \mu})^l] \Lambda_1 = 0 \quad \Rightarrow \quad \frac{c_2}{q_2} = k_1$$

which should be compared to the free-entry condition in the referral market. Next we take derivatives with respect to u_0 and u_1 :

$$\begin{aligned} \frac{\partial H}{\partial u_0} &= \frac{\lambda(\theta)y_0}{r + \delta} + z - c\theta - \delta\Lambda_0 - \lambda(\theta)\Lambda_0 = r\Lambda_0 \\ \Lambda_0(r + \delta) &= z + \lambda(\theta) \frac{y_0 - \Lambda_0(r + \delta)}{r + \delta} - c\theta = z + \lambda(\theta)k_0 - \lambda(\theta)(1 - \eta)[\gamma k_0 + (1 - \gamma)k_1] \\ &= z + \eta\lambda(\theta)k_0 + \lambda(\theta)(1 - \eta)(1 - \gamma)(k_0 - k_1) \\ \frac{\partial H}{\partial u_1} &= \left(\lambda(\theta) + \frac{alv_2}{1 - \mu} \left(1 - \frac{u_1}{1 - \mu}\right)^{l-1} \right) \frac{y_1}{r + \delta} + z - c\theta - \Lambda_1 \left[\lambda(\theta) + \frac{alv_2}{1 - \mu} \left(1 - \frac{u_1}{1 - \mu}\right)^{l-1} + \delta \right] = r\Lambda_1 \\ \Lambda_1(r + \delta) &= z + \left(\lambda(\theta) + \frac{alv_2}{1 - \mu} \left(1 - \frac{u_1}{1 - \mu}\right)^{l-1} \right) \frac{y_1 - \Lambda_1(r + \delta)}{r + \delta} - c\theta \\ &= z + (\lambda(\theta) + \phi\lambda_2)k_1 - \lambda(\theta)(1 - \eta)[\gamma k_0 + (1 - \gamma)k_1] \\ &= z + (\eta\lambda(\theta) + \phi\lambda_2)k_1 - \lambda(\theta)(1 - \eta)\gamma(k_0 - k_1) \end{aligned}$$

From the above equations we can finally find the surplus values k_0 and k_1 ($\Delta k = k_0 - k_1$):

$$\begin{aligned} k_0(r + \delta) &= y_0 - \Lambda_0(r + \delta) = y_0 - z - \eta\lambda(\theta)k_0 - \lambda(\theta)(1 - \eta)(1 - \gamma)\Delta k \\ k_1(r + \delta) &= y_1 - \Lambda_1(r + \delta) = y_1 - z - (\eta\lambda(\theta) + \phi\lambda_2)k_1 + \lambda(\theta)(1 - \eta)\gamma\Delta k \end{aligned}$$

$$k_0 = \frac{y_0 - z - \lambda(\theta)(1 - \eta)(1 - \gamma)\Delta k}{r + \delta + \eta\lambda(\theta)} \quad k_1 = \frac{y_1 - z + (\eta - \phi)\lambda_2 k_1 + \lambda(\theta)(1 - \eta)\gamma\Delta k}{r + \delta + \eta\lambda(\theta) + \eta\lambda_2}$$

Comparing rU_0 in the decentralized economy with $(r + \delta)\Lambda_0$ of the planner when the Hosios condition is satisfied, we get that:

$$\begin{aligned} \tau_0 &= \lambda(\theta)(1 - \eta)(1 - \gamma)(k_0 - k_1) \\ &= \lambda(\theta)(1 - \eta)(1 - \gamma) \left[\Lambda_1 - \Lambda_0 - \frac{(y_1 - y_0)}{r + \delta} \right], \quad \text{where } \Lambda_1 > \Lambda_0 \end{aligned}$$

If productivity differences between the two types of workers are small, i.e. $y_1 = y_0$, then $\tau_0 > 0$ which implies that the planner would want to give subsidies to low ability workers and impose taxes ($\tau_1 < 0$) on high ability workers. It is because high ability workers have better outside opportunities (due to their parallel search in the referral market) but this is not reflected in their productivity. Instead they demand higher wages which dampens the job-creation in the

regular market and hurts the job-finding chances of low ability workers (negative externality on low ability workers). This is a negative spillover effect between the two markets. However, this result is reversed if y_1 is sufficiently large. In this latter case high ability workers are relatively productive and impose a positive externality on low ability workers by fostering job creation in the regular market. This is a positive spillover within the same (regular) market. So the planner would want to give subsidies to high ability workers ($\tau_1 > 0$) and finance them by taxes on low ability workers ($\tau_0 < 0$).

Next we compare our findings to Blazquez and Jansen (2008). The main difference between the two models is that in our model firms decide ex-ante before matching, whether they post an official vacancy in the regular market or they don't post a vacancy and try to fill the position through a referral. Thus all vacancies are separated between the two markets: regular and referral. And there are two matching processes, one between all unemployed workers and regular vacancies, and second between high ability unemployed and referral vacancies. In contrast, in the model of Blazquez and Jansen (2008) firms do not decide in advance about the type of the vacancy, rather all unemployed workers are matched with all vacancies, so there is one matching process/market. After the match the firm and the worker learn whether the vacancy is simple (with probability ϕ) or complex (with probability $1 - \phi$) and decide if the match continues.

The planner's job creation condition in their model is given by (see equation (35)):

$$c\theta = \theta^{1-\alpha}(1 - \alpha)\bar{\sigma} = \theta^{1-\alpha}(1 - \alpha)[\phi\gamma\sigma(l, n) + (1 - \gamma)\bar{\sigma}(h)]$$

where $\sigma(l, n) = y(n) - \lambda(l)(r + \delta)/(r + \delta)$ which is comparable to the total surplus k_0 generated by low ability workers in our model. And $\bar{\sigma}(h) = \phi\sigma(h, n) + (1 - \phi)\sigma(h, s)$ which should be compared to surplus k_1 generated by high ability workers in our model. Next, the present value of unemployed low ability workers according to the planner is given by (see equation (30)):

$$\begin{aligned} (r + \delta)\lambda(l) &= b + \phi\theta^{1-\alpha}\sigma(l, n) - c\theta = b + \phi\theta^{1-\alpha}\sigma(l, n) - \theta^{1-\alpha}(1 - \alpha)[\phi\gamma\sigma(l, n) + (1 - \gamma)\bar{\sigma}(h)] \\ &= b + \alpha\phi\theta^{1-\alpha}\sigma(l, n) - \theta^{1-\alpha}(1 - \alpha)(1 - \gamma)[\bar{\sigma}(h) - \phi\sigma(l, n)] \end{aligned}$$

This should be compared to the present value in the decentralized market (see eq. (60)):

$$\begin{aligned} rU(l) &= b - \tau(l) + \alpha\phi\theta^{1-\alpha}S(l, n) && \Rightarrow \\ -\tau(l) &= \theta^{1-\alpha}(1 - \alpha)(1 - \gamma)[\phi\sigma(l, n) - \bar{\sigma}(h)] \\ &= \phi\theta^{1-\alpha}(1 - \alpha)(1 - \gamma)\left[\lambda(h) - \lambda(l) - \frac{1 - \phi}{\phi}\sigma(h, s)\right], \quad \text{where } \lambda(h) > \lambda(l) \end{aligned}$$

Note that planner's transfers to low ability workers τ_0 and $\tau(l)$ in the two papers are defined in the opposite way. $\tau_0 > 0$ in our model implies subsidies to low ability workers, whereas $\tau(l) > 0$ is defined as a tax on low ability workers in (BJ). Thus we compare the sign of τ_0 in our model with the sign of $(-\tau(l))$ in (BJ). Specifically, they prove that the term in the square bracket is always negative, hence $-\tau(l) < 0$, which implies taxes on low ability workers in their model and subsidies to the high ability workers. Intuitively, this is because firms with open vacancies anticipate to form a highly productive match with a surplus $\sigma(h, s)$ if they are matched with high ability workers and the job turns out to be a complex one, so these workers always impose

a positive externality on low ability workers. Since there is only one large market in (BJ), they can only identify a positive spillover of high ability workers on low ability workers within this market, but they do not have (by construction) the negative spillover between the two markets which is present in our model if productivity differences between the two types of workers are not too large, for example, $y_1 = y_0$.

Appendix 2.8.3: Proof of Proposition 2.3. Consider the decentralized economy with a vector of policy instruments $\{s = \eta c_2, \tau_0 = \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma), \tau_1 = -\lambda(\theta)\Delta k(1 - \eta)\gamma\}$:

$$\begin{aligned} S_0 &= \frac{y_0 - z - \tau_0}{r + \delta + \beta\lambda(\theta)} = \frac{y_0 - z - \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma)}{r + \delta + \beta\lambda(\theta)} \\ S &= \frac{y_1 - z - \tau_1}{r + \delta + \beta\lambda(\theta) + \beta\lambda_2} = \frac{y_1 - z + \lambda(\theta)\Delta k(1 - \eta)\gamma}{r + \delta + \beta\lambda(\theta) + \beta\lambda_2} \end{aligned}$$

For the case when $\beta = \eta = \phi$ the free-entry conditions become:

$$\frac{c}{q(\theta)} = (1 - \eta)[\gamma S_0 + (1 - \gamma)S] \quad \text{and} \quad \frac{c_2 - \eta c_2}{q_2} = (1 - \eta)S \quad \Rightarrow \quad \frac{c_2}{q_2} = S$$

therefore, it follows that $S_0 = k_0$ and $S = k_1$ and the optimal allocation can be implemented. In addition, note that the two transfers τ_0^* and τ_1^* are purely redistributive:

$$\begin{aligned} u_0\tau_0^* + u_1\tau_1^* &= u_0\lambda(\theta)\Delta k(1 - \eta)(1 - \gamma) - u_1\lambda(\theta)\Delta k(1 - \eta)\gamma \\ &= \lambda(\theta)\Delta k(1 - \eta)[u_0(1 - \gamma) - u_1\gamma] = \lambda(\theta)\Delta k(1 - \eta)\left[\frac{u_0u_1}{u_0 + u_1} - \frac{u_1u_0}{u_0 + u_1}\right] = 0 \end{aligned}$$

Appendix 2.8.4: Suppose the optimal transfers ($\tilde{\tau}_0, \tilde{\tau}_1$) are provided to low- and high-ability workers. In addition, we allow for employment subsidies $s = \eta c_2$, then it holds:

$$\begin{aligned} (r + \delta + \beta\lambda(\theta))S_0 &= y_0 - z - \tilde{\tau}_0 = y_0 - z - \lambda(\theta)\Delta k(1 - \eta)(1 - \gamma) - \frac{(\eta - \beta)}{(1 - \beta)}k_0(r + \delta + \lambda(\theta)) \\ &= k_0(r + \delta + \eta\lambda(\theta)) - \frac{(\eta - \beta)}{(1 - \beta)}k_0(r + \delta + \lambda(\theta)) = \frac{(1 - \eta)}{(1 - \beta)}k_0(r + \delta + \beta\lambda(\theta)) \end{aligned}$$

this means that $(1 - \beta)S_0 = (1 - \eta)k_0$. In a similar way (taking into account that $\phi = \eta$) we get:

$$\begin{aligned} (r + \delta + \beta\lambda(\theta) + \beta\lambda_2)S &= y_1 - z - \tilde{\tau}_1 \\ &= y_1 - z + \lambda(\theta)\Delta k(1 - \eta)\gamma - \frac{(\eta - \beta)}{(1 - \beta)}k_1(r + \delta + \lambda(\theta) + \lambda_2) \\ &= k_1(r + \delta + \eta\lambda(\theta) + \eta\lambda_2) - \frac{(\eta - \beta)}{(1 - \beta)}k_1(r + \delta + \lambda(\theta) + \lambda_2) \\ &= \frac{(1 - \eta)}{(1 - \beta)}k_1(r + \delta + \beta\lambda(\theta) + \beta\lambda_2) \end{aligned}$$

additionally taking into account that $s = \eta c_2$, the final result is:

$$(1 - \eta)[\gamma k_0 + (1 - \gamma)k_1] = \frac{c}{q(\theta)} = (1 - \beta)[\gamma S_0 + (1 - \gamma)S] \quad \text{and} \quad k_1 = \frac{c_2}{q_2} = \frac{(1 - \beta)}{(1 - \eta)}S$$

Appendix 2.8.5: Tables for model parameters after imposing the policy for $y_1 = 1.25$ and

$y_1 = 1.5$

Optimal policy	θ	u_0/μ	$u_1/(1-\mu)$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4609	0.0924	0.0388	0.9770	1.2359	0.5565	0.4810
Only subsidy $\{s\}$	0.4634	0.0923	0.0171	0.9770	1.2437	0.5565	0.4911
Final policy $\{s, \tau_0, \tau_1\}$	0.4562	0.0927	0.0158	0.9775	1.2433	0.5573	0.4907

Table 2.5: Optimal policy $s = 0.288, \tau_0 = 0.014, \tau_1 = -0.124$ in a labour market with $y_1 = 1.25$

Optimal policy	θ	u_0/μ	$u_1/(1-\mu)$	w_0	$w_1 = w_2$	Ω_0	Ω_1
Without policy	0.4722	0.0918	0.0324	0.9772	1.4847	0.5565	0.5798
Only subsidy $\{s\}$	0.4702	0.0919	0.0147	0.9771	1.4930	0.5565	0.5903
Final policy $\{s, \tau_0, \tau_1\}$	0.4623	0.0923	0.0141	0.9775	1.4928	0.5572	0.5901

Table 2.6: Optimal policy $s = 0.288, \tau_0 = 0.012, \tau_1 = -0.084$ in a labour market with $y_1 = 1.5$

Appendix 2.8.6: Tables for values of $\tau_0 + \tau^*$ and $\tau_1 + \tau^*$ as well as Ω_0 and Ω_1 with and without final policy for different values of β in cases of $y_1 = 1.25$ and $y_1 = 1.5$

	$\beta = 0.4$	$\beta = 0.49$	$\beta = 0.62$	$\beta = 0.70$	$\beta = 0.72$	$\beta = 0.8$
$\tilde{\tau}_0$	0.4315	0.3670	0.2200	0.0663	0.0141	-0.2989
$\tilde{\tau}_1$	0.5925	0.4818	0.2293	-0.0348	-0.1244	-0.6621
Ω_0 without policy	0.5492	0.5531	0.5561	0.5566	0.5565	0.5555
Ω_0 with final policy	0.5805	0.5769	0.5687	0.5602	0.5573	0.5399
Ω_1 without policy	0.4837	0.4840	0.4832	0.4816	0.4810	0.4769
Ω_1 with final policy	0.4953	0.4946	0.4930	0.4913	0.4907	0.4873
$\Omega_0 + \Omega_1$ without policy	1.0329	1.0371	1.0393	1.0382	1.0375	1.0325

Table 2.7: The values of $\tilde{\tau}_0$ and $\tilde{\tau}_1$, Ω_0 and Ω_1 with and without final policy as well as $\Omega_0 + \Omega_1$ without policy for different values of β in the case of $y_1 = 1.25$

	$\beta = 0.4$	$\beta = 0.48$	$\beta = 0.61$	$\beta = 0.70$	$\beta = 0.72$	$\beta = 0.8$
$\tilde{\tau}_0$	0.4310	0.3746	0.2338	0.0647	0.0141	-0.3016
$\tilde{\tau}_1$	0.7877	0.6703	0.3769	0.0249	-0.1244	-0.7379
Ω_0 without policy	0.5489	0.5526	0.5559	0.5566	0.5565	0.5556
Ω_0 with final policy	0.5804	0.5773	0.5695	0.5601	0.5572	0.5398
Ω_1 without policy	0.5826	0.5829	0.5821	0.5804	0.5798	0.5756
Ω_1 with final policy	0.5950	0.5944	0.5927	0.5907	0.5901	0.5864
$\Omega_0 + \Omega_1$ without policy	1.1316	1.1355	1.1380	1.1369	1.1363	1.1313

Table 2.8: The values of $\tilde{\tau}_0$ and $\tilde{\tau}_1$, Ω_0 and Ω_1 with and without final policy as well as $\Omega_0 + \Omega_1$ without policy for different values of β in the case of $y_1 = 1.5$

3 Explaining U-shape of the referral hiring pattern in a search model with heterogeneous workers

3.1 Introduction

Several studies show that referrals are mostly used by workers in the tails of the skill distribution, whereas all other workers in the middle are more likely to use a formal channel of job search (Brown et al. (2012) for the US, Corak and Piraino (2011) for Canada, Boxman et al. (1991) for the Netherlands). The purpose of this article is to explain this U-shape referral hiring pattern in a labour market matching model with heterogeneous workers, social networks and referrals.

The ingredients of the model are as follows. Firms are homogeneous at the stage of a vacancy, but workers differ in their productivity which we also interpret as skill heterogeneity. There are two types of social contacts. Family contacts are exogenous in the model and serve as a residual method of search. In addition, every worker has a fixed number of professional contacts¹. Ioannides and Datcher Loury (2004) report that acquired social contacts develop along dimensions such as race, ethnicity, religious affiliation and education. Therefore, in our model we assume a strong degree of network homophily along the productivity or the skill dimension. Thus, the job-finding rate through the network of professional contacts is skill-specific. In this setup, we distinguish between the three job search channels: formal applications to posted vacancies and two informal channels - through family and professional networks. Both informal channels of search are costless for workers, but preparing a formal application is associated with a positive effort cost. Moreover, through the endogenous group-specific advertising intensity firms can direct their network search towards particular groups of incumbent employees. This contrasts with the formal search channel, which is random and undirected.

There are two key predictions of the model which can be described in the following way:

- The model exhibits a strong U-shape referral hiring pattern: workers in the right (left) tail of the productivity distribution have the highest propensity of finding a job with a help of professional (family) contacts, whereas the formal channel of search is mostly utilized by workers in the middle range of the distribution;
- When the two types of social contacts are separated, family contacts are associated with wage penalties, whereas referrals from professional contacts are associated with wage premiums. The average effect of referrals on wages is ambiguous and depends on the relative proportions of high and low productivity workers in the population.

To the best of our knowledge there are no other studies that can generate these two predictions in a unified theoretical framework. First, we explain the mechanism which is generating the U-shape. Low productivity workers expect low wages thus it's not optimal for them to exert costly search effort. At the same time hiring these workers is not profitable for firms, so that firms prefer to direct their search towards more productive worker groups. Hence low productivity workers rely on family referrals as a method of last resort. Further we show that due to the strong homophily of professional networks firms correctly anticipate a high productivity applicant if

¹The importance of relatives for job search is reported by Corak and Piraino (2011) and Kramarz and Skans (2014). For the role of former co-workers see Cingano and Rosolia (2012) and Glitz (2013).

they approach an incumbent employee of the same type. Such a behavior of firms is based on the belief that people usually refer workers who are similar to themselves (Galenianos (2014), Saloner (1985), Montgomery (1991)). Therefore, high productivity workers tend to find their jobs by means of professional referrals. It also implies that average workers mostly use the formal channel: their expected wages are sufficiently high and motivating to exert search effort but their outside opportunities in terms of professional referrals are not yet too good. This describes the U-shape hiring pattern as an outcome of workers' selection across search channels. Selection on productivity is consistent with empirical evidence, for example, when pooling data for 14 European countries, Pelizzari (2010) shows that referrals are associated with a wage penalty of 17.4% before controlling for worker characteristics. However, this effect is reduced to only 4.4% after controlling for observable worker traits and down to 2% when controlling for unobservables.

Next we analyze the effect of referrals on wages. As low productivity types are more likely to rely on family contacts, the equilibrium wage distribution of workers who used this channel is first order stochastically dominated by the distribution of workers who used a formal method. So the model predicts wage penalties associated with family contacts. In contrast, high productivity workers are more likely to rely on professional contacts. Thus the equilibrium earnings distribution corresponding to this channel first order stochastically dominates the distribution of workers who used a formal method. Hence the network of professional contacts is associated with wage premiums. Intuitively, wage penalties/premiums arise due to the self-selection of workers into a specific search channel and are robust to different shapes of the productivity distribution. This is different for the average effect of referrals on wages which is negative (positive) in a labour market with a large fraction of low (high) types. This finding may serve as an explanation for the mixed empirical evidence on referral wages as most studies don't distinguish between family and professional referrals focusing on a unique informal channel².

Yet there are several notable exceptions among empirical papers. For example, Cappellari and Tatsiramos (2015) report that high skilled workers with a better network quality of non-relatives experience wage premiums in the British labour market. In contrast, low skilled workers with a better network quality of relatives are more likely to experience a wage penalty associated with a referral. Likewise, empirical evidence presented in Meliciani and Radicchia (2011) for Italy suggests that workers entering the labour market via professional contacts enjoy a wage bonus, whereas those recruited via referrals from family and close friends receive on average lower wages. Similar results are also reported by Sylos Labini (2004) for Italy and Antoninis (2006) for Egypt which is a direct support for the second prediction of our model.

More empirical evidence in favor of the selection mechanism described by our model is provided by Kramarz and Skans (2014) for Sweden and Kuzubas and Szabo (2014) for Indonesia. For example, the former study finds that parental networks matter more in the job search process for low educated youths even though there is a wage penalty in the first years of employment.

²For example, Staiger (1990), Simon and Warner (1992) and Granovetter (1995) report that referrals are associated with wage premiums in the United States. The hypothesis of wage premiums is also supported by Margolis and Simonnet (2003) and Goos and Salomons (2007) for France and the United Kingdom. In contrast, Bentolila et al. (2010) report wage penalties in the United States and the European Union. This result is supported by Delattre and Sabatier (2007), Pistaferri (1999) as well as Addison and Portugal (2002) for France, Italy and Portugal respectively. This contradicting empirical evidence, which can be well described as a "referral puzzle", is summarized in Pelizzari (2010) who writes that "... in the European Union premiums and penalties to finding jobs through personal contacts are equally frequent and are of about the same size".

Moreover, Kuzubas and Szabo (2014) report that in their sample low educated workers are more likely to find a job through family and close friends (52%) compared to college graduates (34%). In addition, Meliciani and Radicchia (2011) write that "people entering the labor market via relatives and friends contacts have lower levels of education, no specific competencies or training than the average and seem to be generally concentrated into lower occupational groups" (p.521).

Finally, we show that due to networks there can be multiple stationary equilibria in our model. This is particularly the case when professional networks are a dominating channel of job search, whereas formal applications and family referrals are hardly used. We find that in this case there is a stable equilibrium with low unemployment and many vacancies, an unstable equilibrium with high unemployment and fewer vacancies and a stable corner equilibrium with full unemployment. The first two equilibria coexist as firms are facing the same expected profits: there are many (few) employees who can give a recommendation but few (many) applicants per employee in the first (second) equilibrium. There are some other studies highlighting the point that social networks may lead to multiple equilibria, for example, Cahuc and Fontaine (2009) in a dynamic frictional framework as well as Cabrales et al. (2011) and Merlino (2014) in a static network framework. However, in all three papers individual search effort is crucial for the results, whereas in our framework networks alone give rise to the multiplicity of equilibria.

Our study is also related to other theoretical papers analyzing the role of social networks. Early economic studies on social contacts include Simon and Warner (1992), Montgomery (1991, 1992, 1994) and Mortensen and Vishwanath (1994). Both Simon and Warner (1992) and Montgomery (1991) explain that referrals reveal the quality of the match to the employer and should have a positive effect on wages. This result is similar to the positive wage effect of professional referrals in our model, however, family contacts are not included in the early studies. Recent theoretical studies generating wage premiums associated with referrals include Kugler (2003), Ioannides and Soetevent (2006) and Galenianos (2014). Ioannides and Soetevent (2006) show that better connected workers experience lower unemployment rates and receive higher wages. This should be compared with our finding that more productive workers experience lower unemployment rates because they have a lower proportion of unemployed contacts in their network. Note that this result is different from Ioannides and Soetevent (2006) as all workers have the same fixed number of network contacts in our model. So it is the endogenous proportion of employed contacts that differs between the agents, whereas it is the total number of contacts which is different between workers in their study.

The group of papers that can generate wage penalties in a theoretical framework includes Bentolila et al. (2010) as well as Ponzio and Scoppa (2010). Ponzio and Scoppa (2010) argue that recruiters may favor low ability family ties over more talented applicants. This is the idea of favoritism in the recruiting process. Bentolila et al. (2010) find that social contacts can generate a mismatch between occupational choices and productive advantages of workers. This is particularly true for workers who failed to find a job in their occupation and followed a recommendation of a close family member. Horvath (2014) extends the mismatch result of Bentolila et al. (2010). As the probability that ties connect similar agents (homophily) increases, the mismatch level decreases in his model. Moreover, if this probability is sufficiently high, networks provide good matches at higher rate upon arrival than the formal market. Therefore, referrals can generate wage premiums (penalties) if the homophily level in the society is high

(low).

The first idea that positive and negative effects of referrals are simultaneously valid for different types of contacts and can account for differences in the wage effects is due to Sylos Labini (2004) and Datcher Loury (2006) followed by Kuzubas and Szabo (2014). In a theoretical model confirmed by empirical evidence Sylos Labini (2004) shows that workers who find their jobs through professional referrals earn on average higher wages, whereas workers who are recommended by their relatives earn lower wages. Similarly Kuzubas and Szabo (2014) develop a theoretical model of a frictional labour market for Indonesia with two channels of search: inner networks (families) and outer networks defined as the ethnic language group. Using the inner network of relatives is costless for workers, which is also the same in our model, however there is a fixed cost of using the outer network. Thus it is mostly high skilled workers who pay this cost and use a large outer network. These results are similar in our model if the network of professional contacts is merged with potential employers into one large outer network. Nevertheless, our model is more specific as the formal channel is separated from professional contacts, which explains the U-shape referral pattern observed in developed economies.

Other theoretical papers which can explain wage premiums/penalties depending on the parameters of the labour market are Tumen (2013) and Zaharieva (2015). Tumen (2013) considers a population of workers heterogeneous with respect to the cost of maintaining connections. In his model well integrated workers with low costs have higher reservation wages and are able to bargain higher wages. Conversely, workers with higher costs accept wages below the market level. Zaharieva (2015) investigates the role of referrals in a matching model with on-the-job search. On the one hand, in her model better connected workers bargain higher wages for a given level of job-related productivity. This is the positive effect of outside opportunities on wages. On the other hand, employees rationally accept job offers from more productive employers and forward other offers to the unemployed contacts. Therefore, job offers transmitted through social contacts are biased in the direction of less productive employers. This selection mechanism can generate a negative effect of referrals on wages. To sum up, both papers by Tumen (2013) and Zaharieva (2015) can generate wage penalties or premiums in wages associated with social contacts, however, in each paper the mechanism is different from the present study.

The paper is organized as follows. Section 3.2 explains notation and the economic environment. In section 3.3 we investigate the decisions of workers and firms and explain their choice of the search intensity. Section 3.4 illustrates our theoretical results by means of a numerical example. Section 3.5 includes a number of robustness checks and section 3.6 concludes the paper.

3.2 Labour market modeling framework

The labour market is characterized by the following properties. There is a continuum of infinitely lived risk neutral workers and firms discounting future at a common discount rate r . Firms are homogeneous, while workers have heterogeneous productivity y_i , $i = 1..p$, with p being the number of distinct productivity groups and f_i denoting a fraction of a given worker group in the labour force, so that $\sum_{i=1}^p f_i = 1$. Workers are perfectly informed about their productivity y_i and it is revealed to the firm upon the match (e.g. after screening). From the perspective of interpretation, we think of productivity y_i as a function of observable and unobservable worker skills, hence it is positively correlated with schooling, but not directly observable by third parties

(econometrician) without screening. The highest productivity y_p is set to 1, while the lowest productivity y_1 is equal to the unemployment benefit b .

Every worker can be either employed and producing output y_i or unemployed and searching for a job. Let u_i denote the mass of unemployed workers with productivity y_i and e_i – the mass of corresponding employees, so that $e_i + u_i = f_i$, since the total measure of workers is normalized to 1. There are three search channels in the labour market. First, unemployed workers can find a job by sending regular applications to open vacancies, this is the formal channel of job search with an endogenous job-finding rate $\phi(s)$. Variable s is the individual search effort of workers and may differ across agents belonging to different productivity groups, i.e. s_i . The formal channel of search is costly in terms of effort, since it requires preparing and sending job applications. However, a more intensive job search is associated with a higher probability of finding employment. Let $C(s) = s^2/c$ denote the effort cost function, which is identical for all workers in the market.

Further, let all workers have an equal number of professional contacts $n > 0$. Employed workers provide referrals and transmit vacancy information to the unemployed members of their network, this is the second channel of job search. To simplify the model we assume that professional contacts are only formed among workers with the same productivity level y_i . Therefore, the job-finding rate through the network of professional contacts is skill-specific and is denoted by λ_i . Empirical support for this assumption comes from the observation of strong homophily in social networks reported in Rivera et al. (2010). Finally, λ_0 is a constant probability of hearing about a job from family members which is a third search channel in the model. In section 3.5 we endogenize λ_0 as a form of robustness check for the model, however, it is constant in the rest of the paper. Job referrals from professional contacts and family are the informal methods of search and are costless for workers³.

Firms are free to enter the labour market by opening new vacancies. Open vacancies are associated with a flow cost z on the side of the firm⁴. Formal matching between unemployed workers and vacancies is random and discussed below. To model the process of network matching we extend the approach of Cahuc and Fontaine (2009) and assume that firms make a random draw from the pool of incumbent employees with an advertising intensity a per unit time. However, in our model the advertising intensity a is endogenous and can be specific to a given group of employees, i.e. a_i . Intuitively, a_i is an effort level with which the manager of an open vacancy is addressing an incumbent employee of type i to refer one of his/her contacts. This extension allows firms to direct their search more intensively towards the more productive group of workers. The advertising search intensity a is costly for firms with a cost function $K(a) = a^2/k$. Note that the advertising intensity a_i is chosen after the match with an employee

³While there is strong agreement in the literature that getting help from family members is a costless method of search, it is less obvious for professional contacts. One explanation of this assumption is that in this paper we only focus on a group of colleagues and former coworkers of the agent which can be seen as a subgroup of all professional contacts. Empirical studies show that former coworkers are an important source of job-related information for the unemployed (Cingano and Rosolia (2012) and Glitz (2013)). Moreover, due to the recent IT development (such as Facebook, LinkedIn and Xing) it became easy for workers to stay in touch with former coworkers. Therefore, in the model we assume that the cost of keeping professional contacts is negligibly small compared to the formal search channel and normalize it to zero. Yet another advantage of treating former colleagues as professional contacts is a strong degree of skill homophily between coworkers.

⁴It can be understood as financial expenses for making vacancy information visible to the applicants. This includes posting vacancies in the newspapers, registering on the recruitment websites and participating in the job fairs. It may also include the cost of capital depreciation.

and so the cost $K(a_i)$ is unrelated to the cost $K(a_j)$ for $i \neq j$. If the job position is filled with a worker, the firm obtains a flow profit $y_i - w_i$, where the wage w_i is bargained between the firm and the worker upon hiring. We use the Nash bargaining rule to determine wages. Every filled job can be destroyed for exogenous reasons at rate δ .

Let $m(x, v)$ denote a matching function between workers and firms, where $v \leq \bar{v}$ is the number of open vacancies and x is the number of searching workers in efficiency units (either unemployed or employed, transmitting job offers to their unemployed contacts). Following the approach of Gautier et al. (2010) we assume that the matching technology is quadratic, that is $m(x, v) = xv$. This approach has been frequently used in the search literature, for example, Teulings and Gautier (2004) provide a number of explanations why this technology may be the most adequate assumption in a model with worker heterogeneity. The main reason is that this technology avoids congestion externalities between different worker types and jobs.

Consider matching between unemployed workers and open vacancies. The total number of searching unemployed workers weighted by their search intensity is given by $x = \sum s_i u_i$, so the number of contacts created through the formal method of search is $v \sum s_i u_i$. However, only proportion $s_i u_i / \sum s_i u_i$ of these contacts are the matches between open vacancies and unemployed workers of type y_i . Therefore, the number of matches between open vacancies and unemployed workers of type y_i is given by:

$$v \sum s_i u_i \cdot \frac{s_i u_i}{\sum s_i u_i} = v s_i u_i$$

This means that the job-finding rate through the formal channel of search is equal to $\phi_i \equiv \phi(s_i) = v s_i u_i / u_i = v s_i$ and is increasing in the total number of vacancies v and the individual search intensity of unemployed workers s_i . In addition, from the perspective of firms, the probability of filling a job through the formal channel with a worker of type y_i is $\phi_i u_i / v = s_i u_i$.

Next consider matching between employed workers and open vacancies. The total number of employees in efficiency units is given by $x = \sum a_i e_i$, so the number of contacts between vacancies and employees with productivity y_i is equal to $v \sum a_i e_i$. However, only a fraction $a_i e_i / \sum a_i e_i$ of these contacts are the matches with employees of type y_i . Every contacted employed worker transmits vacancy information to exactly one randomly chosen unemployed social contact out of n . Here we assume that job information is only transmitted to the direct social links, so the job offer is lost if all n contacts are employed. The probability of being employed for an arbitrary worker of type y_i is equal to $1 - \mu_i$, where $\mu_i \equiv u_i / f_i$ is the unemployment rate in a group of workers with productivity y_i . So the probability that all n contacts of the employee are also employed is equal to $(1 - \mu_i)^n$. This means that the number of matches between vacancies and unemployed workers of type y_i through the network of contacts is given by:

$$v \sum a_i e_i \cdot \frac{a_i e_i}{\sum a_i e_i} \cdot [1 - (1 - \mu_i)^n] = v a_i e_i [1 - (1 - \mu_i)^n]$$

where expression in the square bracket is the probability of having at least one unemployed contact out of n . The individual job-finding rate through the first informal search channel

(professional contacts) is then equal to:

$$\lambda_i = va_i \frac{e_i}{u_i} [1 - (1 - \mu_i)^n] = va_i \frac{1 - \mu_i}{\mu_i} [1 - (1 - \mu_i)^n]$$

Note that λ_i is increasing in the number of vacancies v and the number of social contacts n . Moreover, a more intensive search by firms directed at workers of type y_i , that is a higher a_i , is raising the probability of finding a job for an unemployed worker of this type. From the perspective of firms, the flow probability of filling a job with a professional contact of an incumbent employee of type y_i is equal to $\lambda_i u_i / v = a_i e_i [1 - (1 - \mu_i)^n]$.

3.3 Analysis of the model

3.3.1 Workers and their choice of search effort

Let U_i (W_i) denote the present value of being unemployed (employed) for the worker with productivity $y_i, i = 1..p$. The present value equation for unemployed workers is given by:

$$rU_i = b + (\lambda_0 + \lambda_i)(W_i - U_i) + v \max_s [s(W_i - U_i) - \frac{1}{c}s^2] \quad (3.1)$$

and reflects simultaneous availability of the three job search channels discussed above. The rent from employment is independent of the search channel and is denoted by $R_i \equiv (W_i - U_i)$. Workers choose costly effort s_i to maximize the present value of unemployment U_i , therefore the optimal level of search effort s_i obtains at the point where the marginal gain $(W_i - U_i)$ is equal to the marginal cost $C'(s)$:

$$s_i = 0.5c(W_i - U_i) = 0.5cR_i \quad \Rightarrow \quad rU_i = b + (\lambda_0 + \lambda_i)R_i + 0.25cvR_i^2$$

The asset value of employment for type y_i workers can be written as:

$$rW_i = w_i - \delta(W_i - U_i) \quad (3.2)$$

and so the worker rent from employment is equal to the discounted net present value of earnings: $R_i = (w_i - rU_i)/(r + \delta)$. Combining this and equation (3.2) allows us to derive the optimal search effort $s_i \equiv s(\lambda_i, w_i)$. These results are summarized in Lemma 3.1:

Lemma 3.1. *Consider workers with productivity y_i . The optimal job-finding rate $\phi(s_i) = vs_i$ through the formal channel of search is given by:*

$$vs_i = \sqrt{(r + \delta + \lambda_0 + \lambda_i)^2 + (w_i - b)cv} - (r + \delta + \lambda_0 + \lambda_i) \quad (3.3)$$

The optimal search intensity $s(\lambda_i, w_i)$ is increasing in the wage w_i but decreasing in λ_i , which is a job-finding rate through professional contacts. The optimal search intensity $s(\lambda_i, w_i)$ is also decreasing in the number of vacancies v .

Proof: Appendix 3.8.1.

Lemma 3.1 shows that a higher wage w_i would motivate workers to exert more effort when applying for jobs. On the contrary, a higher job-finding rate through professional contacts λ_i

improves the outside opportunities of workers, so the total rent from a job R_i is reduced. A lower rent then has a disincentive effect on the intensity of job search. In addition, there is a similar disincentive effect from a higher number of vacancies v , thus vacancies and effort are substitutes in our setting.

3.3.2 Firms and the wage determination

From the perspective of firms, let J_i be the asset value of a job, filled with a worker of type y_i , and V be the present value of the open vacancy. We will come back to the determination of V in section 3.3.5. Once matched firms learn the productivity of the applicant, so J_i is given by:

$$rJ_i = y_i - w_i - \delta(J_i - V) \quad (3.4)$$

The equilibrium wages are determined by means of Nash bargaining with a disagreement-while-bargaining state U_i^D for type y_i worker and with $\alpha \in (0, 1)$ being the workers' bargaining power, for example, as in Gautier (2002) and Hall and Milgrom (2008). This approach is close to the bargaining model with a risk of a negotiation breakdown by Binmore et al. (1986) and allows us to simplify the model, while not influencing qualitatively the results. An unemployed worker gets a present value U_i^D during the disagreement time, while the employer obtains a present value V_i^D . We assume that during the time of negotiation neither the worker nor the firm continue searching for other partners. This is intuitive since there are no reasons for agents to exert costly search effort when they are already in the process of bargaining with a prospective partner. This means that neither the worker nor the firm pays the search cost during the period of negotiation, however, the worker still receives the unemployment benefit from the state. Thus, U_i^D and V_i^D can be written as:

$$rU_i^D = b + \delta(U_i - U_i^D) \quad rV_i^D = \delta(V - V_i^D)$$

These equations imply that vacancies have the same probability δ of being destroyed during the bargaining as do existing jobs. Moreover, if the bargaining process breaks down for an exogenous reason, the worker becomes unemployed with a present value U_i and the position remains vacant with a present value V . The solution is the wage w_i maximizing the Nash objective function $(W_i - U_i^D)^\alpha (J_i - V_i^D)^{1-\alpha}$ which can be written as:

$$\max_{w_i} \left(\frac{w_i + \delta U_i}{r + \delta} - \frac{b + \delta U_i}{r + \delta} \right)^\alpha \left(\frac{y_i - w_i + \delta V}{r + \delta} - \frac{\delta V}{r + \delta} \right)^{1-\alpha} \Rightarrow w_i = \alpha y_i + (1 - \alpha)b$$

This maximization problem shows that the wage is a weighted average between the unemployment benefit b and the productivity y_i . Therefore, wages are heterogeneous in the economy and resemble the productivity distribution in the population of workers. Let g_i denote the equilibrium distribution of wages, such that $\sum g_i = 1$. It is then given by:

$$g_i = \frac{e_i}{e} = \frac{f_i(1 - \mu_i)}{\sum f_i(1 - \mu_i)} = \frac{f_i(1 - \mu_i)}{1 - \sum f_i \mu_i}$$

where $e = 1 - \sum f_i \mu_i$ is the equilibrium employment rate in the economy. Intuitively, if the employment rate of some worker group is smaller than the average, i.e. $(1 - \mu_i) < (1 - \sum f_i \mu_i)$,

then this group is underrepresented in the earnings distribution compared to the initial productivity density f_i . The opposite holds when the employment rate of some worker group is larger than the average, so this group is overrepresented.

3.3.3 Type-specific unemployment rates

Consider workers with productivity y_i . The unemployment rate $\mu_i = u_i/f_i$ can be found from the steady-state equation for unemployed workers. It can be written as:

$$0 = \dot{u}_i = \delta(f_i - u_i) - (\lambda_0 + \lambda_i + s_i v)u_i \quad (3.5)$$

and reflects the fact that the inflow into and the outflow out of unemployment are equalized in the steady state. Thus, the equilibrium unemployment rate μ_i is equal to:

$$\mu_i = \frac{\delta}{\lambda_0 + s_i v + \lambda_i + \delta} = \frac{\delta}{\sqrt{(r + \delta + \lambda_0 + \lambda_i)^2 + \alpha(y_i - b)cv} - r} \Rightarrow \mu_i = \mu(\lambda_i, y_i) \quad (3.6)$$

Hence the equilibrium unemployment rate can be expressed as a function of the job-finding rate λ_i and the productivity y_i . Next, consider a partial relationship between μ_i and λ_i for a fixed productivity y_i . A higher probability of finding a job through professional contacts (that is a higher λ_i), has an indirect disincentive effect on the search intensity $s(\lambda_i, y_i)$. Consequently, a lower level of search effort through the formal channel raises the equilibrium unemployment rate μ_i . This is an indirect effect which is operating through the outside opportunities of workers. At the same time a higher λ_i reduces the unemployment rate μ_i . This is a direct effect since more unemployed workers find jobs by means of referrals. Equation (3.6) shows that the direct effect is dominating and describes a negative relationship between the unemployment rate μ_i and the job-finding rate through professional contacts λ_i :

$$\frac{\partial \mu(\lambda_i, y_i)}{\partial \lambda_i} < 0 \quad \lim_{\lambda_i \rightarrow 0} \mu_i = \frac{\delta}{\sqrt{(r + \delta + \lambda_0)^2 + \alpha(y_i - b)cv} - r} \equiv \bar{\mu}_i > 0 \quad \lim_{\lambda_i \rightarrow \infty} \mu_i = 0$$

This is illustrated in figure 3.1, where $\bar{\mu}_i$ denotes the upper limit of the unemployment rate μ_i for a given fixed level of y_i . The corresponding curve is denoted by (UC).

Further, recall from section 3.2 that the job-finding rate by means of referrals λ_i depends on the unemployment rate in the network μ_i . In particular, it holds that:

$$\lambda_i = va_i \frac{1 - \mu_i}{\mu_i} [1 - (1 - \mu_i)^n] \Rightarrow \lambda_i = \lambda(\mu_i, a_i) \quad (3.7)$$

If more workers of a given type are employed (that is a lower μ_i) the possibilities for firms to communicate with this group of employees arise more frequently. And hence the contact rate between firms and unemployed workers of type y_i is increased. But on the other hand, a lower unemployment rate μ_i implies a lower number of unemployed contacts in the network and therefore, a lower probability that the contacted employee will recommend someone for a job $[1 - (1 - \mu_i)^n]$. Lemma 3.2 shows that the indirect network effect is dominated by the direct effect of a higher contact rate between firms and unemployed workers and so equation (3.7) describes a negative relationship between variables λ_i and μ_i . The corresponding curve is denoted by (NC).

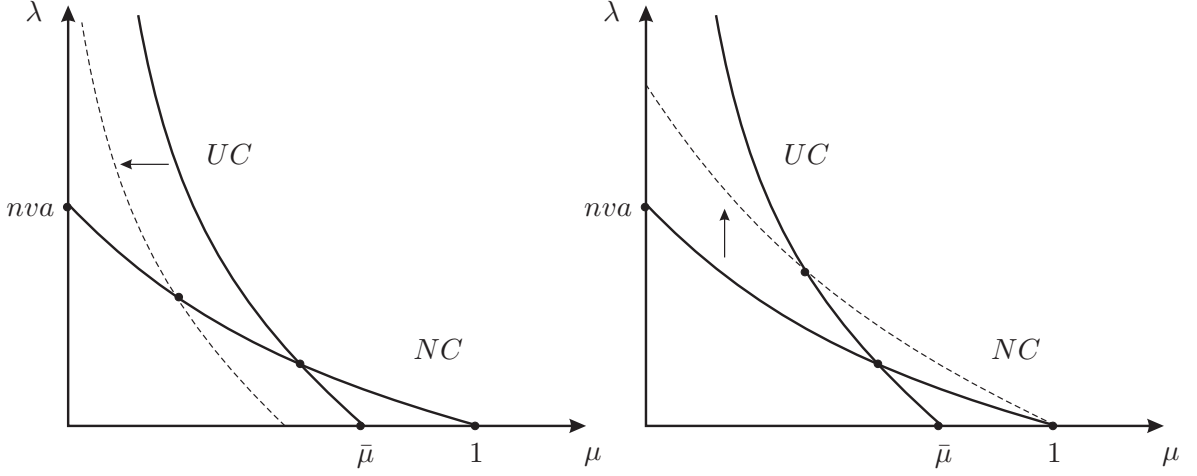


Figure 3.1: Intersection between $\mu(\lambda, y)$ and $\lambda(\mu, a)$ for a given advertising intensity a and a given productivity y . Left panel: changes in $\lambda(\mu, a)$ and $\mu(\lambda, y)$ given a positive shift in y . Right panel: changes in $\lambda(\mu, a)$ and $\mu(\lambda, y)$ given a positive shift in a .

Lemma 3.2. *For a given advertising intensity a_i , a lower unemployment rate μ_i in a group of workers with productivity y_i implies a higher job-finding rate through the informal channel of search λ_i :*

$$\frac{\partial \lambda_i(\mu_i, a_i)}{\partial \mu_i} < 0 \quad \lim_{\mu_i \rightarrow 0} \lambda_i = nva_i \quad \lim_{\mu_i \rightarrow 1} \lambda_i = 0$$

Proof: Appendix 3.8.2.

Based on these results, figure 3.1 shows that there is a unique intersection between the curves $\mu(\lambda_i, y_i)$ and $\lambda(\mu_i, a_i)$. This implies that μ_i is an implicit function of the productivity y_i and the advertising intensity a_i , formally:

$$\mu_i = \frac{\delta}{\sqrt{(r + \delta + \lambda_0 + \lambda(\mu_i, a_i))^2 + \alpha(y_i - b)cv} - r} \quad \Rightarrow \quad \mu_i = m(y_i, a_i)$$

To analyse the intuitive implications of this relationship consider workers with a higher productivity y_i . More productive workers expect to get a higher wage w_i , so the gain from finding a job is increasing in the productivity. This means that more able workers invest more effort in writing applications and preparing for a job interview. More intensive job search through the formal channel improves the job-finding rate $vs(\lambda_i, w(y_i))$ and so the unemployment rate $\mu(\lambda_i, y_i)$ is reduced for every value of λ_i . This is illustrated by the left-ward shift of the curve (UC) on the left panel of figure 3.1. Since productivity does not enter directly into the job-finding rate through the network, there is no shift of the curve (NC). This means that the unemployment rate is unambiguously lower in more productive worker groups. Consequently a larger proportion of employees facilitates informal matching between open vacancies and unemployed workers and therefore the probability of finding a job by recommendation is increasing. These results are summarized in lemma 3.3:

Lemma 3.3. *For a given advertising intensity a_i , the equilibrium unemployment rate $\mu_i =$*

$m(y_i, a_i)$ is lower in more productive worker groups. Further, for every productivity group y_i , the equilibrium unemployment rate falls with a higher search effort by firms, formally:

$$\frac{\partial m(y_i, a_i)}{\partial y_i} < 0 \quad \frac{\partial m(y_i, a_i)}{\partial a_i} < 0 \quad \lim_{a_i \rightarrow 0} m(y_i, a_i) = \lim_{\lambda_i \rightarrow 0} \mu(\lambda_i, y_i) = \bar{\mu}_i$$

Proof: Appendix 3.8.3.

In addition, lemma 3.3 describes consequences of a higher search intensity by firms a_i . If firms exert more effort in contacting their employees, then the probability of finding a job by means of a referral is increased for every unemployment rate μ_i . In figure 3.1 this is illustrated by the up-ward shift of the curve (NC) on the right panel. Since advertising intensity does not enter directly the unemployment equation, there is no shift of the curve (UC). This means that the job finding rate λ_i is unambiguously higher and the equilibrium unemployment rate is reduced.

3.3.4 Endogenous advertising rate for referral hiring

Let us now consider the optimal behavior of a firm with an open vacancy. Apart from formal applications the firm may also fill its vacancy through the informal channel of search. In particular, the firm should choose the optimal advertising intensity a_i for every worker type y_i . Intuitively, at rate a_i the firm is asking type- y_i incumbent employees whether they can recommend a friend for the open vacancy. Similarly to the effort choice of the unemployed, there is a gain and a cost from advertising activity. The expected firm rent from contacting the incumbent employee of type y_i is equal to $a(1 - (1 - \mu_i)^n)(J_i - V)$, which is the probability that the job offer will be transmitted to the unemployed worker of this type times the present value of profits. This gives rise to the following maximization problem:

$$\max_a [a(1 - (1 - \mu_i)^n)(J_i - V) - \frac{1}{k}a^2]$$

The optimal a_i is, thus, given by:

$$a_i = 0.5k(1 - (1 - \mu_i)^n)(J_i - V) \quad \text{where} \quad J_i - V = \frac{(1 - \alpha)(y_i - b) - rV}{r + \delta} \quad (3.8)$$

This first order condition defines the level of advertising a_i as a function of μ_i and y_i , that is $a_i = a(\mu_i, y_i)$. Therefore, for a given y_i , firms exert more advertising effort if they expect a higher proportion of unemployed workers in the network of the incumbent employee. In the following we consider the economy in the steady-state with a free-entry of firms, which means that $V = 0$. Figure 3.2 shows equilibrium for advertising effort and unemployment. Recall that $\mu_i = m(y_i, a_i)$ slopes down in the space (μ, a) : finding jobs becomes easier for unemployed workers if firms increase their advertising activities. Let this curve be denoted by (MA) (see figure 3.2). Equation (3.8) is the advertising curve and slopes up, let it be denoted by (AC). Group-specific equilibrium $(\mu(y_i), a(y_i))$ is at the intersection of the two curves and it is unique.

Next compare the equilibrium vector of variables $(\mu(y_i), a(y_i))$ across different productivity

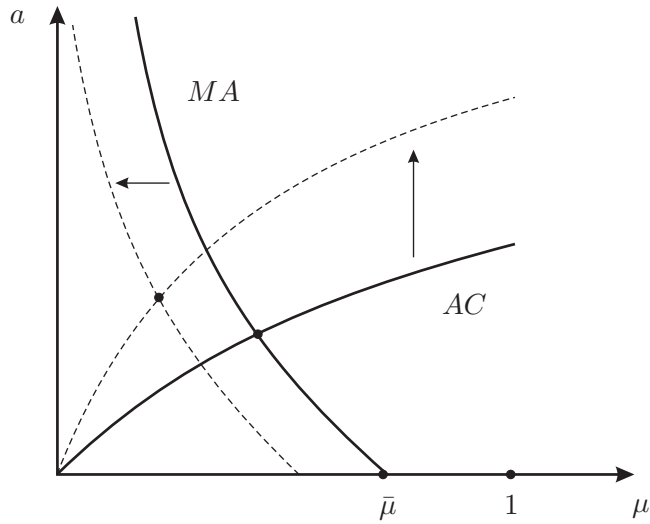


Figure 3.2: Determination of the type-specific unemployment rate $\mu(a, y)$ with the endogenous advertising intensity of firms $a(\mu, y)$. Arrows indicate higher values of y .

groups. On the one hand, more productive workers exert more effort in sending applications and preparing for the job interview, so their unemployment is lower for any advertising intensity a_i . This is illustrated by the inward shift of the curve (MA) (see figure 3.2). On the other hand, for a given μ_i , firms expect to earn higher profits from more productive network applicants, and so their advertising effort is higher when the firm is communicating with a more productive incumbent employee. This implies an upward shift of the advertising curve (AC) since firms' effort is increasing for every level of the unemployment rate μ_i . Considering both changes as a combination shows that the equilibrium unemployment rate is lower in more productive worker groups. This result is described in proposition 3.1:

Proposition 3.1. (i) The group-specific equilibrium unemployment rate μ_i is decreasing in the productivity y_i and vacancies v . (ii) The job-finding rate λ_i and the network advertising intensity a_i are both increasing in y_i if the elasticity of referral probability $\rho(y_i) \equiv [1 - (1 - \mu(y_i))^n]$ with respect to the net productivity $y_i - b$ is less than 1, formally:

$$-\frac{\partial \rho(y_i)}{\partial (y_i - b)} \cdot \frac{(y_i - b)}{\rho(y_i)} = -\frac{n(1 - \mu(y_i))^{n-1}}{1 - (1 - \mu(y_i))^n} \cdot \frac{\partial \mu(y_i)}{\partial y_i} \cdot (y_i - b) < 1 \quad (3.9)$$

Proof: Appendix 3.8.4.

Proposition 3.1 shows that there are two counteracting effects of y_i on the network advertising intensity $a(y_i)$. On the one hand, firms anticipate higher profits from more productive network applicants and direct their search towards worker groups with a higher y_i . But on the other hand, the equilibrium unemployment $\mu(y_i)$ is decreasing in y_i which means that the average proportion of unemployed workers in the network is lower in less productive worker groups. From the perspective of firms this means a lower probability of referral hiring. Condition (3.9) then implies that the first direct effect of higher profits is dominating if the equilibrium unemployment rate is sufficiently inelastic. In a similar way we can show that unemployment is a decreasing function of vacancies v . This is because more vacancies imply higher job-finding rates vs_i and

λ_i , thus the model can reproduce a negative relationship between unemployment and vacancies (the Beveridge curve) which is a standard property of any search model.

Finally, the job-finding rate through professional contacts $\lambda(y_i)$ can be now rewritten as:

$$\lambda(y_i) = \frac{a(y_i)v(1 - \mu(y_i))}{\mu(y_i)}\rho(y_i) = \frac{v(1 - \mu(y_i))}{\mu(y_i)}0.5k\rho^2(y_i)J(y_i) \quad (3.10)$$

Recall that $\lambda(y_i) = \lambda(\mu(y_i), a(y_i))$. If the elasticity condition in proposition 3.1 is satisfied than more productive employees are more intensively approached by firms. So the probability of finding a job through professional contacts is increasing in the productivity. In addition, since the unemployment rate is decreasing in y_i , the probability that a randomly chosen employee is of type y_i is increasing in the productivity. Both of these factors imply that the probability of finding a job by recommendation is an increasing function of y_i , that is $\partial\lambda(y_i)/\partial y_i > 0$ if $\partial a(y_i)/\partial y_i > 0$.

The primary purpose of this paper is to analyze which groups of workers are more likely to use family and professional contacts in the process of job search. To address this question we define the following new variables $d_0(y_i)$ and $d(y_i)$. The former variable is an average proportion of workers with productivity y_i using family contacts in order to find a job. In contrast, the latter variable is an average proportion of workers using professional contacts. This means:

$$d_0(y_i) = \frac{\lambda_0}{\lambda_0 + \phi(y_i) + \lambda(y_i)} \quad d(y_i) = \frac{\lambda(y_i)}{\lambda_0 + \phi(y_i) + \lambda(y_i)}$$

The last possibility to find a job in the model is the formal channel of job search, so the average proportion of type y_i workers finding jobs by means of this channel can be found as $1 - d_0(y_i) - d(y_i)$. Which worker group is relying most on family contacts? To answer this question observe that:

$$d_0(y_i) = \frac{\lambda_0}{\lambda_0 + \phi(y_i) + \lambda(y_i)} = \frac{\lambda_0}{\sqrt{(r + \delta + \lambda_0 + \lambda(y_i))^2 + \alpha(y_i - b)cv} - r - \delta}$$

Therefore, $d_0(y_i)$ is decreasing in y_i if the elasticity condition (3.9) is satisfied. On the one hand, more productive workers anticipate a larger present value of wages and exert more effort when preparing applications and, on the other hand, firms are searching more intensively for more productive applicants. Both of these factors imply that the proportion of workers finding jobs through family contacts is a decreasing function of y_i . In addition, observe that $d_0(b) = 1$ (since $\lambda(b) = 0$ and $\phi(b) = 0$) which means that least productive workers rely exclusively on family contacts. Finally, it is not possible to predict in general whether variable $d(y_i)$ is increasing or decreasing in y_i . We investigate this relationship numerically in section 3.4.

3.3.5 Wage distribution and the free-entry condition

The second purpose of our paper is to analyze the effect of referrals on wages. As mentioned in the earlier part of our paper the equilibrium wage distribution is given by $g_i = e_i/e$ and shows the relative proportion of y_i -workers in the pool of employees. So the average wage in the economy can be found as $\bar{w} = \sum g_i w_i$. Next we find average wages conditional on the specific channel of search. To do so let \bar{w}^o , \bar{w}^s and \bar{w}^n be the corresponding average wages conditional

on the search method being the family, the formal application or the network of professional contacts. In addition, let h_i^o , h_i^s and h_i^n , $\forall i = 1..p$, be the respective wage distributions so that $\sum h_i^o = 1$, $\sum h_i^s = 1$ and $\sum h_i^n = 1$. For example, h_i^s is the equilibrium distribution of wages among employed workers who found a job by using the formal method of search. Each of these three distributions can be obtained as:

$$h_i^o = \frac{g_i d_0(y_i)}{\sum g_i d_0(y_i)} \quad h_i^s = \frac{g_i(1 - d_0(y_i) - d(y_i))}{1 - \sum g_i(d_0(y_i) + d(y_i))} \quad h_i^n = \frac{g_i d(y_i)}{\sum g_i d(y_i)} \quad \forall i = 1..p$$

Variable $\sum g_i d_0(y_i)$ is the proportion of employees who found a job with a help of a family member. It is also the total measure of these workers since the total population size is normalized to 1. In a similar way, $\sum g_i d(y_i)$ is the fraction of employees who found a job with a help of a professional contact. And the remaining part $1 - \sum g_i(d_0(y_i) + d(y_i))$ is the proportion of workers who found a job through the formal method of search. Therefore, the three average wages for each of the search channels can be found as:

$$\bar{w}^o = \sum w_i h_i^o \quad \bar{w}^s = \sum w_i h_i^s \quad \bar{w}^n = \sum w_i h_i^n$$

These equations allow us to compare the average wages \bar{w}^o , \bar{w}^s and \bar{w}^n and to predict whether family and/or professional referrals are associated with a wage premium or a wage penalty relative to the formal method. Either of these results will depend on the self-selection of workers into the specific channels of search. For example, we expect that family contacts will be associated with a wage penalty as this search channel is the most prevalent among the groups of workers with low wages. Formally, one can show that family contacts are associated with a wage penalty if the distribution h_i^s first order stochastically dominates the distribution h_i^o :

$$\bar{w}^o = \sum_{i=1}^p w_i h_i^o = w_1 + \sum_{i=1}^{p-1} \Delta w_i (1 - H_i^o) \quad \text{and} \quad \bar{w}^s = \sum_{i=1}^p w_i h_i^s = w_1 + \sum_{i=1}^{p-1} \Delta w_i (1 - H_i^s)$$

so that $\bar{w}^o < \bar{w}^s$ if $H_i^s \leq H_i^o, \forall i = 1..p$

where $\Delta w_i = w_{i+1} - w_i > 0$ since the wage is an increasing function of the productivity, and variables H_i^o , H_i^s are the cumulative density functions so that $H_i^o = \sum_{j=1}^i h_j^o$ and $H_i^s = \sum_{j=1}^i h_j^s$. The proof is presented in appendix 3.8.5. In a similar way, define $H_i^n = \sum_{j=1}^i h_j^n$ to be the cumulative density function of wages obtained with a help of professional contacts. This channel of search is then associated with a wage premium relative to the formal method, i.e $\bar{w}^s < \bar{w}^n$, if the distribution h_i^n first order stochastically dominates the distribution h_i^s , which is equivalent to $H_i^n \leq H_i^s, \forall i = 1..p$. Intuitively, this condition holds when firms rely on professional recommendations to match with high ability workers, which is the case in our model.

Finally, the last component of the model is the Bellman equation for an open vacancy with a present value denoted by V . Same as workers, firms are simultaneously using each of the three search channels to fill an open vacancy. At rate $\lambda_0 u_i/v$ the firm is matched with an unemployed worker of type y_i as a consequence of a family referral and at rate $\phi(s_i)u_i/v = s_i u_i$ the firm is matched with a similar worker by means of a formal application. Note that a higher measure of unemployed workers with the productivity y_i and a more intensive job search s_i increase the probability of filling a vacancy with this type of worker. In addition, firms may contact one of

the incumbent employees to ask for the referral. An applicant of type y_i is hired through this channel with a job-filling rate $a_i e_i \rho_i$, where we use notation $\rho_i = \rho(y_i) = [1 - (1 - \mu(y_i))^n]$. This latter term is the probability that the contacted employee will recommend an applicant for the open position. Thus, the value of an open vacancy is given by:

$$rV = -z + \frac{\lambda_0}{v} \sum u_i(J_i - V) + \sum s_i u_i(J_i - V) + \sum e_i \left(a_i \rho_i(J(y_i) - V) - a_i^2/k \right) \quad (3.11)$$

where z is the flow cost of filling a vacancy. Note also that the choice of the advertising intensity $a_i \forall i = 1..p$ is compatible with the maximization of the present value of an open vacancy V . The free-entry condition of firms implies that $V = 0$ in the steady-state equilibrium. Substituting the present value of profits J_i and the optimal advertising intensity a_i gives us the equilibrium number of vacancies:

$$v = \lambda_0 \frac{1 - \alpha}{r + \delta} \sum u_i(y_i - b) \left[z - \frac{1 - \alpha}{r + \delta} \sum s_i u_i(y_i - b) - 0.25k \frac{(1 - \alpha)^2}{(r + \delta)^2} \sum e_i \rho_i^2(y_i - b)^2 \right]^{-1}$$

This is the last equilibrium equation. So the equilibrium can be defined in the following way:

Definition 3.1. *Search equilibrium is a vector of variables $(U_i, W_i, J_i, w_i, s_i, a_i, \mu_i), \forall i = 1..p$ as well as the number of vacancies v and the present value of an open vacancy V , satisfying the asset value equations for workers (3.1) and (3.2), for firms (3.4) and (3.11), the wage equations $w_i = \alpha y_i + (1 - \alpha)b$, the optimal effort equations (3.3) and (3.8), the stationary unemployment conditions (3.6) and the free-entry condition $V = 0$.*

To analyze whether the equilibrium defined above is unique, consider first an economy without professional referrals, that is $a_i = 0 \forall i = 1..p$. The free-entry condition $V = 0$ is then:

$$z = \sum \left(\frac{\lambda_0}{v} + s_i \right) \mu_i f_i J_i \quad \text{where} \quad \mu_i = \frac{\delta}{\sqrt{(r + \delta + \lambda_0)^2 + \alpha(y_i - b)cv} - r} \quad (3.12)$$

Note that a larger number of vacancies has a negative effect on the recruiting rate through families λ_0/v and on the individual search intensity s_i (see the result from lemma 3.1). On the one hand, firms compete stronger for applicants, and on the other hand, workers are demotivated and exert less effort in searching for jobs. At the same time, the unemployment rate μ_i is reduced, which makes it even more difficult for firms to hire workers. So the right hand side of the free-entry condition (expected profits) is a decreasing function of v , whereas the left hand side is a fixed cost of hiring z . So there exists a unique equilibrium in this economy. These results are formalized in the following proposition:

Proposition 3.2. *Consider the labour market described in definition 3.1. When professional networks are not utilized, i.e. $a_i = 0 \forall i = 1..p$, there exists a unique equilibrium, where the number of vacancies v is given by the free-entry condition (3.12).*

Proof: Appendix 3.8.6.

Corollary 3.1. *In this equilibrium the number of vacancies is decreasing in the hiring cost z , whereas all unemployment rates $\mu_i, i > 1$, are increasing in z , formally $\partial v / \partial z < 0$ and $\partial \mu_i / \partial z > 0$ for $i = 2..p$*

This corollary shows that a higher recruitment cost z leads to lower vacancies and higher unemployment. Thus the model without professional networks captures the standard dynamics of the labour market even if there are many heterogeneous groups of workers. In the opposite case when professional networks are utilized and are a dominant search channel then we find that there is a possibility of multiple equilibria in our model. Their existence and properties are investigated in the next section of the paper.

3.3.6 Multiple equilibria

To see that multiple equilibria can arise in our model consider a labour market with only one worker type ($y_i > b$), where the formal search channel is not used, that is $s_i = 0$. The free-entry condition ($V = 0$) in this economy simplifies to yield:

$$z = \frac{\lambda_0}{v} \mu_i J_i + 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2$$

The term on the right hand side is the expected profit of firms. Let first $\lambda_0 = 0$, then the term on the right-hand side is equal to zero for $\mu_i = 0$ and $\mu_i = 1$ with an internal maximum for some intermediate value of μ_i . Intuitively, it means that a larger number of unemployed agents raises the probability that a randomly contacted employee will recommend his/her contact for the job, so firm profits are increasing in μ_i as long as μ_i is relatively low. But when the number of unemployed workers is increasing further, then there are fewer employees who can give a recommendation, which has a negative effect on profits. In the extreme case when $\mu_i = 1$, no one is employed and there is no hiring. Note that graphically expression $0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2$ is represented by a one-dimensional hump and thus may have at most two roots at the intersection with a horizontal line z . Let these roots be denoted by μ_i' and μ_i'' . Then we get:

$$\begin{aligned} z > 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2 & \text{ for } 0 < \mu_i < \mu_i' & \Rightarrow & v = 0 \\ z < 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2 & \text{ for } \mu_i' < \mu_i < \mu_i'' & \Rightarrow & v = \bar{v} \\ z > 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2 & \text{ for } \mu_i'' < \mu_i < 1 & \Rightarrow & v = 0 \end{aligned}$$

Intuitively, if unemployment is too low or too high, then the cost of creating jobs z is larger than the expected profit and vacancies immediately drop down to zero. In contrast, for intermediate values of unemployment the cost of creating jobs is lower than the expected profit and a maximum number of vacancies \bar{v} is created. This is illustrated on the left panel of figure 3.3. Next we turn to the investigation of the equilibrium unemployment equation, which is given by:

$$\mu_i = \frac{\delta}{\delta + \lambda_0 + v(1 - \mu_i)0.5k(1 - (1 - \mu_i)^n)^2 J_i / \mu_i}$$

Proposition 3.1 proves that unemployment is a decreasing function of vacancies v which is also illustrated on the left panel of figure 3.3 for $\lambda_0 = 0$. Note that unemployment is a backward-looking variable with a gradual adjustment to the new level, whereas the number of vacancies v is a forward-looking jump variable which adjusts immediately. Figure 3.3 shows that there are three stationary equilibria: $\{\mu_i', v(\mu_i')\}$, $\{\mu_i'', v(\mu_i'')\}$ and $\{\mu_i = 1, v = 0\}$. Let them be denoted by A , B and C respectively. Next we analyze which of these equilibria are stable. If unemployment

is low, that is $\mu_i < \mu'_i$, then vacancies drop down to zero so that unemployment is increasing $\dot{\mu} > 0$. If unemployment is moderate, that is $\mu'_i < \mu_i < \mu''_i$, vacancies jump up to the maximum level, thus $\dot{\mu} < 0$. And finally, if unemployment is too high ($\mu''_i < \mu_i$) we get $\dot{\mu} > 0$. So we find that there are two stable stationary equilibria A and C , whereas B is unstable. These results are summarized in lemma 3.4:

Lemma 3.4. *Consider a labour market with only one worker type $y_i > b$ and professional networks as the only channel of job search. Let condition $z < kn^2 J_i^2 / (1 + 2n)^{2+1/n}$ be satisfied and $0 < \mu'_i < \mu''_i < 1$ denote the two roots of the free-entry condition $z = 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2$. Then there exist three stationary equilibria $\{\mu'_i, v(\mu'_i) = (1 - \mu'_i)0.5\delta J/z\}$, $\{\mu''_i, v(\mu''_i) = (1 - \mu''_i)0.5\delta J/z\}$ and $\{\mu_i = 1, v = 0\}$ if $v(\mu'_i) < \bar{v}$. The first and last equilibria are stable, whereas the second equilibrium is unstable.*

Proof: Appendix 3.8.6.

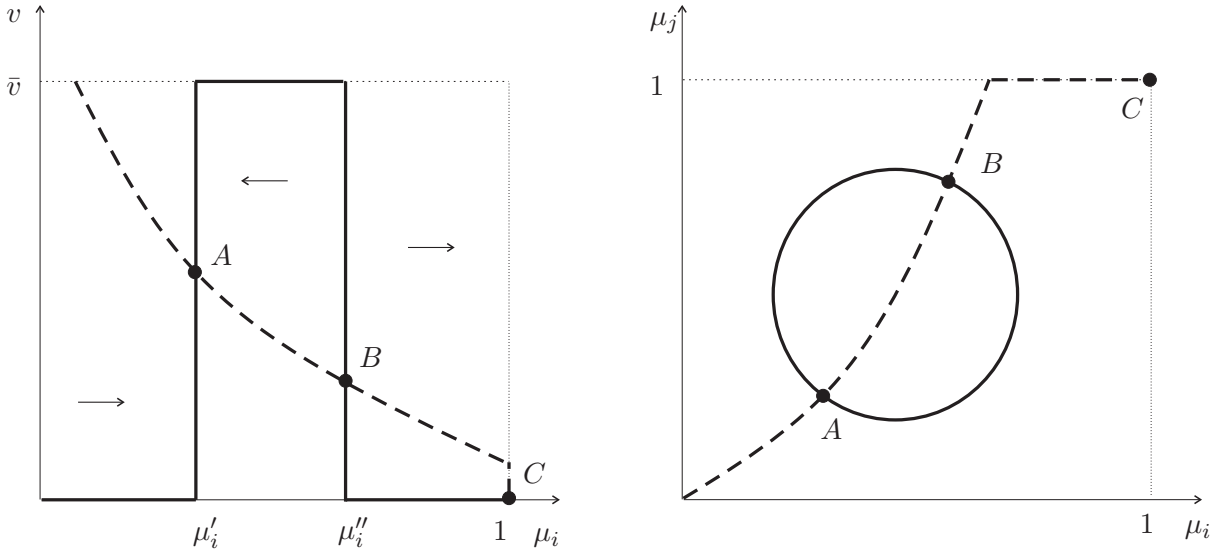


Figure 3.3: Stationary equilibria in the economy without formal job search and $\lambda_0 = 0$ Left panel: multiple equilibria with one worker type y_i . Right panel: multiple equilibria with two worker types $y_i > y_j$

Next we show that multiplicity of equilibria is not an artifact of a single worker type, but can also appear in the model with several worker groups. In order to illustrate this graphically we consider two types of workers i and j where group j is less productive than group i : $y_j < y_i$. For λ_0 equal to zero, the free-entry condition becomes:

$$z = 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2 f_i + 0.25k(1 - \mu_j)(1 - (1 - \mu_j)^n)^2 J_j^2 f_j$$

where $f_j = 1 - f_i$. Graphically expression on the right hand side is a two-dimensional hump. Cutting this hump with a horizontal plane z creates an ellipse-like projection in the space $\{\mu_i, \mu_j\}$ which is illustrated on the right panel of figure 3.3. Expected profits are the same at every point of this ellipse-like curve and equal to z . Next we consider the equilibrium unemployment equation

for μ_j , express μ_j in terms of v and insert it into the equilibrium equation for μ_i (keeping $\lambda_0 = 0$):

$$v = \frac{\delta}{0.5k(1 - (1 - \mu_j)^n)^2 J_j} \Rightarrow \mu_j = 1 - \left[1 - (1 - (1 - \mu_i)^n) \sqrt{\frac{J_i}{J_j}} \right]^{1/n}$$

This equation captures an indirect relationship between the two unemployment rates. Intuitively, if there are more (less) vacancies both unemployment rates μ_i and μ_j are decreasing (increasing). Thus the two unemployment rates are always moving in the same direction, which is illustrated by the increasing curve on the right panel of figure 3.3. This figure illustrates that even in the labour market with two worker groups there are at most three types of stationary equilibria. Either both unemployment rates are low which makes hiring easy to firms as there are many employed workers who can give a recommendation, this is a stable stationary equilibrium A . Alternatively, both unemployment rates can be high, which is still profitable for firms as there is a large pool of unemployed workers willing to take a job. This is the unstable stationary equilibrium B . Finally, there is a stable equilibrium C with $\{\mu_i = 1, \mu_j = 1, v = 0\}$. Note that increasing the number of worker groups does not create conceptually new equilibria.

Next we analyze the case $\lambda_0 > 0$. Expressing v from the free-entry condition we get:

$$\begin{aligned} v &= \lambda_0 \mu_i J_i / [z - 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2] \quad \text{for } 0 < \mu_i < \mu'_i \\ v &= \bar{v} \quad \text{for } \mu'_i < \mu_i < \mu''_i \\ v &= \lambda_0 \mu_i J_i / [z - 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2] \quad \text{for } \mu''_i < \mu_i < 1 \end{aligned}$$

This is due to the fact that μ'_i and μ''_i are the asymptotes of the free-entry condition. The corresponding curve and its intersection with the equilibrium unemployment curve is illustrated on the left panel of figure 3.4. One can see that equilibria B and C move closer to each other. Moreover C is shifted inwards and involves a positive measure of vacancies and the equilibrium unemployment rate less than 1. This is because with $\lambda_0 > 0$ workers can find jobs with a help of their relatives (for example, in family business), thus full unemployment can never be a stationary equilibrium with $\lambda_0 > 0$. Altogether equilibrium C is a network trap for the labour market, it is stable with low vacancies and high unemployment. Raising λ_0 further we find that equilibria B and C first coincide (this is a degenerate case with two equilibria) and then disappear. This is illustrated on the right panel of figure 3.4. Hence we find that larger values of λ_0 are not compatible with multiple equilibria. The same effect appears if we allow for the formal job search which is reducing the dominance of professional networks.

Closing this section, we note that the first study investigating multiple equilibria in a frictional labour market framework is Cahuc and Fontaine (2009). However, our result is different from theirs as in their model there are two equilibria with and without networks whereas in our model professional networks are used in both equilibria A and B . Finally, several equilibria may prevail even if all three search channels are used simultaneously, but we do not find it for realistic parameter values in the next section.

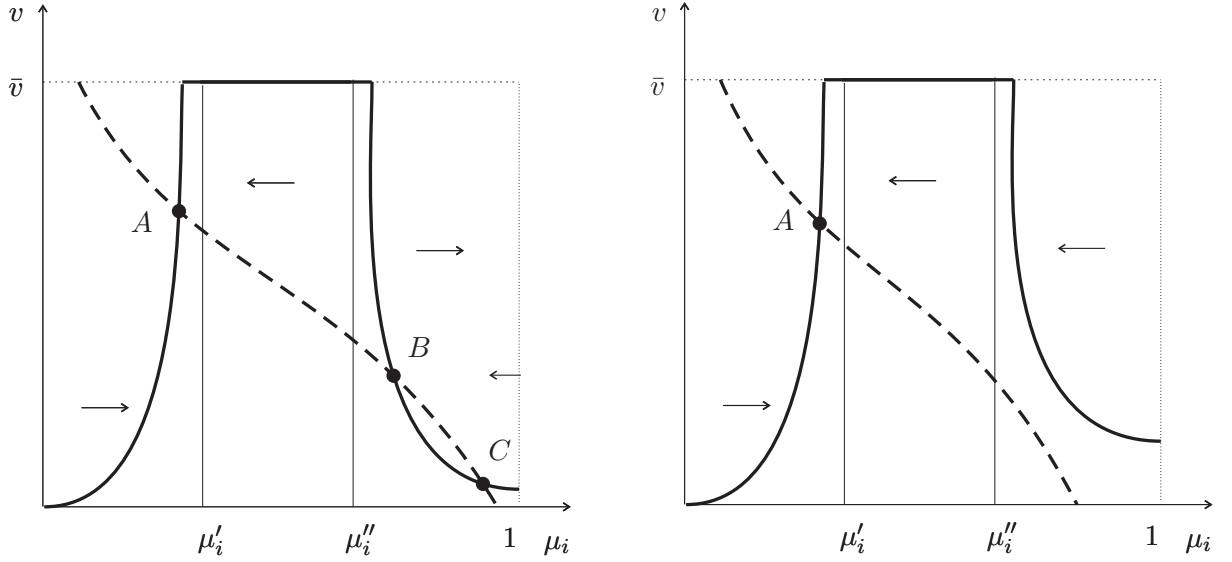


Figure 3.4: Stationary equilibria in the economy with one worker type y_i and no formal job search, $\lambda_0 > 0$ Left panel: multiple equilibria when λ_0 is small. Right panel: unique stable equilibrium when λ_0 is large.

3.4 Numerical example

3.4.1 Search effort and the equilibrium unemployment curve

This section parameterizes the model to match the average labour market indicators in the OECD countries. We choose a unit period of time to be six months and set $r = 0.01$ which corresponds to the annual discount rate of 2%. Further, we follow Shimer (2005) and set the unemployment benefit b equal to 0.4. Fontaine (2008) uses the value of 0.15 for the U.S. economy and 0.4 for the French economy. Gautier (2002) and Cahuc and Fontaine (2009) set b equal to 0.2. At the same time, Hall and Milgrom (2008) obtain a larger value of 0.71. Therefore, our choice of b is in the middle range of the typical values in the literature. Given $b = 0.4$, the range of productivities in the model becomes $[0.4..1]$. The number of worker groups p is not relevant for the results, so we set $p = 25$, which corresponds to the productivity gap between two consequent worker groups equal to 0.025. Intuitively we interpret $y_1 = 0.4$ as unproductive workers, $y_7 = 0.55$ as median workers and $y_{25} = 1$ as most productive workers.

Next, we take the value of the separation rate $\delta = 0.15$ which corresponds to the average job duration of $1/(2 \cdot 0.15) = 3.3$ years. Pissarides (2009) and Shimer (2005) choose δ equal to 0.1 with one unit of time being a quarter. This corresponds to the average job duration of $1/(4 \cdot 0.1) = 2.5$ years. Hall and Milgrom (2008) choose the value of 3%, whereas Hobijn und Sahin (2009) report a value of 2.3% per month, so the average job duration becomes $1/(12 \cdot 0.03) = 2.78$ years and $1/(12 * 0.023) = 3.6$ years respectively, which is similar to our choice. The job-finding rate through family contacts λ_0 is chosen to be 0.3 and is defined by the unemployment rate of the least productive workers being equal to $\mu(b) = \delta/(\delta + \lambda_0) = 0.33$. Note that the productivity of these workers is equal to b and so the gain from finding a job is zero, and firm profits from hiring these workers are also zero since $w(b) = b$. For this reason unproductive workers rely exclusively on family referrals with a longest average unemployment duration of 1.6 years.

We choose the number of workers' professional contacts equal to 50 as in Cahuc and Fontaine (2009), while Fontaine (2008) uses $n = 40$ in a benchmark model of his paper. These numbers are in line with the empirical evidence, for example, in their recent study Cingano and Rosolia (2012) find that the median number of professional contacts in Italy is equal to 32. This number is higher in Germany and is equal to 43 according to Glitz (2013). The workers' bargaining power α is set equal to 0.72 as in Shimer (2005).

Variable	Value	Explanation, source and target
n	50	Network size (Cahuc and Fontaine (2009))
r	0.01	Annual interest rate of 2%
λ_0	0.3	Unemployment of the least able worker=33%
δ	0.15	Average job duration of 3.3 years
α	0.72	Worker's bargaining power (Shimer (2005))
b	0.4	Unemployment benefit (Shimer (2005))
p	25	Number of productivity types
cv	22.07	Unemployment of the median worker=8.7%
kv	0.24	Referral hiring of the median worker=40%

Table 3.1: Values of the model parameters

An important feature of our model is its invariance to the shape of the productivity distribution and the number of vacancies. Recall that the two key variables in the model $d_0(y_i)$ and $d(y_i)$ are defined in relative terms and are independent of the productivity distribution f_i . Moreover, the total number of vacancies only enters in the two multiplicative terms kv and cv , where k and c are the unobservable parameters of the two cost functions. To identify variables kv and cv we target $d_0(y_7) + d(y_7) = 0.4$ and $\mu(y_7) = 0.087$. The first of these conditions implies that 40% of workers in the median group find employment by means of referrals. This assumption is in line with the empirical observation that 30% to 60% of the employees in developed countries rely on social contacts in order to find a job (see Ioannides and Datcher Loury (2004) for an overview). The second condition implies that the unemployment rate in the median group of workers is equal to 8.7%. This number is the average unemployment rate in the United States in the recent years (BLS, 2009-2013). This yields $kv = 0.24$ and $cv = 22.07$. Table 3.1 presents our parameter choices for the benchmark case.

Next we describe our results. Figure 3.5 (left panel) presents variables $d_0(y_i)$, $1 - d_0(y_i) - d(y_i)$ and $d(y_i)$ for every worker group $i = 1..25$. These are the average proportions of workers finding employment by means of family contacts, formal applications and professional contacts respectively. As we proved in the theoretical part of the paper the average fraction of workers using family contacts to find a job, $d_0(y_i)$, is a decreasing curve and the lowest productivity group never finds jobs through channels other than family contacts. Therefore, the reliance on family contacts falls down from 100% for the least able workers to only 7% for the most productive group. Intuitively, even though family contacts become less important for more productive workers, our model does not exclude situations when talented employees are recommended and work for the same employer as their parents.

Now consider professional relations. Figure 3.6 (the right panel) shows that firms exert more advertising effort $a(y_i)$ when targeting more productive groups of incumbent employees. This means that the elasticity condition in proposition 3.1 is satisfied and the positive effect of

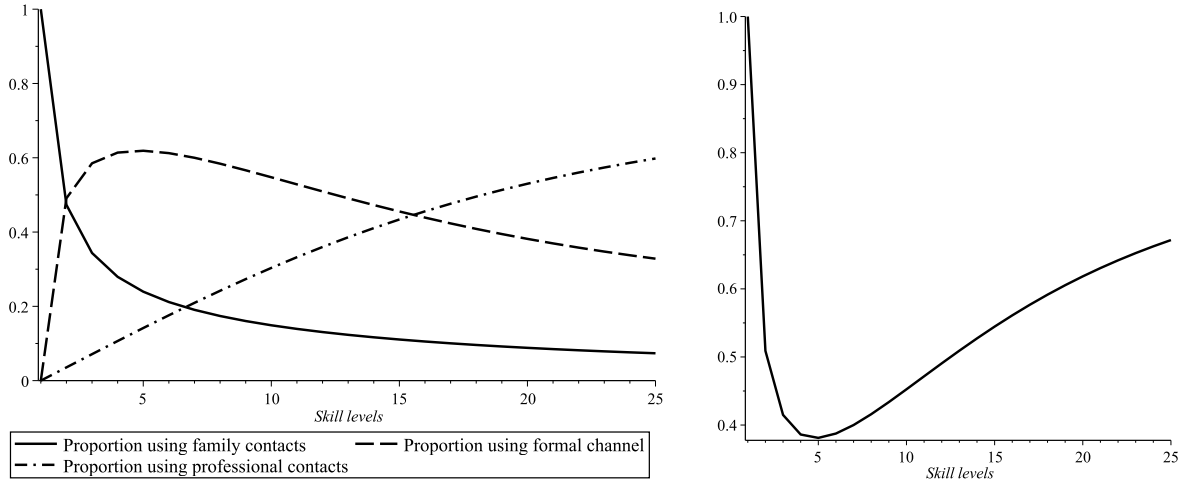


Figure 3.5: Left panel: The graph for average proportions of workers using family, professional contacts and the formal application in the job search process for different productivity levels. Right panel: The graph for the average proportions of workers using social contacts in the job search for different productivity levels (U-shape).

higher profits is dominating for firms. This in turn implies that the job finding rate $\lambda(y_i)$ is an increasing function of productivity. On the one hand, even if firms contacted their incumbent employees in a random and undirected manner they would be more likely to be in contact with a more productive worker as the equilibrium unemployment rate is decreasing with y_i (see the right panel of figure 3.6). On the other hand, it is profitable for firms to direct their search towards more productive groups of incumbent employees in the expectation of a good applicant. Therefore, both effects are reinforcing and amplifying each other and the network job-finding rate $\lambda(y_i)$ is an increasing and a convex function of y_i (see the left panel of figure 3.6). Thus the average proportion of workers using professional contacts to find a job, $d(y_i)$ is increasing from 0% for the least productive group up to 60% for the most productive group. Moreover, professional referrals are a dominating channel of search for workers with a productivity above $y_{15} = 0.75$.

Finally, consider formal applications as a search channel. Figure 3.5 (left panel) shows that the relative fraction of workers finding jobs through this channel, $1 - d_0(y_i) - d(y_i)$ is increasing for productivities below $y_5 = 0.5$ and decreasing thereafter. Intuitively, for the less able workers the probability of being referred for a job $\lambda(y_i)$ is still relatively low, but the wage w_i is already sufficiently large to motivate these workers for preparing formal applications. However, as the productivity is increasing, workers' chances of being referred for a job are improving and the incentives to invest costly effort and time in preparing applications are mitigated. In line with this reasoning figure 3.6 (left panel) shows that the search effort $s(w_i, \lambda_i)$ is an increasing but a concave function of y_i as the positive effect of a higher wage is partially neutralized by the negative effect of a higher λ_i . In addition, figure 3.5 (left panel) illustrates that the formal channel of search is dominating for workers in the middle range of productivities between $y_2 = 0.425$ and $y_{15} = 0.75$ reaching a maximum of 62% for workers with a productivity $y_5 = 0.5$.

To sum up, our model is able to jointly replicate a number of empirical observations. First, without separating social contacts into different types the model shows that the reliance on

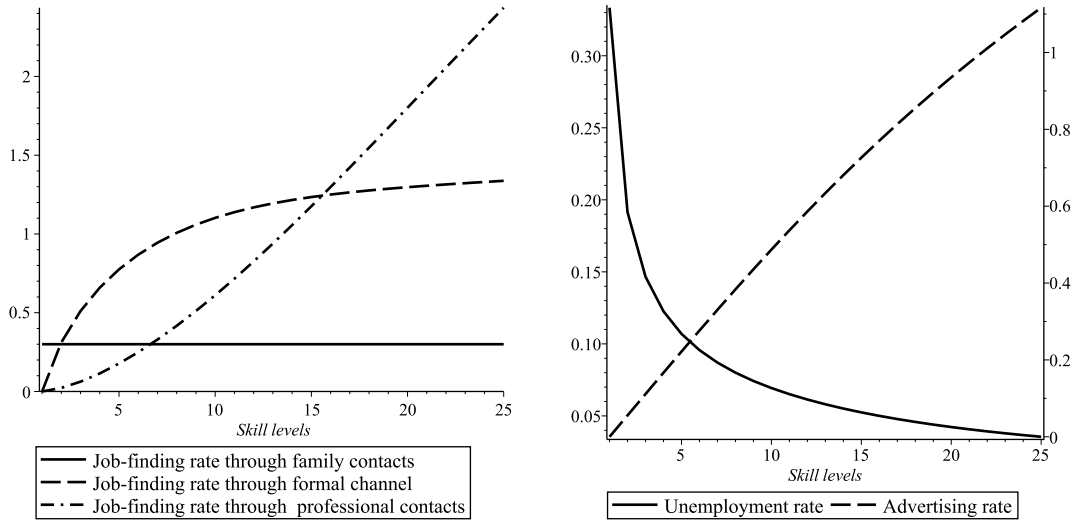


Figure 3.6: Left panel: The graphs for λ_0 , $\phi(s(y_i))$ and for $\lambda(y_i)$ for different productivity levels. Right panel: The optimal advertising rate $a(y_i)$ by firms and the graph for the unemployment rate $\mu(y_i)$ for different productivity levels.

social contacts $d_0(y_i) + d(y_i)$ has a distinct U-shape pattern falling down from 100% to 38% for workers with $y_5 = 0.5$ and rising again to the level of 67% for the most productive workers (see figure 3.5 (right panel)). In the next section we continue our analysis by comparing average wages associated with each of the three search channels.

3.4.2 Wage and productivity distributions

It is a well documented empirical fact (see Neal and Rosen (2000) and Mortensen (2003)) that a typical earnings distribution is hump-shaped and positively skewed with a mean value larger than the median. Therefore, it is often well approximated by the log-normal distribution. In our model the distribution f_i is discrete, so we use the Negative Binomial productivity distribution which is a discrete counterpart of the log-normal distribution. In particular, we rely on a special case of the density which is known as the Polya distribution. Given that this distribution has an infinite range we truncate it at $i = 25$. The productivity density f_i is then characterized by two parameters t and π and takes the form:

$$f_i = \frac{\tilde{f}_i}{\sum_{i=1}^{25} \tilde{f}_i} \quad \text{where} \quad \tilde{f}_i = \binom{i+t-2}{i-1} (1-\pi)^t \pi^{i-1} = \frac{\Gamma(i+t-1)}{(i-1)! \Gamma(t)} (1-\pi)^t \pi^{i-1}, \quad i = 1, 2, \dots$$

We set $t = 3$ to guarantee that the earnings distribution is hump-shaped and positively skewed in line with empirical evidence. To identify π we exploit the definition of the median worker having the productivity $y_7 = 0.55$, therefore, we set $\sum_{i=1}^7 f_i = 0.5$ which yields $\pi = 0.717$. The productivity density function f_i with $t = 3$ and $\pi = 0.717$ is illustrated by the solid curve on figure 3.7 (left panel). The equilibrium wage distribution g_i , defined in the theoretical part of the paper, is shown by the dashed curve on the same figure. The wage distribution g_i first order stochastically dominates the productivity distribution f_i . This is because the unemployment rate is higher than the average among the less productive types and lower among the more productive. Both distributions are, however, very close to each other.

Next, we consider the free-entry condition. To identify the cost of an open vacancy z we set the market tightness v/u equal to 1, where $u = \sum_{i=1}^p \mu_i f_i$ is the equilibrium unemployment rate in the economy. This value coincides with the calibration of Shimer (2005). We get $z = 0.39$ (flow cost of an open vacancy) in equilibrium, which is close to the value of 0.4 chosen for the formal search method by Cahuc and Fontaine (2009). The choice of Pissarides (2009) is also close to our value and is equal to 0.36.

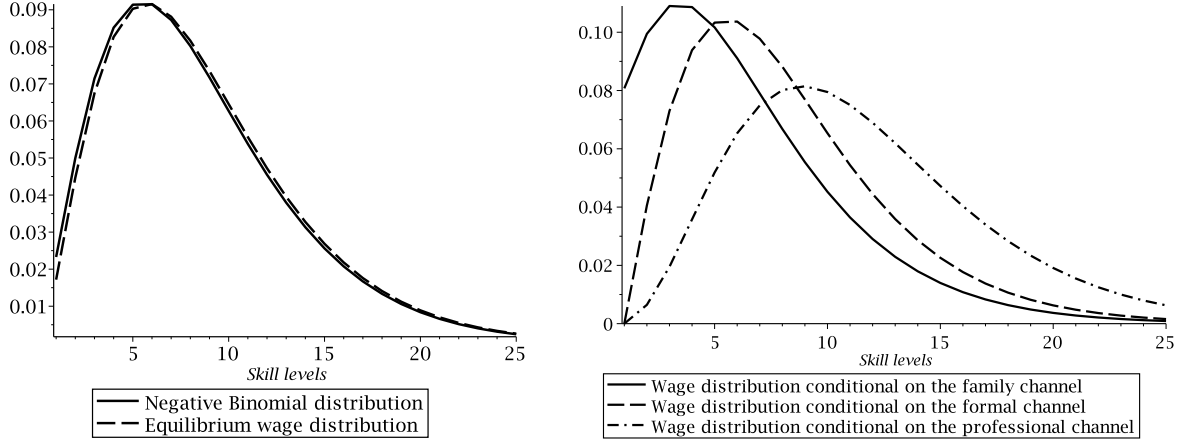


Figure 3.7: Left panel: The graphs for the probability mass functions of the Negative Binomial distribution and of the equilibrium wage distribution. Right panel: The graphs for the CDFs of the Negative Binomial distribution and of the equilibrium wage distribution.

The equilibrium wage/productivity distributions conditional on the job search channel h_i^o , h_i^s and h_i^n are presented on the right panel of figure 3.7. The wage distribution of workers finding jobs through their families, h_i^o , is first order stochastically dominated by the wage distribution of employees who used a formal application, h_i^s . The probability mass of the distribution h_i^o is mostly concentrated in the lower productivity range and so most of the employees in this group are the low productivity types with low wages. The average productivity of workers using the family channel is equal to 0.536 and the average wage is $\bar{w}^o = 0.498$. In contrast, the average productivity of workers using the formal channel is equal to 0.580 and the average wage is $\bar{w}^s = 0.530$. Therefore, we conclude that the family search channel is associated with a *wage penalty* of 6% compared to the formal channel.

The second distribution, h_i^s , is in turn first order stochastically dominated by the wage distribution of workers who used professional contacts, h_i^n . Here the average productivity is equal to 0.656 and the average wage is $\bar{w}^n = 0.584$ (see table 3.2). The probability mass of the distribution h_i^n is shifted to the right and so this density is mostly concentrated in the middle range of the productivities. This is due to the fact that the proportion of high productivity workers in the population is relatively low and almost all of them are employed. Finally, observe that finding a job with a help of professional contacts is associated with a *wage premium* of 10%. Therefore, we can rank $\bar{w}^o < \bar{w}^s < \bar{w}^n$, which is the second prediction of the paper: when the two types of contacts are separated, then family contacts are more likely to have a negative effect on wages, whereas professional contacts are more likely to have a positive effect. Cappellari and Tatsiramos (2015) confirm this result with their empirical finding for the UK that high-skilled individuals, whose employed friends are non-relatives, earn higher wages and

low-skilled individuals, whose employed friends are relatives, experience a wage penalty. Sylos Labini (2004) confirm this finding for Italy.

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.536	0.580	0.656	0.590
Average wage	0.498	0.530	0.584	0.536
Proportion of employees	0.213	0.541	0.247	1

Table 3.2: Model-generated statistics for the benchmark case

Based on table 3.2 we can also calculate the average wage of employees who found a job by means of referrals, i.e. both types of social contacts. Let this variable be denoted by \bar{w}^c :

$$\bar{w}^c = \frac{\bar{w}^o \sum g_i d_0(y_i) + \bar{w}^n \sum g_i d(y_i)}{\sum g_i d_0(y_i) + \sum g_i d(y_i)} = \frac{0.498 \cdot 0.213 + 0.584 \cdot 0.247}{0.213 + 0.247} = 0.544$$

$\bar{w}^c = 0.544$ is higher than $\bar{w}^s = 0.530$. Thus, in the benchmark case the positive effect of professional networks is dominating the negative effect of family contacts and job referrals are associated with a wage premium of 2.6%. However, this result is sensitive to the relative proportions of workers relying on family and professional relations. To elaborate on this point we perform comparative statics analysis with respect to parameter t which is a shift parameter of the distribution. Intuitively, a lower value of t corresponds to labour markets with a larger proportion of low skilled workers. In the first step, we find t^* for the neutral scenario when the effect of referrals on wages is zero. Other parameters remain unchanged. We get the value $t^* = 2.21$. Our results for the neutral scenario are presented in table 3.3. In the second step, we recognize that the effect of referrals should be negative for $t < t^*$. Therefore, we consider a wage penalty scenario as a symmetric case around t^* : $t = 1.4 = 2.2 - (3 - 2.2)$, subtracting from t^* the difference between the benchmark value of $t = 3$ and t^* . Our results for the wage penalty scenario ($t = 1.4$) are presented in table 3.4:

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.493	0.545	0.617	0.545
Average wage	0.467	0.504	0.556	0.504
Proportion of employees	0.270	0.538	0.193	1

Table 3.3: Model-generated statistics with $t = 2.2$

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.454	0.511	0.575	0.498
Average wage	0.439	0.480	0.526	0.471
Proportion of employees	0.371	0.496	0.134	1

Table 3.4: Model-generated statistics with $t = 1.4$

f_i in the case of the neutral scenario ($t = 2.2$) is close to our benchmark productivity distribution, although shifted to the left. Compared to the benchmark scenario, more workers are relying on their families in the job search process ($0.270 > 0.231$) and less workers find

jobs by means of professional contacts ($0.193 < 0.247$). From table 3.3 we also see that family contacts have a negative effect of wages, whereas professional contacts are associated with a wage premium: $\bar{w}^o = 0.467 < \bar{w}^s = 0.504 < \bar{w}^n = 0.556$. Therefore, we conclude that these results are due to the self-selection of workers into channels and are robust to different specifications of the productivity distribution. The average referral wage can be calculated from table 3.3:

$$\bar{w}^c = \frac{0.467 \cdot 0.270 + 0.556 \cdot 0.193}{0.270 + 0.193} = 0.504$$

This value is equal to \bar{w}^s and so the average effect of referrals on wages is equal to zero.

The scenario for the wage penalty ($t = 1.4$) is associated with a further increase in the proportion of workers relying on families ($0.371 > 0.270$) and a lower importance of professional contacts ($0.134 < 0.193$). As before, table 3.4 confirms that family contacts are associated with a wage penalty, whereas professional networks with wage premium, since $\bar{w}^o = 0.439 < \bar{w}^s = 0.480 < \bar{w}^n = 0.526$. The average referral wage can be calculated from table 3.4:

$$\bar{w}^c = \frac{0.439 \cdot 0.371 + 0.526 \cdot 0.134}{0.371 + 0.134} = 0.462$$

This value is lower than $0.480 = \bar{w}^s$ and so there is a wage penalty equal to 1.9%.

This section shows that the negative effect of family contacts and the positive effect of professional contacts are both robust to the exact specification of the productivity distribution in the population. However, the average effect of referrals on wages is sensitive to the specific distribution and can be positive or negative depending on the relative proportions of high and low productivity groups. Thus, our model provides an additional explanation for the ambiguous results reported in the empirical literature, which were summarized in the introduction.

3.5 Robustness checks

3.5.1 Family contacts

In this section we analyze whether the model is robust with respect to the modeling of the family search channel. For simplicity suppose that every worker has exactly one family member, for example, a parent or a spouse. If this family member is employed, he/she continues searching for jobs in the formal way with a constant search intensity s_0 . At rate $\phi(s_0) = vs_0$ this family member is matched with an open vacancy and forwards this information to the unemployed relative. Thus we can extend the model, where the modified job-finding rate through the family channel, λ_0^i , is equal to the matching rate vs_0 multiplied by the employment probability of the helping family member. Further, the probability of being employed depends on the skill level of the helping relative. In the benchmark model of the paper we assumed that the job-finding rate λ_0 was constant across groups which can only be in the absence of skill homophily within families. However, this case is not completely satisfactory as there exists empirical evidence of positive skill correlation between parents and grown-up children and between spouses⁵. In order

⁵See Hertz et al. (2007) and Black et al. (2011) for the intergenerational schooling correlation and Smits (2003) for the educational homogamy of spouses

to account for this correlation we propose the following equation for λ_0^i :

$$\lambda_0^i = vs_0[\beta e_i + (1 - \beta)\bar{e}]$$

where $0 \leq \beta \leq 1$ is a mixing parameter, e_i is the employment rate of group i and \bar{e} is the employment rate in the median skill group ($i = 7$). To understand this equation consider the two extreme cases. If $\beta = 1$, then the job-finding rate λ_0^i is equal to vs_0e_i , this is the case of strong homophily between family members. Note that in this case family members are situated in the same skill group i and so their employment rate is equal to e_i . In the opposite case, when $\beta = 0$, the job-finding rate is constant across groups, $\lambda_0^i = vs_0\bar{e}$, implying the absence of skill homophily. This is the benchmark case of the model, so we set $vs_0\bar{e} = \lambda_0$. Following the calibration above, the equilibrium employment rate of the median worker group ($i = 7$) is equal to $\bar{e} = 1 - 0.087 = 0.913$, which gives us an estimate of the formal matching rate between firms and family members: $\phi(s_0) = 0.33$. Note that s_0 is relatively low given that the individual matching rate of unemployed workers is ranged between 0 and 1.33 for $i = 1..25$.

Variable β can be seen as a fraction of type i workers with family members in the same group. Thus a larger value of β is associated with a stronger homophily of family members and a stronger correlation of skills within families. In order to find an estimate of β we target the correlation coefficient between family members equal to 0.46, which is the empirical estimate of Hertz et al. (2007). This correlation coefficient can be derived from the corresponding probability matrix, where the measure βf_i of type i workers are linked to family members in the same skill group. In contrast, a measure $(1 - \beta)f_i$ of these workers are linked to family members with a median skill level $y_7 = 0.55$. In the special case $\beta = 1$, this matrix has zero entries off the diagonal as families are exclusively formed within the same skill group. Based on this probability matrix we find that a correlation coefficient of 0.46 corresponds to $\beta = 0.225$.

With a stronger homophily within the family, there is a higher probability that family members of unproductive workers are also unproductive. This makes their help in the search process less likely, thus the job-finding rate λ_0^i falls below $\lambda_0 = 0.3$. This drop is particularly pronounced for the least productive group ($i = 1$) as the job-finding rate falls down to 0.28 for the realistic scenario $\beta = 0.225$ and down to 0.18 for the case of full homophily $\beta = 1$. At the same time, the unemployment rate in this worker group rises from 0.33 to 0.35 for the realistic scenario $\beta = 0.225$ and up to 0.46 for the case of full homophily. However, we find that the change in unemployment is relatively small for all other groups.

Further, we have calculated average wages for each of the three search channels in the model for the extreme case of perfect skill correlation between family members (case $\beta = 1$). They are given in table 3.5. In this scenario, unproductive workers are worse off as on average they are less likely to get help from their family members. This is reflected in the lower proportion of workers finding jobs through families compared to the benchmark scenario in table 3.2 ($0.207 < 0.213$). As family contacts become less relevant, workers exert more effort in the formal channel, so there is a small increase in the proportion of workers finding jobs through the formal channel ($0.545 > 0.541$). Despite these changes, the overall intuitive result of the model remains unchanged: family contacts are associated with wage penalties, whereas professional contacts are associated with wage premiums. Given that $\beta = 1$ is an unrealistic hypothetical scenario, the changes are

even smaller for the realistic case $\beta = 0.225$. Thus we conclude that our results are robust to the constant specification of λ_0 .

Variable	Specific search channel			All channels
	Family	Formal	Professional	
Average productivity	0.541	0.579	0.656	0.590
Average wage	0.501	0.529	0.584	0.537
Proportion of employees	0.207	0.545	0.247	1

Table 3.5: Perfect skill correlation between family members

3.5.2 Hiring costs

As a final robustness check we present comparative statics results with respect to the hiring cost parameter z . This parameter primarily includes the costs of posting job ads in the media (newspapers, Internet, etc.). In the benchmark case we have chosen $z = 0.39$ to achieve a market tightness ratio (v/u) equal to 1. Corollary of proposition 3.2 proves that vacancies (unemployment) are decreasing (increasing) in z without professional networks. In this section we numerically investigate whether this relationship also holds in the presence of professional referrals.

The left panel of figure 3.8 shows changes in v and in the average unemployment rate u . As the cost z is increasing from 0.2 to the benchmark case 0.39, firms post less vacancies and the average unemployment rate is increasing from 0.072 to the benchmark case 0.096. Thus the model captures the macroeconomic dynamics of the labour market as vacancies and unemployment are moving in the opposite directions. The right panel of figure 3.8 shows changes in the U-shape of the referral hiring pattern. If the cost parameter is decreasing, then firms have more vacancies which should be filled. This improves the formal job-finding rates $\phi(s_i) = vs_i$ and the network matching rates λ_i . However, as the chances of finding jobs through professional contacts are improving (that is λ_i is increasing) workers optimally reduce their individual search effort s_i . Thus the initial rise in $\phi(s_i)$ is moderated by the lower search intensity s_i . Overall, this implies that the ratio of workers finding jobs through professional contacts is increasing with a larger number of vacancies and the U-shape pattern becomes more pronounced. So the model predicts that networks are relatively more (less) utilized in the periods of expansions (recessions) compared to the formal search channel.

Investigating the link between vacancies and referral hiring is a relatively new research direction. To the best of our knowledge there are only two other studies dealing with this issue. First, in a theoretical model Horvath (2012) finds that in economic upturn the neighbors of an individual are more likely to be employed in high paying jobs and hence, it is more likely that the individual hears about a high paying job through them. This finding is intuitively similar to our result if a higher number of vacancies in our model is understood as an economic upturn in Horvath (2012). And second, Galeotti and Merlino (2014) find a U-shape relationship between the job-destruction rate and the network matching rate. This is an empirical finding which is in line with their theoretical model. It means that improving economic conditions are associated with a higher utilization of networks in the beginning. But as economic conditions improve further, network matching becomes less relevant. The first part of the effect is compatible with

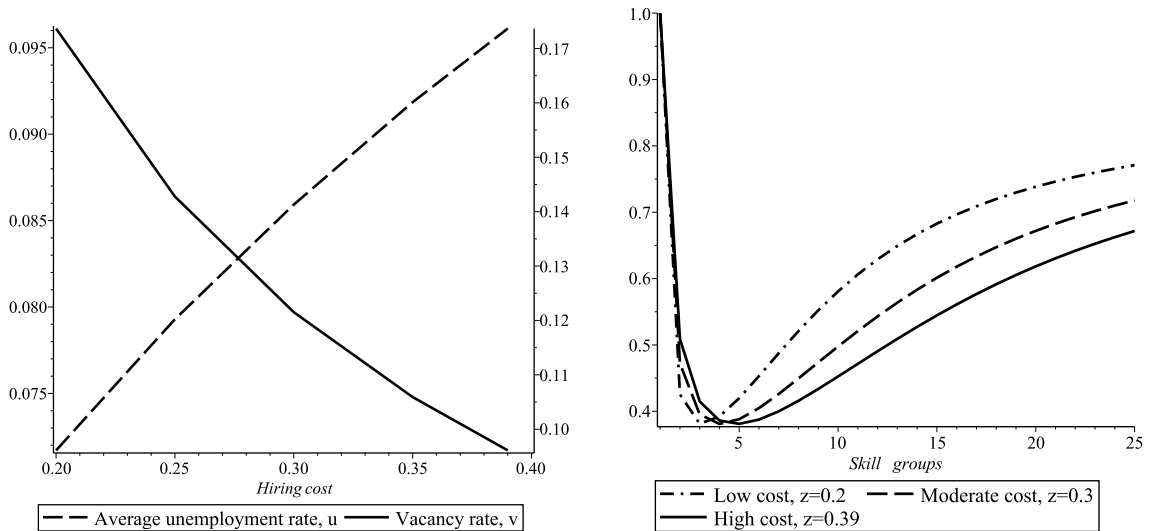


Figure 3.8: Left panel: Changes in the vacancy rate v and in the average unemployment rate u with the increase in vacancy cost z . Right panel: Changes in the U-shape of the referral hiring pattern for different vacancy costs z .

our model, even though it does not predict a lower network matching rate at the pick of the economic expansion. Overall, we conclude that empirical evidence on this question is rather sparse and more empirical and theoretical work should be done in the future to investigate this issue in more details.

3.6 Conclusions

This paper develops a labour market matching model with a finite number of heterogeneous worker groups and three channels of job search: family contacts, formal applications and professional contacts. Moreover, the model relies on the assumption of network homophily meaning that workers connected in the network are all of the same productivity type. In this framework, we are able to generate a significant U-shape relationship between the frequency of referral hiring and the productivity/skill level of the worker.

We show that the gain from preparing applications is increasing in the worker type and so it is relatively costly for low productivity workers to rely on the formal channel. On the other hand, firms with open vacancies direct their network search towards more productive incumbent employees in the anticipation of higher profits. Therefore, the family channel of search is predominantly employed by unproductive workers as a method of last resort, whereas the network of professional contacts is largely used by most productive workers. These two mechanisms explain the U-shaped referral hiring pattern, which implies that professional (family) referrals are associated with wage premiums (penalties) compared to the average wage. This is due to the endogenous sorting of workers across the three channels and implies that combining family and professional referrals into one informal channel may generate spurious empirical findings. Our results shed some light on the contradicting empirical evidence and may serve as a further step in the explanation of the "referral puzzle", at least from a theoretical perspective.

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3.8 Appendix

Appendix 3.8.1. Proof of Lemma 3.1:

The rent R_i can be obtained as a solution of the following quadratic equation:

$$0.25cvR_i^2 + (r + \delta + \lambda_0 + \lambda_i)R_i - (w_i - b) = 0$$

Since workers will only accept the job with $R_i \geq 0$ it holds that:

$$R_i = \frac{2}{cv} [\sqrt{(r + \delta + \lambda_0 + \lambda(y_i))^2 + (w_i - b)cv} - (r + \delta + \lambda_0 + \lambda_i)]$$

therefore the optimal effort is given by $s_i = 0.5cR_i$, where R_i is increasing in w_i but decreasing in λ_i :

$$\frac{\partial R_i}{\partial \lambda_i} = \frac{2}{cv} \left[\sqrt{\frac{(r + \delta + \lambda_0 + \lambda_i)^2}{(r + \delta + \lambda_0 + \lambda_i)^2 + (w_i - b)cv}} - 1 \right] < 0$$

To reduce notation in the following let $D_i \equiv r + \delta + \lambda_0 + \lambda_i$. To prove that search effort s_i is a decreasing function of the number of vacancies v , differentiate it with respect to v to obtain:

$$\frac{\partial s_i}{\partial v} = \frac{1}{v^2} \left[\frac{0.5(w_i - b)cv}{\sqrt{D_i^2 + (w_i - b)cv}} - \left(\sqrt{D_i^2 + (w_i - b)cv} - D_i \right) \right]$$

The function in the square bracket takes value zero at $v = 0$. It turns out that there are no other values of v delivering a zero to this function. To see this, differentiate expression in the square bracket to get:

$$-\frac{0.25(w_i - b)^2 c^2 v}{(D_i^2 + (w_i - b)cv)\sqrt{D_i^2 + (w_i - b)cv}} + \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)cv}} - \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)c}} < 0$$

Thus the function in the square bracket starts at zero and is downward sloping for any $v > 0$. However, this means that it is negative for any $v > 0$, so the derivative $\partial s_i / \partial v$ is also negative for any $v > 0$.

Appendix 3.8.2. Proof of Lemma 3.2:

Differentiate λ_i with respect to μ_i for a given fixed advertising intensity a_i :

$$\begin{aligned}\frac{\partial \lambda(\mu_i, a_i)}{\partial \mu_i} &= va_i \left[-\frac{1}{\mu_i^2} [1 - (1 - \mu_i)^n] + \frac{1 - \mu_i}{\mu_i} n(1 - \mu_i)^{n-1} \right] \\ &= \frac{va_i}{\mu_i^2} \left[-1 + (1 - \mu_i)^n + n\mu_i(1 - \mu_i)^n \right] = \frac{va_i}{\mu_i^2} [(1 - \mu_i)^n (n\mu_i + 1) - 1]\end{aligned}$$

Let $\sigma(\mu)$ denote the first term in the square bracket (suppressing the subindex), i.e. $\sigma(\mu) = (1 - \mu)^n (n\mu + 1)$. Note that $\sigma(0) = 1$ and $\sigma(1) = 0$. Moreover, $\sigma(\mu)$ is a decreasing function of μ for $0 < \mu < 1$:

$$\begin{aligned}\frac{\partial \sigma}{\partial \mu} &= -n(1 - \mu)^{n-1} (n\mu + 1 - \mu + \mu) + (1 - \mu)^n n \\ &= n[-(1 - \mu)^n - (1 - \mu)^{n-1} \mu(n + 1) + (1 - \mu)^n] = -n(1 - \mu)^{n-1} \mu(n + 1) < 0\end{aligned}$$

This proves that $\sigma(\mu) = (1 - \mu)^n (n\mu + 1) < 1$ and, therefore, $\partial \lambda(\mu_i, a_i) / \partial \mu_i < 0$ for $0 < \mu < 1$. Next, applying the L'Hopital's rule one can show that:

$$\lim_{\mu_i \rightarrow 0} \lambda_i = va_i \frac{\lim_{\mu_i \rightarrow 0} (1 - (1 - \mu_i)^n)}{\lim_{\mu_i \rightarrow 0} \mu_i} = va_i \lim_{\mu_i \rightarrow 0} n(1 - \mu_i)^{n-1} = nva_i$$

and also $\lim_{\mu_i \rightarrow 1} \lambda_i = 0$. This completes the proof of lemma 3.2.

Appendix 3.8.3: Proof of Lemma 3.3

First, note that at the intersection between the curves $\mu(\lambda, y)$ and $\lambda(\mu, a)$ (see figure), the latter curve (NC) is flatter than the former curve (UC), this means:

$$0 > \frac{\partial \lambda(\mu, a)}{\partial \mu} > \left[\frac{\partial \mu(\lambda, y)}{\partial \lambda} \right]^{-1} \quad \Rightarrow \quad 0 < \frac{\partial \lambda(\mu, a)}{\partial \mu} \cdot \frac{\partial \mu(\lambda, y)}{\partial \lambda} < 1$$

Taking a total derivative of $\mu(\lambda, y)$ with respect to y yields the following:

$$d\mu = \frac{\partial \mu(\lambda, y)}{\partial \lambda} d\lambda + \frac{\partial \mu(\lambda, y)}{\partial y} dy \quad \text{and} \quad d\lambda = \frac{\partial \lambda(\mu, a)}{\partial \mu} d\mu$$

Therefore, we get the following result:

$$\frac{\partial m(y, a)}{\partial y} = \frac{d\mu}{dy} = \frac{\partial \mu(\lambda, y)}{\partial y} \left[1 - \frac{\partial \mu(\lambda, y)}{\partial \lambda} \cdot \frac{\partial \lambda(\mu, a)}{\partial \mu} \right]^{-1} < 0$$

since $\partial \mu(\lambda, y) / \partial y < 0$ and expression in the square bracket is positive. Similarly, we can show that $\partial m(y, a) / \partial a < 0$.

Appendix 3.8.4: Proof of Proposition 3.1:

(i) First, note the following results from before:

$$\frac{\partial m(y, a)}{\partial y} < 0 \quad \frac{\partial m(y, a)}{\partial a} < 0 \quad \frac{\partial a(m, y)}{\partial m} > 0 \quad \frac{\partial a(m, y)}{\partial y} > 0$$

Taking a total derivative of $m(a, y)$ with respect to y yields the following:

$$dm = \frac{\partial m(y, a)}{\partial y} dy + \frac{\partial m(y, a)}{\partial a} da = \frac{\partial m(y, a)}{\partial y} dy + \frac{\partial m(y, a)}{\partial a} \left[\frac{\partial a(m, y)}{\partial m} dm + \frac{\partial a(m, y)}{\partial y} dy \right]$$

$$dm \underbrace{\left[1 - \frac{\partial m(y, a)}{\partial a} \frac{\partial a(m, y)}{\partial m}\right]}_{>0} = \underbrace{\left[\frac{\partial m(y, a)}{\partial y} + \frac{\partial m(y, a)}{\partial a} \frac{\partial a(m, y)}{\partial y}\right]}_{<0} dy \Rightarrow \frac{dm}{dy} < 0$$

Next we show that unemployment is decreasing in vacancies v . First note that:

$$d\mu = \frac{\partial \mu(\lambda, y)}{\partial \lambda} d\lambda + \frac{\partial \mu(\lambda, y)}{\partial v} dv = \frac{\partial \mu(\lambda, y)}{\partial \lambda} \left[\frac{\partial \lambda(\mu, a)}{\partial \mu} d\mu + \frac{\partial \lambda}{\partial v} dv \right] + \frac{\partial \mu(\lambda, y)}{\partial v} dv$$

$$\frac{\partial m(y, a)}{\partial v} = \frac{d\mu}{dv} = \left[\frac{\partial \mu(\lambda, y)}{\partial \lambda} \frac{\partial \lambda}{\partial v} + \frac{\partial \mu(\lambda, y)}{\partial v} \right] \left[1 - \frac{\partial \mu(\lambda, y)}{\partial \lambda} \cdot \frac{\partial \lambda(\mu, a)}{\partial \mu} \right]^{-1} < 0$$

because $\partial \mu(\lambda, y)/\partial \lambda < 0$, $\partial \lambda/\partial v > 0$ and $\partial \mu(\lambda, y)/\partial v < 0$. Then we consider the total change in unemployment $m(y, a)$:

$$dm = \frac{\partial m(y, a)}{\partial v} dv + \frac{\partial m(y, a)}{\partial a} da = \frac{\partial m(y, a)}{\partial v} dv + \frac{\partial m(y, a)}{\partial a} \frac{\partial a}{\partial m} dm$$

Finally, we can show that unemployment is a decreasing function of vacancies v :

$$dm \underbrace{\left[1 - \frac{\partial m(y, a)}{\partial a} \frac{\partial a(m, y)}{\partial m}\right]}_{>0} = \frac{\partial m(y, a)}{\partial v} dv < 0$$

(ii) Let $\rho(y_i) \equiv [1 - (1 - \mu(y_i))^n]$ denote the probability of a referral, it then holds that $a(y_i) = 0.5k\rho(y_i)J(y_i)$, where $J(y_i) = (1 - \alpha)(y_i - b)/(r + \delta)$ given the free-entry condition $V = 0$. Next differentiate $a(y_i)$ with respect to y_i to obtain:

$$\frac{\partial a(y_i)}{\partial (y_i - b)} = \frac{\partial \rho(y_i)}{\partial (y_i - b)} J(y_i) + \rho(y_i) \frac{\partial J(y_i)}{\partial (y_i - b)} = \frac{\partial \rho(y_i)}{\partial (y_i - b)} \frac{(1 - \alpha)(y_i - b)}{r + \delta} + \rho(y_i) \frac{(1 - \alpha)}{r + \delta}$$

$$\text{Therefore, } \frac{\partial a(y_i)}{\partial y_i} = \frac{\partial a(y_i)}{\partial (y_i - b)} > 0 \quad \text{if} \quad \frac{\partial \rho(y_i)}{\partial (y_i - b)} \cdot \frac{(y_i - b)}{\rho(y_i)} > -1$$

Appendix 3.8.5. Suppose the distribution h_i^s first order stochastically dominates the distribution h_i^o , then it holds $H_i^s \leq H_i^o, \forall i = 1..p$. The average wage \bar{w}^o can be written as:

$$\begin{aligned} \bar{w}^o &= \sum_{i=1}^p w_i h_i^o = w_1 h_1^o + w_2 h_2^o + w_3 h_3^o + \dots + w_{p-1} h_{p-1}^o + w_p h_p^o \\ &= w_1 (h_1^o + h_2^o + \dots + h_p^o) + (w_2 - w_1)(h_2^o + \dots + h_p^o) + \dots + (w_p - w_{p-1}) h_p^o \\ &= w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) \sum_{j=i+1}^p h_j^o = w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) \left(1 - \sum_{j=1}^i h_j^o\right) = w_1 + \sum_{i=1}^{p-1} (w_{i+1} - w_i) (1 - H_i^o) \end{aligned}$$

In a similar way, one can derive an equation for \bar{w}^s . So that $\bar{w}^o < \bar{w}^s$ if $H_i^s \leq H_i^o, \forall i = 1..p$.

Appendix 3.8.6: Proof of proposition 3.2.

Consider the free-entry condition $z = \sum \left(\frac{\lambda_0}{v} + s_i\right) \mu_i f_i J_i$. In the absence of professional

networks we get that $D_i = r + \delta + \lambda_0$. Next, applying the L'Hopital's rule one can show that:

$$\lim_{v \rightarrow 0} s_i = \frac{\lim_{v \rightarrow 0} \sqrt{D_i^2 + (w_i - b)cv} - D_i}{\lim_{v \rightarrow 0} v} = \lim_{v \rightarrow 0} \frac{0.5(w_i - b)c}{\sqrt{D_i^2 + (w_i - b)cv}} = \frac{0.5(w_i - b)c}{r + \delta + \lambda_0}$$

Similarly one can show that $\lim_{v \rightarrow \infty} s_i = 0$, $\lim_{v \rightarrow 0} \mu_i = \delta/(\delta + \lambda_0)$ and $\lim_{v \rightarrow \infty} \mu_i = 0$. Then we know that the right hand side of the free-entry condition is a decreasing function such that $\lim_{v \rightarrow 0} (\frac{\lambda_0}{v} + s_i)\mu_i = \infty$ and $\lim_{v \rightarrow \infty} (\frac{\lambda_0}{v} + s_i)\mu_i = 0$. Thus there exists a unique intersection between the cost z on the left hand side and the expected profit on the right hand side.

To show that vacancies v are decreasing in the hiring cost z we find $\partial v/\partial z$ from equation (3.12):

$$\frac{\partial v}{\partial z} = \left[\sum \left(\left(-\frac{\lambda_0}{v^2} + \frac{\partial s_i}{\partial v} \right) \mu_i + \left(\frac{\lambda_0}{v} + s_i \right) \frac{\partial \mu_i}{\partial v} \right) f_i J_i \right]^{-1} < 0 \quad \text{since} \quad \frac{\partial s_i}{\partial v} \leq 0 \quad \text{and} \quad \frac{\partial \mu_i}{\partial v} \leq 0$$

Next we prove lemma 3.4. Consider an economy with only one worker type ($y_i > b$), where professional networks are the only channel of search. The free-entry condition ($V = 0$) in this economy simplifies to yield $z = 0.25k(1 - \mu_i)(1 - (1 - \mu_i)^n)^2 J_i^2$. The first order derivative of $(1 - \mu_i)(1 - (1 - \mu_i)^n)^2$ is given by:

$$\frac{\partial(1 - \mu_i)(1 - (1 - \mu_i)^n)^2}{\partial \mu_i} = (1 - (1 - \mu_i)^n)[(1 - \mu_i)^n(1 + 2n) - 1]$$

Thus this function takes value zero at $\mu_i = 0$, it is then increasing to the unique maximum point at $\mu_i^* = 1 - (1 + 2n)^{-1/n}$ and then falls down to zero for $\mu_i = 1$. So there exist two roots of the free-entry condition if z is smaller than the maximum of this function which is given by:

$$0.25k(1 - \mu_i^*)(1 - (1 - \mu_i^*)^n)^2 J_i^2 = 0.25kJ_i^2(1 + 2n)^{-1/n} \left(1 - \frac{1}{1 + 2n}\right)^2 = \frac{kn^2 J_i^2}{(1 + 2n)^{2+1/n}}$$

Next we find the corresponding vacancies from the equilibrium unemployment equation:

$$v = \frac{\delta}{0.5k(1 - (1 - \mu_i^*)^n)^2 J_i} = \delta(1 - \mu_i^*)0.5J_i/z$$

since from the free-entry condition we have $0.5k(1 - (1 - \mu_i^*)^n)^2 J_i = z/(1 - \mu_i^*)0.5J_i$.

4 Asymmetric information in a search model with social contacts

4.1 Introduction

Uncertainty with respect to worker characteristics is one of the most important problems in the hiring process and has been intensively studied in the literature since the works of Akerlof (1970) and Spence (1973) on asymmetric information about the worker's productivity (for instance, Guash and Weiss (1980), Samuelson (1984), Myerson and Satterthwaite (1983), Alonso (2014)). In addition, there are many recent models studying the issue of asymmetric information in a search-theoretical framework. For example, in their paper, Guerrieri et al. (2010) add search and matching frictions to the classical problems in economies with adverse selection. Kennan (2010) extends the Mortensen and Pissarides (1994) model to allow for private information about the match productivity on the firms' side. Moreover, Bruegemann and Moscarini (2010) investigate the search model with two-sided private information about gains from a match. Related papers are, for example, Dao (2009), Delacroix and Wasmer (2009), Michelacci and Suarez (2006), Acemoglu (1995), Kugler and Saint-Paul (2004).

In addition, there is strong empirical evidence that 30 - 60% of new employees find jobs through their social contacts (see for example Staiger (1990), Granovetter (1995), Pistaferri (1999), Kugler (2003), Pelizarri (2010), Bentolila et. al. (2010) for different countries). Moreover, the popularity of electronic Social Network Systems (SNS) like LinkedIn, Xing or Facebook in the last decade made it easier to maintain social connections and use them for the job search.¹ These observations increase the importance of incorporating the social contacts' aspects into the theoretical models of labour market.

Montgomery (1991) is a first theoretical paper which includes both the uncertainty of firms with respect to worker's productivity and the presence of job referrals. In this model, it was found that social contacts' use in the job search may lead to a lower mismatch and therefore, to a higher production efficiency. Due to the pioneering assumption that referrals reveal the quality of the match, also emphasized, for example, in the theoretical paper of Simon and Warner (1992) and approved in the empirical work of Hensvik and Skans (2013), firms with more ability-sensitive technology will hire through social contacts. Therefore, there will be more good matches due to referrals. The classical view in the models on uncertainty not including networks, however, is that asymmetric information generally leads to more inefficiency in the economy (for example, Akerlof (1970), Guerrieri et al. (2010), Myerson and Satterthwaite (1983), Bruegemann and Moscarini (2010), Delacroix and Wasmer (2009)). These mixed results, thus, motivate for more research on these issues.

In particular, it would be natural to think, that the presence of firm's uncertainty with respect to workers' possibilities to find jobs through social contacts in addition to the asymmetric information about their productivity will create even larger inefficiency. One of the present paper's main objectives is, thus, to investigate this question, which is novel to the literature. More precisely, this paper studies the consequences of the former uncertainty while allowing the

¹According to Statista (2014), 87% (23%) of U.S. Internet users, who are 18-29 years old, 73% (31%) of 30-49 years old and 63% (30%) of 50-64 years old used Facebook (LinkedIn) in September 2014. Similar numbers are presented also by Pew Research Center (2014). These numbers are larger than in the previous years.

worker's productivity to be the common knowledge for simplicity. Let us now shortly describe the main ingredients of the theoretical model.

In this paper, the random matching model is proposed, in which homogeneous firms face uncertainty about the social capital of job seekers, who have equal productivity and all other observable characteristics. The term social capital denotes the number of actual contacts of a worker, who typically can help her to find a job in addition to her own job search through the formal channel of public offers. Thus, workers who have a larger social capital, have also larger outside options (reservation wage) in terms of job search. This number is known perfectly to workers. For simplicity, only two worker types are considered - with a low and high number of actual contacts. Wages are assumed to be offered only by firms in a form of a take-it-or-leave-it offer during the interview with a job seeker. A worker accepts a wage contract if it is at least as large as her outside options. In addition, wages are set in such a way that workers will have an incentive to exert an endogenous effort which increases the duration of a match.

A firm knows only the distribution of workers' social capital (or of worker types) in the economy and has to offer such a wage contract to a worker, so that she accepts it, otherwise a firm is left with an open vacancy and receives zero profit in the equilibrium. An important feature of the model is that firms also check the worker's profile and her public number of non-fictional social contacts in the Social Network Systems in the Internet during the interview. This number is assumed to be correlated to the actual number of friends and, therefore, serves as a noisy signal of the social capital for firms when they decide about the wage offer. A worker knows that her profile is being checked.²

This ingredient of the model is supported by the recent empirical evidence. For instance, in the nationwide survey in the U.S., which was conducted on behalf of CareerBuilder in February 2013, and included more than 2,100 hiring managers and human resource professionals, it was found that "nearly 39 percent of firms use SNS to research job candidates, up from 37 percent in 2012" (CareerBuilder.com (2013))³. Among other general personal characteristics of an individual, the firms pay attention, whether the job seeker has great communication skills and whether other people posted great references about the candidate (CareerBuilder.com (2013)). Moreover, Roulin and Bangarter (2013) find from the 96 HR managers' survey, that recruiters also focus on the number of friends generated by the SNS, since it may reflect the applicants network⁴.

In addition, Bohnert and Ross (2010) have conducted the laboratory experiment, where it was found that the candidates having alcohol-oriented pictures in their profile were offered 7 percent less salary than candidates having family-oriented pictures. At the same time, Utz (2010) proposes a sociological experiment, which shows, that the person's profile, profile pictures

²Vicknair et al. (2010) report that 45.3% students believe that employers and recruiters look at job candidates' social networking profiles all of the time.

³Further, Manant et al. (2014) conduct an experiment with two fictitious Facebook profiles of applicants that differ in their origin, in which they find the strong evidence (40% difference) that employers rely on the online information when deciding to call an applicant back for interview. "IT is the industry using it the most, at a whopping 52 percent. The least? Health care, at 28 percent. Employers are primarily using Facebook (65 percent) and LinkedIn (63 percent) to research candidates" (CareerBuilder.com (2012)). Employers are using all the tools available to them to assure they make the correct hiring decision, and the use of social media continues to grow", says Rosemary Haefner, vice president of human resources at CareerBuilder (CareerBuilder.com (2013)).

⁴They also report that professional SNS (e.g., LinkedIn) is perceived as a potential antecedent of Person-Job fit and personal SNS (e.g., Facebook) - of Person-Organization fit.

of the friends and number of friends jointly influence others' impressions, since the number of contacts is more likely to be manipulated.⁵ Hence, the public number of contacts, which a firm can look up in an SNS is not necessarily the one generated by the system, but rather the approximate number of contacts who would be ready and able to help the person to find a job from the firm's point of view⁶.

In the present model, it is also assumed for simplicity that all job-seekers have SNS profiles and all firms look them up. Another simplifying assumption is that workers cannot increase their public number of contacts (make the overall impression and social attractiveness better) only for the signalling purpose or, equivalently, firms can identify this manipulation quite easily.

In this paper, there are two wage contracts for simplicity, that are intended at workers with low and high number of actual friends, respectively. A (partially) separating equilibrium is considered, which follows a threshold rule w.r.t. a signal according to a firms' indifference condition. In this equilibrium, the higher wage will be accepted by both worker types and the lower wage only by the low types leading only to partial separation.

The model generates a positive relationship between the number of contacts in the Social Network System in the Internet and the wage offered by firms in the equilibrium. Thus, there will be a wage dispersion between equally productive workers with different number of contacts in the Internet, which extends the classical result on wage dispersion with respect to the signal in the literature on uncertainty about the worker's productivity (see, for example, Spence (1973))⁷. Moreover, this model gives an additional explanation for the empirically observed log wage dispersion between workers with equal productivity and other observable characteristics of about 70% (e.g. Mortensen (2003)). In addition, this model is in line with the theoretical literature emphasizing the positive effect of referrals on wages and the wage dispersion due to the difference in the number of social contacts. For instance, such theoretical papers as Montgomery (1992) and Ioannides and Soetevent (2006) incorporate the similar mechanism as in the present paper, namely, that social contacts increase the reservation wage. This positive effect is found also in many empirical works, for example, in Staiger (1990), Simon and Warner (1992), Granovetter (1995), Margolis and Simonnet (2003) and Goos and Salomons (2007) for different countries. Overall, however, the theory and evidence on the effect of social contacts on wages are mixed⁸.

In addition, the comparative statics w.r.t. the firms' uncertainty level increase was conducted. Moreover, the equilibrium outcomes were compared numerically with the two extreme cases: the case of perfect information, when workers' social capital is observed perfectly, and the case of a full information asymmetry, when firms don't have any reliable signal to make inferences about workers' outside options. It was found that (reservation) wages, the overall average firm's profit and average workers' income levels in the benchmark case lead to those arising in the case of

⁵Therefore, one can conclude, that hiring managers try to get an overall impression about the candidates personality, communal orientation, social attractiveness etc. and about her possibilities to be referred for a job.

⁶Indeed, among adults, on average, 37% of their total Facebook friends are reported to be actual friends (Ellison et al. (2014)) and, among undergraduate students, 25% are their actual friends (Ellison et al. (2011)).

⁷The present framework can be easily changed in order to analyze the relationship between the wage offered and the test score during the interview with a worker, where the test score is a noisy signal of a worker's productivity. In this case workers will be different in the productivity, but not in the job-finding rate. The present model, however, aims at analyzing the opposite case, when workers differ in the job-finding rate, but not in the productivity.

⁸Bentolila, Michelacci and Suarez (2010) report wage penalties in the United States and the European Union. This result is supported by Delattre and Sabatier (2007), Pistaferri (1999) as well as Addison and Portugal (2002) for France, Italy and Portugal respectively.

a full information asymmetry as the firms' uncertainty level increases. Thus, naturally, the equilibrium outcomes in the asymmetric information case are in between of these two extreme cases.

The overall average workers' income is decreasing since the average workers' income of low types is increasing slower than that of high types is decreasing. The overall average firms' profit is increasing since the number of vacancies decreases. The overall social welfare is increasing and is larger than those in the two extreme cases for the large level of uncertainty since the overall average firms' profit is increasing faster the overall average workers' income is decreasing. This result may seem counterintuitive.

One of the reasons for this is that firms anticipate that expected profits from an open vacancy will decrease due to more mismatched wages offered and open less vacancies thus decreasing their overall cost and leading to the welfare increase. So the information asymmetry turns out to be welfare improving as firms, by chance, will employ less workers which they would not like to employ. In the standard search theory with perfect information (for example, Pissarides (2000)), the social welfare is maximized when the workers' bargaining power is equal to the elasticity of the job-filling rate. This result is known as the Hosios condition. Otherwise, when their bargaining power is too low (high), firms will open too many (few) vacancies due to low (high) wages leading to more inefficiency. Since in the present model the wage is offered only by firms, the workers' wages are relatively low. This gives an intuition why the social welfare in the perfect information case is not the largest since the Hosios condition is not satisfied.

It is interesting to compare this finding to the conclusion of Montgomery (1991) that social contacts use leads to a higher level of social welfare due to a lower mismatch between firms and workers as referrals reveal the quality of the match. In the present paper, a higher level of mismatched wages offered contributes to the increase in the welfare.

The paper is organized as follows. Section 4.2 explains notation and the general labour market environment. In section 4.3 the decisions of workers and firms are investigated and the equilibrium outcome is presented. Section 4.4 compares the benchmark case of the model to the perfect information case and section 4.5 compares it to the another extreme case of a full information asymmetry. Section 4.6 discusses the issue of social welfare comparison between these three cases. Section 4.7 illustrates the theoretical results and comparative statics by means of a numerical example, while section 4.8 concludes the paper.

4.2 Labour market modeling framework

The labour market is characterized by the following properties. There is a continuum of infinitely lived risk neutral workers and firms discounting future at a common discount rate r . Firms are homogeneous and free to enter the labour market by opening a new vacancy with the flow cost c of travelling and accommodation of job seekers and advertising job offers in the Internet, newspapers, job fairs etc.

All the workers in the economy have the same productivity y , but differ only in the number of actual contacts (or social capital), which is their private information in the benchmark case of the model. These contacts may help them in the job search (informal channel) in addition to the formal search in a public job market and therefore, influence their outside options. It is assumed that the search is costless through both channels for simplicity. The matching between

workers and firms is random. Wages are assumed to be only offered by firms (take-it-or-leave-it offer) during the interview and a worker accepts the wage contract if it exceeds or is equal to her outside options. For simplicity, let there be only two types of workers: with low and high social capital, i.e. with the number of actual contacts n_L or n_H , respectively. They are further also referred to as type i workers, where $i = L, H$. Firms are aware about the values of n_L and n_H and about the distribution of worker types. Denote the fraction of workers who have a low social capital as $Pr(n_L)$. Then, the fraction of workers with the high social capital is equal to $Pr(n_H) = 1 - Pr(n_L)$.

Let the total measure of workers be equal to 1. A worker can be either employed or unemployed. Let u_i be the mass of unemployed workers of type i (with $\mu_i = u_i/Pr(n_i)$ being their unemployment rate) and e_i - the mass of corresponding employed workers in the benchmark case, so that $e_i + u_i = Pr(n_i)$. In addition, firms can see the worker's public number of contacts n'_i in the Social Network Systems in the Internet which is correlated to n_i and therefore serves as a noisy signal of n_i for firms when they decide about the wage offer during the interview. It is assumed for simplicity that workers cannot increase their number n'_i only for the signalling purpose.

Moreover, firms intend to provide workers with correct incentives. When employed the worker of a particular type chooses an optimal effort level $g \geq 0$ conditional on the contract wage offered her by the firm. This effort is unobservable to the firm. The cost of exerting effort g is $k(g)$, where it is assumed that $k(0) = 0$, $k'(g) > 0$. Every firm-worker match is subject to the separation rate $\delta(g)$, which is modeled as a decreasing function of g ($\delta'(g) < 0$) as in Zaharieva (2010) in such a way inducing workers to exert more effort in order to increase the match duration. Let us consider throughout the paper the example when the separation rate $\delta(g)$ takes the following form: $\delta(g) = 1/(\sqrt{g} + d_0)$ (diminishing returns of effort to the job duration). The constant $d_0 > 0$ denotes the minimal job duration corresponding to zero effort. Let also the effort cost function take a usual quadratic form, i.e. $k(g) = k_0g^2$, where $k_0 > 0$ is a constant multiplier.

Let $m(u, v)$ denote a matching function between workers and firms, where v is the number of open vacancies and $u = \sum_i u_i$ is the overall number of unemployed workers. For simplicity, let us assume that the matching technology is quadratic, that is $m(u, v) = uv$.⁹ Consider first the formal search channel. The number of matches between open vacancies and unemployed workers of type i is given by $vu \cdot \frac{u_i}{u} = vu_i$, meaning that their job-finding rate through this channel of job search is equal to $\frac{vu_i}{u_i} = v$.

In addition, unemployed workers can hear about open vacancies through their actual social contacts. First, an employee hears about a new job opening at an exogenous rate a per unit time as in Cahuc and Fontaine (2009). Then, this employee transmits the vacancy information to one randomly chosen unemployed friend out of a pool of her actual contacts. Thus, the firm is not aware whether the person has found a job in the formal way or received vacancy information through the network. Let us introduce the additional parameter γ denoting the level of homophily between the actual social contacts of a worker, i.e. when $\gamma = 1$ all the workers with low (high) number of friends are in contact only with also low (high) types and

⁹This approach is also used, for example, in Gautier et al. (2010). Moreover, this technology may be, due to Teulings and Gautier (2004), the most adequate assumption in models with heterogeneous workers.

when $\gamma = 0.5$ there is no homophily. In general, for $\gamma \in [0.5, 1]$, γn_i contacts of a type- i worker are of the same type and $(1 - \gamma)n_i$ contacts are of the opposite type.

For the special case of the full homophily, $\gamma = 1$, the job finding rate of a type- i worker $\lambda_i = ave_i \frac{[1 - (1 - \mu_i)^{n_i}]}{u_i} = av(1 - \mu_i) \frac{[1 - (1 - \mu_i)^{n_i}]}{\mu_i}$ as in Stupnytska and Zaharieva (2015). The expression in square brackets is the probability that there is at least one unemployed worker among the n_i contacts of an employed worker. This case will be taken as a benchmark throughout the paper for the sake of simplicity. According to Rivera et al. (2010), social networks tend to exhibit a high level of homophily with respect to such characteristics as age, gender, religion, ethnicity, values, intelligence, and education. Indeed, this case may, for instance, capture the situation when foreigners (natives) are more likely to be in contact with other foreigners (natives) and, thus, to be members of a network with low (high) number of contacts. Thus, in this case, the larger is the number of actual contacts of the worker of type i the larger is her λ_i . The expression for λ_i can be also easily modified for the case of $\gamma < 1$.¹⁰

As it is mentioned above, firms don't observe the actual number of contacts n_i of a worker but look up the number of contacts n'_i in the Internet during the interview in order to make an inference about n_i (and, hence, about her outside options) and to offer on this basis such a wage contract that a worker of type i will accept. This noisy signal n'_i is assumed to be correlated to n_i , i.e. $n'_i = n_i + \epsilon$, where the observation error of the firm ϵ is normally distributed with the mean 0 and the standard deviation σ_ϵ , i.e. $N(0, \sigma_\epsilon^2)$.

Therefore, a worker with n_i actual contacts knows that a firm will draw the number of contacts n'_i from the conditional distribution with the c.d.f. $F(n'|n_i)$ and the density $f(n'|n_i)$ having the mean n_i and the standard deviation σ_ϵ :

$$F(n'|n_i) : N(n_i, \sigma_\epsilon^2)$$

On the other hand, a firm infers the probability that the unemployed worker has n_L actual contacts conditional on the observed signal n' . It can be found from the Bayes' rule:

$$Pr(n_L|n') = 1 - Pr(n_H|n') = \frac{f(n'|n_L) \cdot \beta}{f(n'|n_L) \cdot \beta + f(n'|n_H) \cdot (1 - \beta)}$$

where $\beta = \frac{u_L}{u_L + u_H}$ is the probability that the worker met is of type- L and $1 - \beta = \frac{u_H}{u_L + u_H}$ is

¹⁰Let us denote the type opposite to the type i by j . The probability that there is at least one unemployed worker among the n_i contacts of a type- i employed worker is equal to $[1 - (1 - \mu_i)^{\gamma n_i} (1 - \mu_j)^{(1-\gamma)n_i}]$ as $(1 - \mu_i)^{\gamma n_i} (1 - \mu_j)^{(1-\gamma)n_i}$ is the probability that all her contacts are employed. Let the average unemployment rate in the network of type- i workers be equal to $\bar{\mu}_i = \gamma \mu_i + (1 - \gamma) \mu_j$. With the probability $\frac{\gamma \mu_i}{\bar{\mu}_i}$ the unemployed worker, to whom the vacancy information is transmitted, is also of type i and with the probability $\frac{(1 - \gamma) \mu_j}{\bar{\mu}_i}$ this worker is of the opposite type. The probability that there will be a match between a firm and a given type- i unemployed worker through any of her type- i employed contacts is then equal to $ave_i [1 - (1 - \mu_i)^{\gamma n_i} (1 - \mu_j)^{(1-\gamma)n_i}] \frac{\gamma \mu_i}{\bar{\mu}_i}$. Analogously, the probability that there will be a match between a firm and this worker through any of her type- j employed contacts is then equal to $ave_j [1 - (1 - \mu_j)^{\gamma n_j} (1 - \mu_i)^{(1-\gamma)n_j}] \frac{(1 - \gamma) \mu_j}{\bar{\mu}_j}$. The job finding rate of a type- i worker, λ_i , through this channel is then the sum of these two expressions divided by u_i :

$$\lambda_i = av[(1 - \mu_i)[1 - (1 - \mu_i)^{\gamma n_i} (1 - \mu_j)^{(1-\gamma)n_i}] \frac{\gamma}{\bar{\mu}_i} + (1 - \mu_j)[1 - (1 - \mu_j)^{\gamma n_j} (1 - \mu_i)^{(1-\gamma)n_j}] \frac{(1 - \gamma)}{\bar{\mu}_j}]$$

the probability that this worker is of type- H .

Assume for simplicity that, when offering wage contracts, firms follow the threshold rule w.r.t. a signal according to the ex-post indifference condition and then let us check whether there will be such a separating equilibrium in this economy. Denote the two wage contracts that are intended at the workers with n_L and n_H number of actual friends by w_L and w_H , respectively. If the outside options of type- H workers are larger than those of type- L workers only because of the higher job-finding rate for a given wage, the wage w_H offered must be larger than w_L (which itself must be less than the reservation wage of high types in the separating equilibrium) for the workers with n_H contacts to accept. Otherwise, the position which met the high type worker remains vacant and the firm receives zero profit in the equilibrium. Denote the threshold value of the signal n' , for which firms are indifferent between offering the wage w_L and w_H , by \bar{n}' . This means that after observing $n' \leq \bar{n}'$ a firm will offer the wage w_L and, in the opposite case, it will offer w_H . Thus, there will be a positive correlation between the number of contacts in the Social Network System and the wage offered by firms in the equilibrium. In this equilibrium, the wage w_H will be accepted by both worker types and the wage w_L - only by the low types leading only to partial separation. In order to fully characterize this equilibrium outcome, let us first consider the workers' and then the firms' side.

4.3 Analysis of the model

4.3.1 Workers: effort choice

Consider first the partial equilibrium case when the number of vacancies v is exogenously given. Let U_i denote the present value of an unemployed worker of type $i = L, H$ or her outside options. In addition, let W_L and W_{LH} denote the asset value of a type- L worker employed at the wage w_L and, by the firm's mistake, at the wage w_H , respectively, and let W_H be the present value of a type- H worker employed at wage w_H .

Both firms and workers are interested in more effort to be exerted on the job, since it increases the match duration, which is profitable for workers as well as for firms as in Zaharieva (2010). Denote the effort level of the type- i worker induced by the wage w_i offered as g_i and the effort level of type- L worker in response to the wage w_H offered as g_{LH} in case when the firm has made a mistake. W_i and W_{LH} then also denote the asset values of an employed worker exerting the optimal effort level g_i and g_{LH} , respectively. The Bellman equations for the employed workers choosing different effort levels can be then written as:

$$rW_i = \max_{g_i} \{w_i - k(g_i) - \delta(g_i)(W_i - U_i)\} \quad rW_{LH} = \max_{g_{LH}} \{w_H - k(g_{LH}) - \delta(g_{LH})(W_{LH} - U_L)\} \quad (4.1)$$

Equations (4.1) show that workers face a tradeoff between the gain from a lower separation rate $\delta(g)$ and the cost of exerting effort $k(g)$. A worker of type i employed at the wage w_i maximizes the rent $W_i - U_i$ w.r.t. g_i given U_i and a worker of type L employed at the wage w_H maximizes the job surplus $(W_{LH} - U_L)$ w.r.t. g_{LH} given U_L . The first order conditions for these two problems can be written as:

$$W_i - U_i = \frac{w_i - k(g_i) - rU_i}{r + \delta(g_i)} = \left| \frac{k'(g_i)}{\delta'(g_i)} \right| \quad W_{LH} - U_L = \frac{w_H - k(g_{LH}) - rU_L}{r + \delta(g_{LH})} = \left| \frac{k'(g_{LH})}{\delta'(g_{LH})} \right| \quad (4.2)$$

Thus, from equations (4.2) (incentive compatibility constraints) the optimal effort level g_i can be expressed as a function of $w_i - rU_i$ and g_{LH} as a function of $w_H - rU_L$. Then, analogously to Lemma 1 in Zaharieva (2010) one can show that for the convex cost function $k(g)$, $g_i(g_{LH})$ is increasing in $w_i - rU_i$ ($w_H - rU_L$) for a given U_i (U_L) when $\delta''(g_i) < 0$ ($\delta''(g_{LH}) < 0$). Moreover, the optimal effort level $g_i(g_{LH})$ is equal to 0 when $w_i = rU_i$ ($w_H = rU_L$). These conditions hold for the assumed functional forms of $k(g)$ and $\delta(g)$.

This mechanism of the optimal effort choice ensures that conditions $W_i - U_i > 0 \Leftrightarrow rU_i < w_i - k(g_i)$ and, therefore, $rU_i < w_i$ hold. Hence, for the existence of the semi-separating equilibrium discussed above assume that the condition $w_L < rU_H$, which prevents high types from accepting the low wage, holds. To summarize, for this equilibrium to exist, the following condition should hold:

$$rU_L < w_L < rU_H < w_H \quad (4.3)$$

In the numerical example (section 4.7) it is checked that the condition $w_L < rU_H$ holds for the realistic parameter values and that it will be indeed optimal for firms to offer wages according to the threshold rule.

All unemployed workers receive the unemployment benefit z and can find a job through the both search channels with the rate $\lambda_i + v$ depending on the type. In the equilibrium, workers correctly anticipate the threshold value \bar{n}' . A type- L unemployed, therefore, expects to be employed at the wage w_L when the signal n' drawn by the firm is less than \bar{n}' and at the wage w_H otherwise. The Bellman equation for U_L can be, thus, written as follows:

$$rU_L = z + (\lambda_L + v)[Pr(n' \leq \bar{n}'|n_L)(W_L - U_L) + (1 - Pr(n' \leq \bar{n}'|n_L))(W_{LH} - U_L)] \quad (4.4)$$

where the probability $Pr(n' \leq \bar{n}'|n_L)$ is equivalent to $F(\bar{n}'|n_L)$.

A type- H unemployed accepts only the wage w_H in the equilibrium. However, this wage is offered by a firm only when the signal n' drawn is larger than \bar{n}' . The present value of unemployment for the worker with n_H contacts can, thus, be written as follows:

$$rU_H = z + (\lambda_H + v)(1 - Pr(n' \leq \bar{n}'|n_H))(W_H - U_H) \quad (4.5)$$

where the probability $Pr(n' \leq \bar{n}'|n_H)$ is equivalent to $F(\bar{n}'|n_H)$.

4.3.2 Firms: wage determination

Let V denote the present value of the open vacancy, which will be defined later. In the equilibrium it is equal to 0 (the free-entry condition). Assume that, when choosing wages, firms maximize their ex-ante expected profit (before the realization of a signal) with respect to wages w_L and w_H and the threshold value of the signal \bar{n}' subject to their ex-post indifference condition (after the realization of a signal):

$$\max_{w_L, w_H, \bar{n}'} \{ \beta(Pr(n' \leq \bar{n}'|n_L)J_L + Pr(n' > \bar{n}'|n_L)J_{LH}) + (1 - \beta)Pr(n' > \bar{n}'|n_H)J_H \} \quad (4.6)$$

$$s.t. \quad Pr(n_L|\bar{n}')J_L = Pr(n_L|\bar{n}')J_{LH} + (1 - Pr(n_L|\bar{n}'))J_H \quad (4.7)$$

Firms take β parametrically.

J_L denotes the firm's present value of profits from a worker employed at wage w_L and therefore exerting the effort level g_L , which results in the separation rate $\delta(g_L)$. J_H (J_{LH}) is the firm's present value of profits from the high (low) type worker employed at wage w_H and, thus, exerting the effort level g_H (g_{LH}). The Bellman equations for J_L , J_{LH} and J_H can be then written as follows¹¹:

$$rJ_L = y - w_L - \delta(g_L)(J_L - V) \quad (4.8)$$

$$rJ_{LH} = y - w_H - \delta(g_{LH})(J_{LH} - V) \quad rJ_H = y - w_H - \delta(g_H)(J_H - V) \quad (4.9)$$

The maximization problem of a firm is intuitive. With probability $\beta Pr(n' \leq \bar{n}'|n_L)$ the worker met by the firm is of type L and the signal n' drawn by the firm is lower than the threshold value \bar{n}' . In this case, the firm receives the asset value J_L from the job filled by the low type worker who gets the wage w_L . With probability $\beta Pr(n' > \bar{n}'|n_L)$ the worker met by the firm is of type L , but the signal n' drawn by the firm is higher than \bar{n}' . In this case, the firm receives the asset value J_{LH} from the job filled by the low type worker who gets the wage w_H . With probability $(1 - \beta)Pr(n' \leq \bar{n}'|n_H)$ this worker is of H -type and the signal drawn is smaller than \bar{n}' . In this case, the firm offers the wage w_L and is left with an open vacancy (receives zero profit) since the high type will not accept. With probability $(1 - \beta)Pr(n' > \bar{n}'|n_H)$ this worker is of H -type and the signal was correctly drawn larger than \bar{n}' . In this case, the firm receives the asset value J_H from the job filled by the high type worker who gets the wage w_H .

The left hand side of the indifference condition is the ex-post expected profit of a firm (after the realization of a signal) from offering the low wage w_L to a worker with a signal \bar{n}' , which will be accepted only when the worker is a low type. The right hand side is the expected profit from proposing the high wage w_H to a worker with a signal \bar{n}' , which is always accepted. With the probability $Pr(n_L|\bar{n}')$ this worker will be of type L and with the opposite probability - of type H .

In the numerical example (section 4.7) the values for optimal w_L , w_H and \bar{n}' are found.

4.3.3 Steady-state equations and the free-entry condition

Denote the number of low types employed at high wage by e_{LH} and at low wage by e_{LL} so that $e_{LL} + e_{LH} = e_L$. Expressions for e_{LH} and e_{LL} can be found from the respective steady-state equations:

$$\dot{e}_{LH} = 0 = u_L(\lambda_L + v)(1 - Pr(n' \leq \bar{n}'|n_L)) - \delta(g_{LH})e_{LH} \quad (4.10)$$

$$\dot{e}_{LL} = 0 = u_L(\lambda_L + v)Pr(n' \leq \bar{n}'|n_L) - \delta(g_L)e_{LL} \quad (4.11)$$

The mass of unemployed workers with n_L actual contacts, u_L , can find a job with probability $\lambda_L + v$ through both search channels and with probability $(1 - Pr(n' \leq \bar{n}'|n_L))$ this job pays a high wage due to the firm's mistake. This is the inflow into the state e_{LH} . At the same time, the mass of workers of type L employed at a high wage, e_{LH} , can lose the job with probability $\delta(g_{LH})$. This is the outflow out of this state. On the other hand, with the opposite probability $Pr(n' \leq \bar{n}'|n_L)$ the job found by these unemployed workers pays a low wage. This forms the inflow into the state e_{LL} . Similarly, the mass of workers of type L employed at a low wage, e_{LL} ,

¹¹It is easy to see that g_{LH} is always larger than g_H in the equilibrium, and therefore, J_{LH} is always larger than J_H .

can loose the job with probability $\delta(g_L)$ determining the outflow out of this state.

The steady-state equation for the number of unemployed low types can be then written as:

$$\dot{u}_L = 0 = \delta(g_L)e_{LL} + \delta(g_{LH})e_{LH} - u_L(\lambda_L + v) \quad (4.12)$$

The mass of workers of type L employed at a low and high wage, e_{LL} and e_{LH} , can loose a job with probabilities $\delta(g_L)$ and $\delta(g_{LH})$, respectively, leading to the inflow into the state u_L . However, the unemployed low types, u_L , can find any job with probability $\lambda_L + v$ through both search channels and form in such a way the outflow out of this state.

Therefore, from these three equations, the expressions for e_{LL} , e_{LH} and u_L can be written as:

$$e_{LL} = \frac{Pr(n_L)\delta(g_{LH})(\lambda_L + v)Pr(n' \leq \bar{n}'|n_L)}{(\lambda_L + v)[(1 - Pr(n' \leq \bar{n}'|n_L))\delta(g_L) + Pr(n' \leq \bar{n}'|n_L)\delta(g_{LH})] + \delta(g_L)\delta(g_{LH})}$$

$$e_{LH} = \frac{Pr(n_L)\delta(g_L)(\lambda_L + v)(1 - Pr(n' \leq \bar{n}'|n_L))}{(\lambda_L + v)[(1 - Pr(n' \leq \bar{n}'|n_L))\delta(g_L) + Pr(n' \leq \bar{n}'|n_L)\delta(g_{LH})] + \delta(g_L)\delta(g_{LH})}$$

$$u_L = \frac{Pr(n_L)\delta(g_L)\delta(g_{LH})}{(\lambda_L + v)[(1 - Pr(n' \leq \bar{n}'|n_L))\delta(g_L) + Pr(n' \leq \bar{n}'|n_L)\delta(g_{LH})] + \delta(g_L)\delta(g_{LH})}$$

On the other hand, the steady-state equation for workers with n_H actual contacts can be written as follows:

$$\dot{u}_H = 0 = (Pr(n_H) - u_H)\delta(g_H) - u_H(\lambda_H + v)(1 - Pr(n' \leq \bar{n}'|n_H)) \quad (4.13)$$

The mass of employed workers of type H can loose a job with probability $\delta(g_H)$ leading to the inflow into the state u_H . However, the unemployed high types can find a job with probability $\lambda_L + v$ through both search channels and accept it with probability $(1 - Pr(n' \leq \bar{n}'|n_H))$ when a high wage is offered. This determines the outflow out of this state. Thus, the number of unemployed type- H workers, u_H , is equal to:

$$u_H = \frac{Pr(n_H)\delta(g_H)}{(\lambda_H + v)(1 - Pr(n' \leq \bar{n}'|n_H)) + \delta(g_H)}$$

Finally, a present value of an open vacancy denoted by V can be defined as follows. To fill an open vacancy, firms are also using both search channels at the same time. At rate $q_i = \lambda_i u_i / v = aPr(n_i)(1 - \mu_i)[1 - (1 - \mu_i)^{n_i}]$ a match between a firm and an unemployed worker of type i is formed due to her social contacts and at rate u_i the firm is matched with an unemployed worker of type i through a formal channel. Since firms don't know the worker's type and whether the worker has found a job in a formal way or through the network information transmission, they expect to be matched with some unemployed worker with a rate $q_L + q_H + u$. Then, with probability β this will be a low type. The firm will offer her the wage w_L with probability $Pr(n' \leq \bar{n}'|n_L)$ and the wage w_H with the opposite probability, and a worker will always accept. On the other hand, with probability $1 - \beta$ this will be a worker of high type, and a firm will employ her at a wage w_H only with probability $Pr(n' > \bar{n}'|n_H)$, i.e. when it infers her type

correctly. The asset value of an open vacancy is then equal to:

$$rV = 0 = -c + (q_L + q_H + u)[\beta(Pr(n' \leq \bar{n}'|n_L)(J_L - V) + (1 - Pr(n' \leq \bar{n}'|n_L))(J_{LH} - V)) + (1 - \beta)(1 - Pr(n' \leq \bar{n}'|n_H))(J_H - V)]$$

The expression in the square brackets is the expected profit of a firm from the maximization problem (4.6). Hence the optimal firm strategy is chosen so that it maximizes the present value of a vacancy V . In the equilibrium, V is equal to 0 (the free entry condition). This allows us to find the last equilibrium variable, the number of vacancies v entering through the unemployment rates. Thus, the described equilibrium can be formally defined in a following way:

Definition 4.1. *Search equilibrium with asymmetric information and with the partial separation of types is a vector of variables $(U_i, W_i, W_{LH}, J_i, J_{LH}, g_i, g_{LH}, \bar{n}', w_i, \mu_i, e_{LL}, e_{LH})$, $i = L, H$ as well as the number of vacancies v and the present value of an open vacancy V , satisfying the asset value equations for workers (4.4), (4.5) and (4.1), for firms (4.8) and (4.9), the firm's maximization problem (4.6), the optimal effort equations (4.2), the steady-state conditions (4.10), (4.11), (4.12) and (4.13), the condition (4.3) and the free-entry condition $V = 0$.*

In the numerical example (section 4.7) it is checked that this equilibrium exists for the realistic parameter values.

4.4 Perfect information case

Let us now compare the equilibrium outcomes in a model with asymmetric information from the previous section to those arising in the situation when firms are perfectly informed about the worker type, i.e. when $\sigma_\epsilon = 0$. When the actual number of contacts is observed perfectly, the wage $w_i^1, i = L, H$, inducing the optimal effort level g_i^1 is offered to the workers with n_i actual contacts, which they always accept. To characterize these equilibrium variables, let us consider the workers' and the firms' side for the case of perfect information.

Let v^1 be the number of open vacancies in this case. Hence, analogously to the asymmetric information case, the job-finding rate through the formal channel is now equal to v^1 . In addition, let u_i^1 and e_i^1 denote the amounts of unemployed and employed workers of type i , respectively, so that $u_i^1 + e_i^1 = Pr(n_i)$ and the unemployment rate of a worker of this type $\mu_i^1 = u_i^1 / Pr(n_i)$. Let the overall number of unemployed in this case be $u^1 = \sum_i u_i^1$. Then, the job finding rate through the social contacts' channel is equal in this case to:

$$\lambda_i^1 = av^1(1 - \mu_i^1) \frac{[1 - (1 - \mu_i^1)^{n_i}]}{\mu_i^1}$$

Hence, the equation for the present value of unemployment for type i workers can be written in this case similarly to the equation for U_i as follows:

$$rU_i^1 = z + (\lambda_i^1 + v^1)(W_i^1 - U_i^1) \quad (4.14)$$

where, analogously to W_i for the case of asymmetric information, W_i^1 denotes the asset value

of employment at the wage w_i^1 for the workers with n_i actual contacts and is equal to:

$$rW_i^1 = \max_{g_i^1} \{w_i^1 - k(g_i^1) - \delta(g_i^1)(W_i^1 - U_i^1)\} \quad (4.15)$$

Thus, from the first order conditions, one can show that the optimal effort level chosen by type i workers, g_i^1 is a function of $w_i^1 - rU_i^1$ with the similar properties as g_i , i.e.:

$$W_i^1 - U_i^1 = \frac{w_i^1 - k(g_i^1) - rU_i^1}{r + \delta(g_i^1)} = \left| \frac{k'(g_i^1)}{\delta'(g_i^1)} \right| \quad (4.16)$$

From the point of view of the firms, let V^1 denote the present value of the open vacancy, defined later. Moreover, let J_i^1 denote the firm's present value from hiring the type i worker at wage w_i^1 and, thus, inducing the effort level g_i^1 leading to the separation rate $\delta(g_i^1)$. The Bellman equation for J_i^1 can be then written as follows:

$$rJ_i^1 = y - w_i^1 - \delta(g_i^1)(J_i^1 - V^1) \quad (4.17)$$

The values of w_L^1 and w_H^1 offered by firms can be found from the firm's expected profit maximization problem:

$$\max_{w_L^1, w_H^1} \{ \beta^1 J_L^1 + (1 - \beta^1) J_H^1 \} \quad (4.18)$$

where β^1 is the analogue of β for the perfect information case. Firms take β^1 parametrically. Thus, the solution to this problem gives rise to the following Proposition:

Proposition 4.1. *The wage $w_i^1, i = L, H$ offered by firms is equal to:*

$$w_i^1 = y - \frac{(r + \delta(g_i^1))^2 (k''(g_i^1) \delta'(g_i^1) - \delta''(g_i^1) k'(g_i^1))}{\delta'(g_i^1)^3}$$

Proof: Appendix 4.10.1.

Since this wage is always accepted by a worker with n_i number of contacts, the steady-state condition for this worker type can be written as follows:

$$\dot{u}_i^1 = 0 = (Pr(n_i) - u_i^1) \delta(g_i^1) - u_i^1 (\lambda_i^1 + v^1) \quad (4.19)$$

Therefore the unemployment rate of type i workers, μ_i^1 , is equal to:

$$\mu_i^1 = \frac{\delta(g_i^1)}{\lambda_i^1 + v^1 + \delta(g_i^1)}$$

To find the last equilibrium variable, the number of vacancies v^1 from the free-entry condition, let us define a present value of an open vacancy V^1 analogously to V from the previous section. The job-filling rate through the social contacts in this case, q_i^1 , is the analogue of q_i and is equal to $\lambda_i^1 u_i^1 / v^1$. When a worker met is of low type (with probability β^1), a firm will always figure it out correctly and offer her the wage w_L^1 . On the other hand, when a firm meets a worker of high type (with probability $1 - \beta^1$), it offers her the wage w_H^1 . The asset value of an open vacancy

in the perfect information case is then equal to:

$$rV^1 = 0 = -c + (q_L^1 + q_H^1 + u^1)(\beta^1 J_L^1 + (1 - \beta^1)J_H^1)$$

Thus, the perfect information equilibrium can be formally defined in a following way:

Definition 4.2. *Search equilibrium with perfect information is a vector of variables $(U_i^1, W_i^1, J_i^1, V^1, g_i^1, w_i^1, \mu_i^1, v^1)$, $i = L, H$, satisfying the asset value equations for workers (4.14) and (4.15), for firms (4.17), the wage determination equation (4.18), the optimal effort equations (4.16), the steady-state conditions (4.19) and the free-entry condition $V^1 = 0$.*

This equilibrium is the special case of the equilibrium in the asymmetric information case for $\sigma_\epsilon = 0$. In section 4.7 below, it is checked that this equilibrium exists for the realistic parameter values and the equilibrium outcomes in the perfect and asymmetric information case are numerically compared.

4.5 Case of a full information asymmetry

This section now compares the benchmark model with asymmetric information to the other extreme case when firms do not have any reliable signal about the worker type, i.e. when $\sigma_\epsilon \rightarrow \infty$. This is the case of a full information asymmetry. In this case, the only one wage w^0 is offered by firms to all workers, which induces the workers with n_i , $i = L, H$ actual contacts to exert the optimal effort level g_i^0 .

Let v^0 be the number of open vacancies in this case. Hence, analogously to the benchmark case, the job-finding rate through the formal channel is now equal to v^0 . In addition, let u_i^0 and e_i^0 denote the amounts of unemployed and employed workers of type i , respectively, so that $u_i^0 + e_i^0 = Pr(n_i)$ and the unemployment rate of a worker of this type $\mu_i^0 = u_i^0 / Pr(n_i)$. Let the overall number of unemployed in this case be $u^0 = \sum_i u_i^0$. Then, the job finding rate through the social contacts' channel is equal in this case to:

$$\lambda_i^0 = av^0(1 - \mu_i^0) \frac{[1 - (1 - \mu_i^0)^{n_i}]}{\mu_i^0}$$

Hence, the expression for the present value of unemployment for type i workers, U_i^0 , can be written in this case similarly to the equation for U_i as follows:

$$rU_i^0 = z + (\lambda_i^0 + v^0)(W_i^0 - U_i^0) \quad (4.20)$$

where, analogously to W_i for the case of asymmetric information, W_i^0 denotes the asset value of employment at the wage w^0 for the workers with n_i actual contacts and is equal to:

$$rW_i^0 = \max_{g_i^0} \{w^0 - k(g_i^0) - \delta(g_i^0)(W_i^0 - U_i^0)\} \quad (4.21)$$

Thus, from the first order conditions, one can show that the optimal effort level chosen by type i workers, g_i^0 is a function of $w^0 - rU_i^0$ with the similar properties as g_i , i.e.:

$$W_i^0 - U_i^0 = \frac{w^0 - k(g_i^0) - rU_i^0}{r + \delta(g_i^0)} = \left| \frac{k'(g_i^0)}{\delta'(g_i^0)} \right| \quad (4.22)$$

From the point of view of the firms, let V^0 denote the present value of the open vacancy, defined later. Moreover, let J_i^0 denote the firm's present value from hiring the type i worker and, thus, inducing the effort level g_i^0 . The Bellman equation for J_i^0 can be then written as follows:

$$rJ_i^0 = y - w^0 - \delta(g_i^0)(J_i^0 - V^0) \quad (4.23)$$

The steady-state condition for this worker type can be written as follows:

$$\dot{u}_i^0 = 0 = (Pr(n_i) - u_i^0)\delta(g_i^0) - u_i^0(\lambda_i^0 + v^0) \quad (4.24)$$

Therefore the unemployment rate of type i workers, μ_i^0 , is equal to:

$$\mu_i^0 = \frac{\delta(g_i^0)}{\lambda_i^0 + v^0 + \delta(g_i^0)}$$

The value of w^0 offered by a firm which has met the type i worker can be found from the firm's expected profit maximization problem:

$$\max_{w^0} \{ \beta^0 J_L^0 + (1 - \beta^0) J_H^0 \} \quad (4.25)$$

where β^0 is the analogue of β for the case of a full information asymmetry. Firms take β^0 parametrically.

The solution to this maximization problem gives rise to the following Proposition:

Proposition 4.2. *Let $x^0(g_i^0) = (k''(g_i^0)\delta'(g_i^0) - \delta''(g_i^0)k'(g_i^0))(r + \delta(g_i^0))^3$, where $i = L, H$. Then, the wage w^0 offered by firms is equal to:*

$$w^0 = y - \frac{x^0(g_L^0)x^0(g_H^0) \cdot \left[\frac{\beta^0}{r + \delta(g_L^0)} + \frac{(1 - \beta^0)}{r + \delta(g_H^0)} \right]}{\beta^0(\delta'(g_L^0))^3 x^0(g_H^0) + (1 - \beta^0)(\delta'(g_H^0))^3 x^0(g_L^0)}$$

Proof: Appendix 4.10.2.

To find the last equilibrium variable, the number of vacancies v^0 , from the free-entry condition, let us define a present value of an open vacancy V^0 analogously to V^1 from the previous section. The job-filling rate through the social contacts in this case, q_i^0 , is the analogue of q_i^1 and is equal to $\lambda_i^0 u_i^0 / v^0$. The asset value of an open vacancy in this case is then equal to:

$$rV^0 = 0 = -c + (q_L^0 + q_H^0 + u^0)(\beta^0 J_L^0 + (1 - \beta^0) J_H^0)$$

Thus, the equilibrium with a full information asymmetry can be formally defined in a following way:

Definition 4.3. *Search equilibrium with a full information asymmetry is a vector of variables $(U_i^0, W_i^0, J_i^0, V^0, g_i^0, \mu_i^0, v^0)$, $i = L, H$, as well as the wage offer w^0 , satisfying the asset value equations for workers (4.20) and (4.21), for firms (4.23), the wage determination equation (4.25), the optimal effort equations (4.22), the steady-state conditions (4.24) and the free-entry condition $V^0 = 0$.*

In section 4.7 below, it is checked that this equilibrium exists for the realistic parameter values and these equilibrium outcomes are also numerically compared with those in the perfect and asymmetric information cases.

4.6 Social welfare comparison

The natural question in this model is, in which of the three cases considered above the overall social welfare as well as workers' income and firms' profits are larger. First, denote by Λ_{WL} (Λ_{WH}) the average income of low (high) type workers so that $\Lambda_W = \Lambda_{WL} + \Lambda_{WH}$ is the average income of all workers. More precisely, these variables are defined as follows:

$$\Lambda_{WL} = (w_L - k(g_L))e_{LL} + (w_H - k(g_{LH}))e_{LH} + zu_L \quad \Lambda_{WH} = (w_H - k(g_H))e_H + zu_H$$

The average income of low types is equal to the sum of their wages net of the effort cost at low and high wage jobs when employed plus the utility of unemployed workers of this type. Analogously, the average income of high types is their wage net of the effort cost when employed plus the utility of unemployed workers of this type. In addition, let Λ_F be the average profit of all firms, defined as:

$$\Lambda_F = (y - w_L)e_{LL} + (y - w_H)(e_{LH} + e_H) - cv$$

It is equal to the firms' profits after hiring low types at both wage contracts and high types at a high wage contract minus the cost of vacancies' creation. It is then easy to see that $\Lambda_W + \Lambda_F = \Lambda$, which is the overall social welfare level.

Analogously, let us define variables similar to Λ_{WL} , Λ_{WH} , Λ_W , Λ_F and Λ for the perfect information case and the case of a full information asymmetry with upper indexes 1 and 0, respectively. Expressions for average incomes of low and high types in the perfect information case, Λ_{WL}^1 and Λ_{WH}^1 , respectively, can be written as follows:

$$\Lambda_{WL}^1 = (w_L^1 - k(g_L^1))e_L^1 + zu_L^1 \quad \Lambda_{WH}^1 = (w_H^1 - k(g_H^1))e_H^1 + zu_H^1$$

Intuitively, the average income of low (high) type workers in the perfect information case is equal to the low (high) wage net of the low (high) effort cost when employed plus the utility of unemployed low (high) types. Expressions for average incomes of low and high types in the case of the full information asymmetry, Λ_{WL}^0 and Λ_{WH}^0 , respectively, can be written analogously as follows:

$$\Lambda_{WL}^0 = (w^0 - k(g_L^0))e_L^0 + zu_L^0 \quad \Lambda_{WH}^0 = (w^0 - k(g_H^0))e_H^0 + zu_H^0$$

Similarly, the average income of low (high) type workers in the case of the full information asymmetry is equal to the wage w^0 net of the low (high) effort cost when employed plus the utility of unemployed low (high) types. Expressions for average profits of firms in the perfect information case and in the case of the full information asymmetry, Λ_F^1 and Λ_F^0 , respectively, can be written as follows:

$$\Lambda_F^1 = (y - w_L^1)e_L^1 + (y - w_H^1)e_H^1 - cv^1 \quad \Lambda_F^0 = (y - w^0)(e_L^0 + e_H^0) - cv^0$$

Λ_F^1 equals to the firms' profits after hiring low and high types at low and high wage, respectively, minus the cost of vacancies' creation. Analogously, Λ_F^0 is equal to the firms' profits after hiring workers at the wage w^0 , minus the cost of vacancies' creation.

Similarly, the average incomes of all workers in the perfect information case and in the case of a full information asymmetry are $\Lambda_{WL}^1 + \Lambda_{WH}^1 = \Lambda_W^1$ and $\Lambda_{WL}^0 + \Lambda_{WH}^0 = \Lambda_W^0$, respectively. Moreover, the overall social welfare levels in cases of perfect information and a full information asymmetry are $\Lambda_W^1 + \Lambda_F^1 = \Lambda^1$ and $\Lambda_W^0 + \Lambda_F^0 = \Lambda^0$, respectively.

In section 4.7 below, the overall social welfare levels, average income levels of workers and average profits of firms in the asymmetric information, perfect information and full information asymmetry case will be numerically compared.

4.7 Numerical example

4.7.1 Calibration

This section parameterizes the model to match the average labour market indicators in the OECD countries. Let the productivity parameter y be normalized to 1. A unit period of time in the model is chosen to be six months and the discount rate r is set to 0.01, which is equivalent to the annual discount rate of 2%. Next, the flow value of leisure z is equal to 0.5, which is in the middle range of values in the literature. Shimer (2005) sets this value to 0.4, while Fontaine (2008) uses the value of 0.15 for the U.S. economy and 0.4 for the French economy. Gautier (2002) and Cahuc and Fontaine (2009) set z equal to 0.2. On the other hand, Hall and Milgrom (2008) obtain a larger value of 0.71. The cost of an open vacancy c is chosen to be 0.5 and is also close to the average in the literature. Hagedorn and Manovskii (2008) set the value of 0.58 for this parameter, while Cahuc and Fontaines (2009) value is 0.4. Shimer (2005) has chosen the value of 0.213 for the cost of vacancies while Fontaine (2008) uses the value of 0.3.

Let the number of actual contacts of type- L workers who can help in the job search be equal to 50 and that of type H - to 90. These choices are in the middle range of numbers in the literature. First, Ellison et al. (2014) report that, for adults in 2011, the mean number of actual friends among their Facebook contacts was 76 out of the 207 total Facebook contacts on average (37% are actual friends). Tong et al. (2008) report the mean of 395 total Facebook contacts for undergraduate students. According to Statista (2014), the average number of Facebook friends for all age groups is 350. Moreover, Ellison et al. (2011) report that, for undergraduate students, the average fraction of actual friends on Facebook is 25%. Thus, considering relatively younger generations, the average numbers of Facebook contacts of low and high types are equal to $50/0.25=200$ and $90/0.25=360$, respectively. In addition, let the fraction of type L workers, $Pr(n_L)$, be equal to 0.5 for easier comparability. Moreover, Rostila (2013) reports that the percentage of people with poor social contacts varies between 21.7% and 65.1% for different education levels in European countries. Then, the average numbers of actual and Facebook friends are equal to 70 and 280, respectively. These numbers are naturally a bit lower than those in the literature since not all contacts usually help in the job search, in particular, workers of the same profession tend to help more. Cingano and Rosolia (2012) find that the median number of only professional contacts in Italy is equal to 32. This number is higher in Germany and is equal to 43 (Glitz, 2013). At the same time, the mean number of actual contacts is equal to 50

in Cahuc and Fontaine (2009). Fontaine (2008) uses the number of 40 in a benchmark model of his paper. Thus, 70 is a reasonable estimation for an average number of actual contacts who can help to find a job. Moreover, let the standard deviation for the firms' error σ_ϵ take the value of 10 in the benchmark case so that, for low types, a firm can receive a signal approximately between 0 and 100 and, for high types, between 40 and 140 as on figure 4.1 (right panel). This value is the maximal one for the signal to be positive in most of cases.

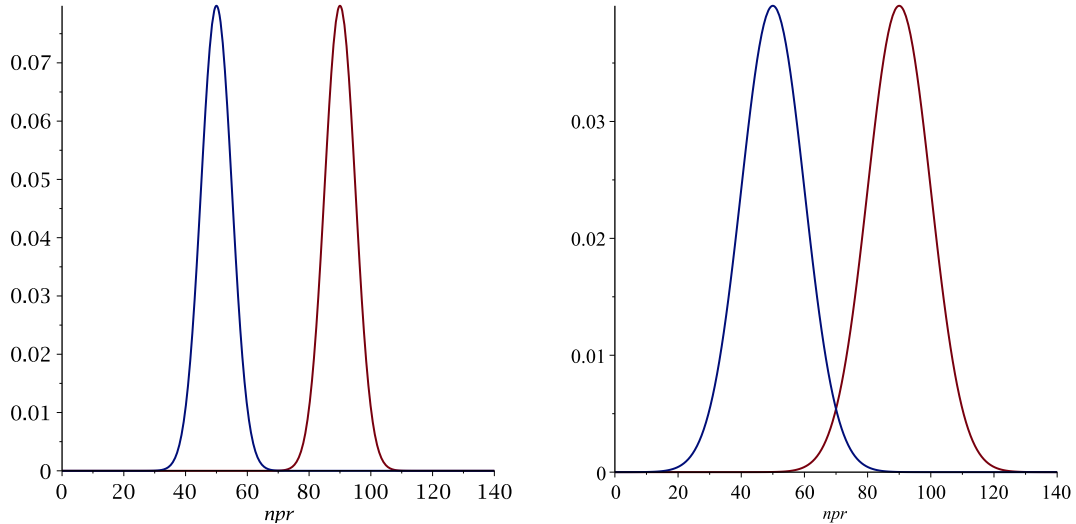


Figure 4.1: The densities $f(n'|n_L)$ (blue) and $f(n'|n_H)$ (red) for $n_L = 50$ and $n_H = 90$. $\sigma_\epsilon = 5$ (left) and $\sigma_\epsilon = 10$ (right).

In addition let the rate a with which employees hear about job vacancies be equal to 0.5. This number is chosen for the average equilibrium job-finding rate to be close to values in the literature. Hobijn und Sahin (2009) report using OECD (2006) "Employment and Labour Market Statistics" that the highest monthly job-finding rate is in the U.S. and is equal to 56.3%, while the lowest one is in Italy, 2.58%. Therefore, the annual job finding rate varies between $0.00258 \cdot 12 = 0.03$ and $0.563 \cdot 12 = 6.75$. Parameter d_0 is set to 5 so that $\delta(0) = 1/5 = 0.2$. This value corresponds to the average job duration without effort of $1/(2 \cdot 0.2) = 2.5$ years and is close to the minimal value in the literature. Hall and Milgrom (2008) set the value of 3% per month, so the average job duration is $1/(12 \cdot 0.03) = 2.78$ years. Pissarides (2009) and Shimer (2005) choose the value of δ equal to 0.1 for a unit of time being a quarter. This corresponds to the average job duration of $1/(4 \cdot 0.1) = 2.5$ years. Hobijn and Sahin (2009) report a value of 2.3% per month, so the average job duration becomes $1/(12 \cdot 0.023) = 3.6$ years. Let also $k_0 > 0$ be equal to $2 \cdot 10^{-5}$. This parameter is chosen for average job durations for equilibrium values of effort to be large enough. Table 4.1 presents the calibration for the benchmark case. In Table 4.2, the comparison of equilibrium and social welfare outcomes in asymmetric information case, perfect information case and the case of a full information asymmetry is presented.

In the asymmetric information case, the separation rate $\delta(g_{LH})$ is equal to 0.12069 corresponding to the job duration of $1/(2 \cdot 0.12069) = 4.14285$ years. The separation rate $\delta(g_L)$ is larger and is equal to 0.12922, which corresponds to the job duration of $1/(2 \cdot 0.12922) = 3.86937$ years. The separation rate $\delta(g_H)$ is the largest and is equal to 0.13188 corresponding to the job duration of $1/(2 \cdot 0.13188) = 3.79139$ years. These high effort levels will naturally cause low

Variable	Value	Explanation
n_L	50	Number of actual contacts of type L workers
n_H	90	Number of actual contacts of type H workers
$Pr(n_L)$	0.5	The share of type L workers
r	0.01	Annual interest rate of 2%
z	0.5	Unemployment benefit
σ_ϵ	10	Standard deviation for the firms' error
$\delta(0)$	1/5	Average job duration of 2.5 years (without effort)
k_0	$2 \cdot 10^{-5}$	$\delta(g)$ is close to the minimal for equilibrium effort values
c	0.5	Vacancy cost
γ	1	Degree of network homophily
a	0.5	The rate of hearing about job openings by employees
y	1	Workers' productivity

Table 4.1: Values of the model parameters

overall number of unemployed and a large amount of vacancies¹².

For the case of the full information asymmetry, the wage offered is in between the low and high wages in the asymmetric and perfect information cases. The overall social welfare is the biggest in the asymmetric information case and the smallest in the perfect information case. This may seem counterintuitive, since the asymmetric information case should naturally be in between of the two extreme cases. In the next subsection, the reasons for this will be discussed.

Variable	Asymmetric	Perfect	Full asymmetry
Low wage	0.84533	0.83732	0.85122
High wage	0.85829	0.8699	0.85122
Type- L reservation wage	0.8305	0.82157	0.83481
Type- H reservation wage	0.84616	0.85784	0.83946
Effort of low types at low wage	7.50096	7.76799	7.95924
Effort of low types at high wage	10.79608	-	-
Effort of high types	6.67079	6.64786	6.55044
Overall unemployment rate	3.49426%	3.42372%	3.46373%
Unemployment rate of low types	4.08832%	4.02538%	4.02738%
Unemployment rate of high types	2.93084%	2.82206%	2.90009%
Threshold value of the signal	61.77651	-	-
Number of vacancies	0.26662	0.26874	0.26752
Overall social welfare	0.8481	0.8475	0.8479

Table 4.2: Comparison of equilibrium and social optimum outcomes in asymmetric information case, perfect information case and the case of a full information asymmetry

4.7.2 Comparative statics

The most important comparative statics question in this model is what happens with the change of σ_ϵ ? Consider the case of σ_ϵ changing from 5 to 10. The signal distributions in the cases of σ_ϵ equal to 5 and 10 are illustrated on figure 4.1. The threshold number of contacts \bar{n}' decreases

¹²The average unemployment rate in the United States in the recent years is 8.7% (BLS, 2009-2013). In addition, Elsbey et al. (2013) report that, for OECD countries, the unemployment rate varies between 3.3% for Japan and 15.4% for Spain in 1968-2009.

from 68.93801 to 61.77651. Therefore, the probability to hire a low type worker at the low wage after a match, $Pr(n' \leq \bar{n}'|n_L)$, decreases from 0.97087 to 0.88053. Similarly, the probability not to hire a high type worker after a match, $Pr(n' \leq \bar{n}'|n_H)$, decreases slightly from 0.01759 to 0.00238 as the change in \bar{n}' has the smaller effect in this case.

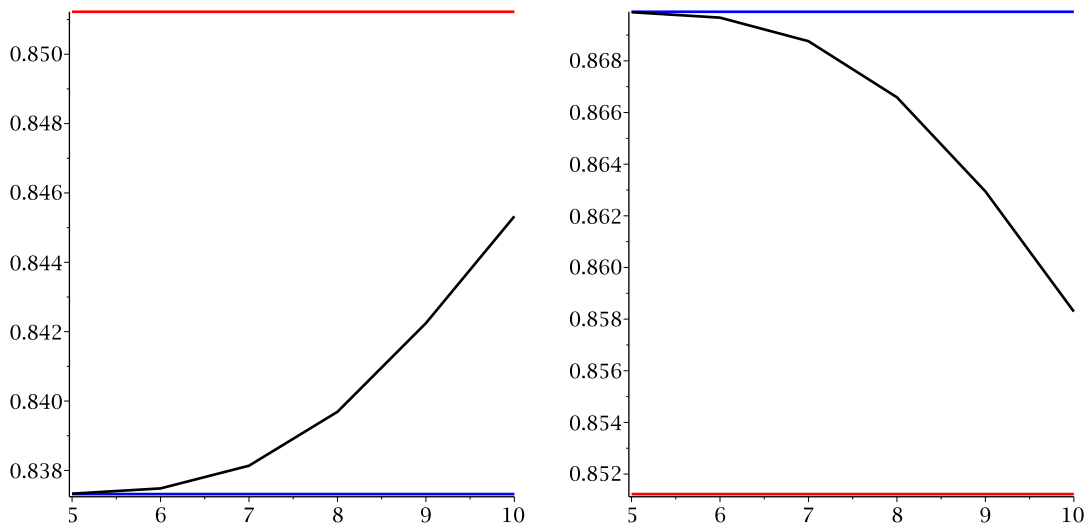


Figure 4.2: Left panel: Change in w_L (black) with the increase in σ_ϵ , w_L^1 (blue) and w^0 (red). Right panel: Change in w_H (black) with the increase in σ_ϵ , w_H^1 (blue) and w^0 (red).

Changes in wage contracts and reservation wages are illustrated on figures 4.2 and 4.3, respectively. The wage contract w_L (w_H) increases (decreases) from the value very close to w_L^1 (w_H^1) to the value close to w^0 and the reservation wage rU_L (rU_H) increases (decreases) from the value very close to rU_L^1 (rU_H^1) to the value close to rU_L^0 (rU_H^0).

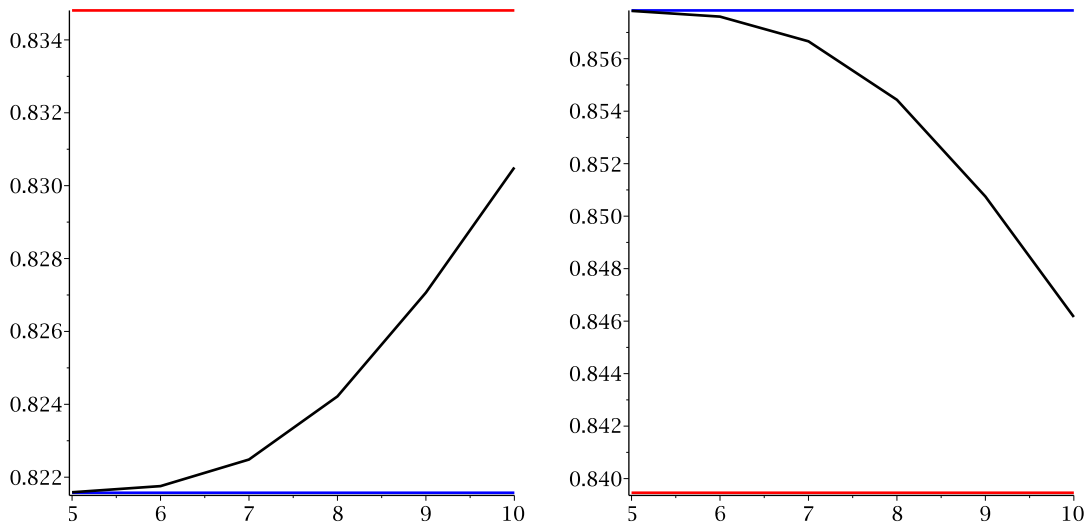


Figure 4.3: Left panel: Change in rU_L (black) with the increase in σ_ϵ , rU_L^1 (blue) and rU_L^0 (red). Right panel: Change in rU_H (black) with the increase in σ_ϵ , rU_H^1 (blue), rU_H^0 (red).

This is intuitive, as a larger uncertainty of firms makes low types better off in terms of reservation wages, and therefore, in wages since the probability of a low type to be considered

as a high type is higher and they always accept. High types are worse off in terms of wages since they have to accept more offers competing with low types more which leads also to lower reservation wages.

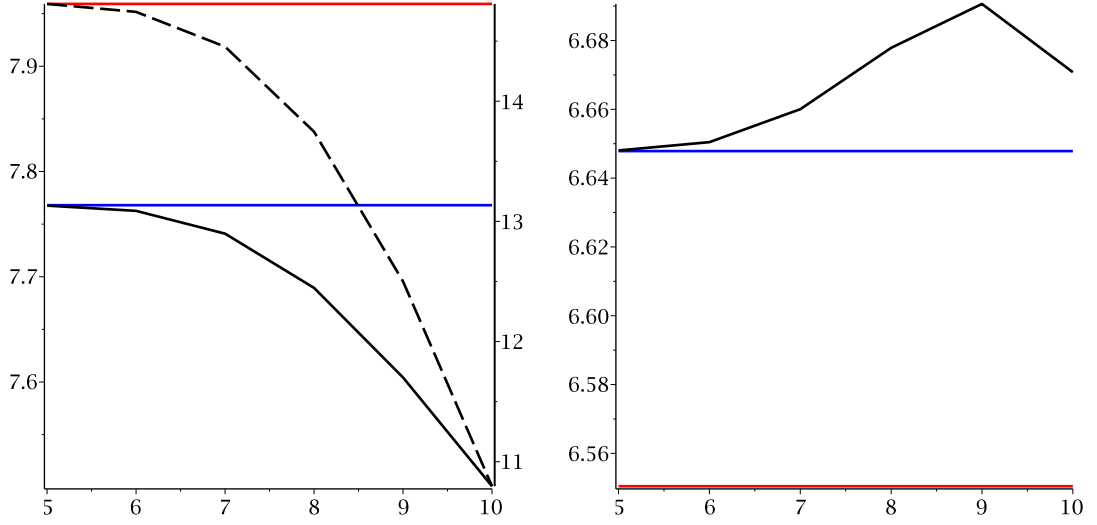


Figure 4.4: Left panel: Change in g_L (solid) and in g_{LH} (dashed, right axis) (black) with the increase in σ_ϵ , g_L^1 (blue), g_L^0 (red). Right panel: Change in g_H (black) with the increase in σ_ϵ , g_H^1 (blue), g_H^0 (red).

Changes in effort levels are illustrated on figure 4.4. The effort level g_L is always larger than g_H . Therefore, $J_L > J_H$, and firms will always get more profit from more low type workers employed.

The effort levels of low types, g_L and g_{LH} , decrease and the effort level of high types, g_H , first increases and then decreases.

Intuitively, as the firms uncertainty increases, high type workers will be more interested in exerting a higher effort level to increase the duration of a match, otherwise, when unemployed, they will compete with low types more often. On the other hand, their wage will decrease, which has a negative effect on effort. The latter effect dominates when a firm's uncertainty is already large. On the contrary, low types will be less concerned about losing the job and exert less effort. This effect dominates the effect of a wage increase.

From the pictures it can be seen that the change in g_{LH} is much larger than in g_L and g_H . The change in g_L is naturally larger than the change in g_H as \bar{n}' decreases.

Both unemployment rates, μ_L and μ_H , increase from the values very close to μ_L^1 and μ_H^1 . This is illustrated on figure 4.5.

The firms' profit per low type worker employed at the low (high) wage, J_L , (J_{LH}) decreases (increases slightly) from 1.17521 to 1.111 (from 1.05771 to 1.08431). On the other hand, the firms' profit per high type worker, J_H , increases from 0.9166 to 0.9988.

In order to check whether offering of two wage contracts is indeed an equilibrium strategy of firms let us suppose that one firm deviates and offers either the wage $rU_L < \tilde{w}_L < rU_H$ or $\tilde{w}_H > rU_H$ to both types.

More precisely, when a firm adopts the strategy of two wage contracts w_L and w_H considered in the model, it receives the following expected profit from hiring a worker as in the maximization

problem (4.6):

$$J^{exp} = \beta Pr(n' \leq \bar{n}'|n_L)J_L + \beta Pr(n' > \bar{n}'|n_L)J_{LH} + (1 - \beta)Pr(n' > \bar{n}'|n_H)J_H$$

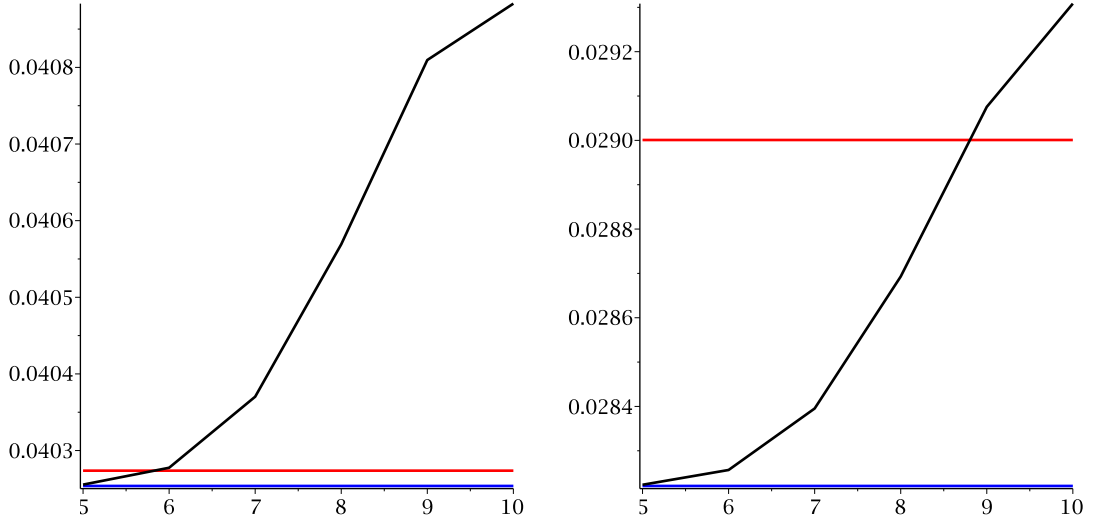


Figure 4.5: Left panel: Change in μ_L (black) with the increase in σ_ϵ , μ_L^1 (blue) and μ_L^0 (red). Right panel: Change in μ_H (black) with the increase in σ_ϵ , μ_H^1 (blue) and μ_H^0 (red).

On the contrary, the expected profit from hiring a worker when a firm deviates to offering either \tilde{w}_L or \tilde{w}_H is, respectively:

$$J_L^{exp} = \beta \max_{\tilde{w}_L} \tilde{J}_L \quad J_H^{exp} = \max_{\tilde{w}_H} \{\beta \tilde{J}_{LH} + (1 - \beta) \tilde{J}_H\}$$

$$\text{where } \tilde{J}_L = \frac{y - \tilde{w}_L}{r + \delta(\tilde{g}_L)}, \tilde{J}_{LH} = \frac{y - \tilde{w}_H}{r + \delta(\tilde{g}_{LH})} \text{ and } \tilde{J}_H = \frac{y - \tilde{w}_H}{r + \delta(\tilde{g}_H)}.$$

In the first maximization problem, the wage \tilde{w}_L can be found analogously to w_L^1 from the Proposition 4.1 and is numerically equivalent to w_L from the maximization problem (4.6) and, therefore, the firm receives $J_L > J_{LH}$ instead of J_{LH} and loses $(1 - \beta)Pr(n' > \bar{n}'|n_H)J_H$ since only low types accept.

In the second maximization problem, the wage \tilde{w}_H can be found analogously to w^0 from the Proposition 4.2.

From the figure 4.6 it can be seen that the strategy of offering two contracts is indeed the optimal one for firms for σ_ϵ from 5 to 10.

The job-finding rate λ_L decreases from 2.7929 to 2.73947 and the job-finding rate λ_H decreases from 4.27473 to 4.1116. The job-filling rate q_L increases from 0.20918 to 0.21004 and the job-filling rate q_H increases from 0.22447 to 0.22599.

Changes in the income levels of low and high type workers are illustrated on figure 4.7. Λ_{WL} naturally increases from the value very close to Λ_{WL}^1 and leads to Λ_{WL}^0 . On the other hand, Λ_{WH} decreases starting from the value very close to Λ_{WH}^1 and leads to Λ_{WH}^0 , which is also intuitive.

The change in the average income of both worker types is illustrated on figure 4.8 (left panel). It decreases from the value very close to Λ_W^1 and naturally leads to Λ_W^0 as Λ_{WH} decreases faster

than Λ_{WL} increases.

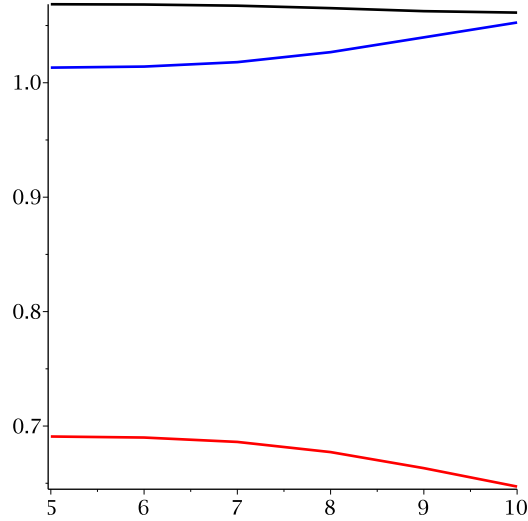


Figure 4.6: Change in J^{exp} (black), J_L^{exp} (red) and J_H^{exp} (blue) with the increase in σ_ϵ .

The number of vacancies v decreases from the value very close to v^1 (figure 4.9) (right panel). Intuitively, firms anticipate that in the asymmetric information case they will offer mismatched wages more often due to the larger probability of firms' mistakes leading to lower expected profits.

More precisely, when the firms' uncertainty increases, as it can be seen from the free-entry condition, there are two direct reinforcing effects influencing the number of vacancies mostly. Both the probability to employ low types at low wages after a match, $Pr(n' \leq \bar{n}' | n_L)$, and the profit per hiring of such a worker, J_L , decrease. Other effects are rather small and are dominated.

The change in the overall average profits of firms, Λ_F , is illustrated on figure 4.8 (right panel). It increases from the value very close to Λ_F^1 and leads to Λ_F^0 . This is intuitive as the number of vacancies decreases.

The overall social welfare, Λ , increases from the value close to Λ^1 , leads to Λ^0 and increases further (figure 4.9 (left panel)) since Λ_F increases faster than Λ_W decreases.

As it was mentioned above, the increase in the overall social welfare with the increase in the uncertainty of firms may seem counterintuitive.

One of the reasons for this is that firms anticipate that expected profits from an open vacancy will decrease due to more mismatched wages offered and open less vacancies. So the information asymmetry turns out to be welfare improving as firms, by chance, will employ less workers which they would not like to employ.

Indeed, in the standard search theory with perfect information (for example, Pissarides (2000)), the social welfare is maximized when the workers' bargaining power is equal to the elasticity of the job-filling rate.

This result is known as the Hosios condition. Otherwise, when the workers' bargaining power is too low (high), firms will open too many (few) vacancies due to low (high) wages leading to more inefficiency.

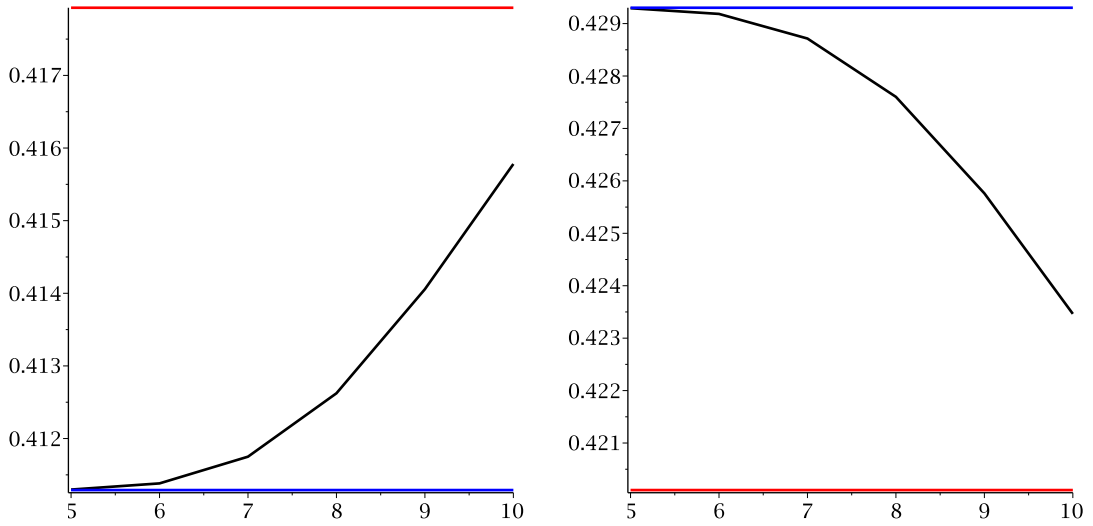


Figure 4.7: Left panel: Change in Λ_{WL} (black) with the increase in σ_ϵ , Λ_{WL}^1 (blue) and Λ_{WL}^0 (red). Right panel: Change in Λ_{WH} (black) with the increase in σ_ϵ , Λ_{WH}^1 (blue) and Λ_{WH}^0 (red).

Since in the present model the wage is offered only by firms, the workers' wages are relatively low. Moreover, the number of vacancies in the perfect information case is larger than in the asymmetric information case. This gives an intuition why the social welfare in the perfect information case is not the largest since the Hosios condition is not satisfied.

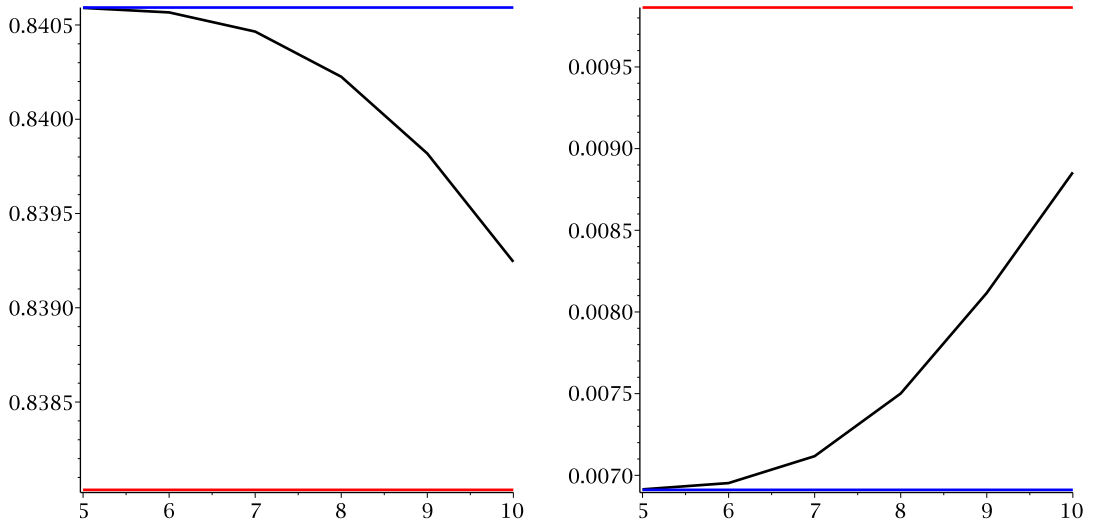


Figure 4.8: Left panel: Change in Λ_W (black) with the increase in σ_ϵ , Λ_W^1 (blue) and Λ_W^0 (red). Right panel: Change in Λ_F (black) with the increase in σ_ϵ , Λ_F^1 (blue) and Λ_F^0 (red).

It is interesting to compare this finding to the conclusion of Montgomery (1991) that social contacts use leads to a higher level of social welfare due to a lower mismatch between firms and workers as referrals reveal the quality of the match. In the present paper, it is the higher level of mismatched wages offered which contributes to the increase in the welfare.

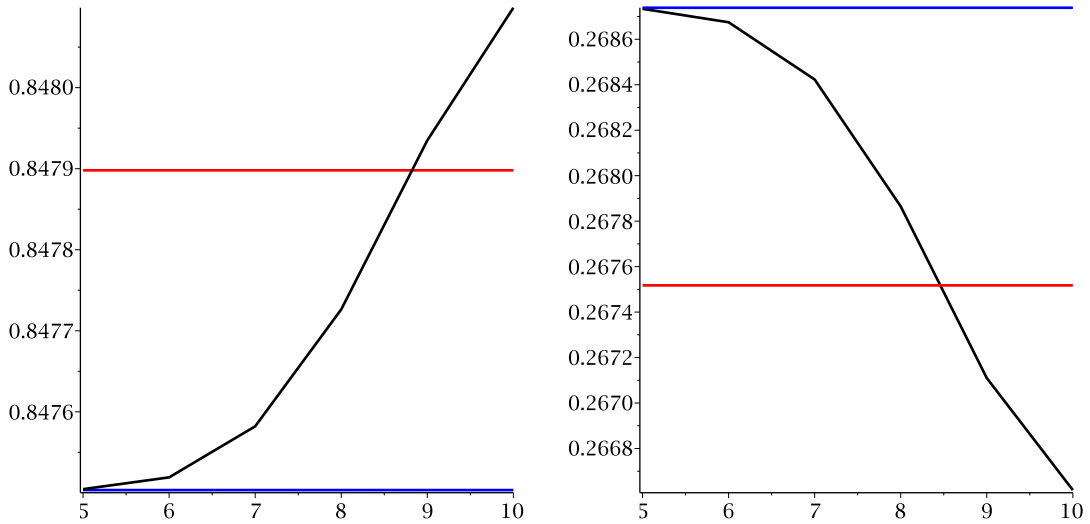


Figure 4.9: Left panel: Change in Λ (black) with the increase in σ_ϵ , Λ^1 (blue) and Λ^0 (red). Right panel: Change in v (black) with the increase in σ_ϵ , v^1 (blue) and v^0 (red).

4.8 Conclusions

In this paper, the random matching model is proposed, in which firms face uncertainty about workers' number of social contacts defining their outside options in the sense of job search through referrals. This number is known perfectly to workers who are homogeneous in all other characteristics. Wages are assumed to be offered only by firms in a form of a take-it-or-leave-it offer during the interview with a job seeker and a worker accepts a wage contract if it is at least as large as her outside options. In addition, wages are set in such a way that workers will have an incentive to exert effort. Firms also check the worker's public number of non-fictitious social contacts in the Social Network Systems in the Internet. This number is assumed to be correlated to the actual number of social contacts and, therefore, serves as a noisy signal of the social capital for firms. For simplicity, only two worker types are considered in the model: with low and high social capital.

The semi-separating equilibrium with two wage contracts, which follows the threshold rule, is considered. In this equilibrium, the higher wage will be accepted by both worker types and the lower wage only by the low types leading only to partial separation. The model generates a positive relationship between the number of contacts in the social media and the wage offered by firms in the equilibrium. Thus, there will be a wage dispersion between equally productive workers with different number of contacts in the Social Network System, which extends the classical result on wage dispersion with respect to the signal in the literature on uncertainty about the worker's productivity. Therefore, this model gives an additional explanation for the empirically observed wage dispersion between workers with equal productivity.

Moreover, the comparative statics w.r.t. firms' uncertainty level was conducted and the equilibrium outcomes of this model were compared numerically with the two extreme cases: the case of a perfect information and the case of a full information asymmetry. It was found that (reservation) wages, the overall average firms' profit and average workers' income levels in the asymmetric information case lead to those arising in the case of a full information asymmetry

as the firms' uncertainty level increases. Thus, naturally, the equilibrium outcomes in the asymmetric information case are in between of these two extreme cases. However, the overall social welfare in the asymmetric information case is increasing which may seem counterintuitive.

One of the reasons for this is that firms anticipate that expected profits from an open vacancy will decrease due to more mismatched wages offered and open less vacancies thus decreasing their overall cost and leading to the welfare increase. So the information asymmetry turns out to be welfare improving as firms, by chance, will employ less workers which they would not like to employ. It is also interesting to compare this finding to the conclusion of Montgomery (1991) that social contacts use leads to a higher level of social welfare due to a lower mismatch between firms and workers as referrals reveal the quality of the match. In the present paper, it is a higher level of mismatched wages offered that contributes to the increase in the welfare.

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4.10 Appendix

Appendix 4.10.1. Proof of Proposition 4.1:

In order to determine the wage $w_i^1, i = L, H$ offered by firms, let us find the first order condition to the firm's maximization problem (4.18), taking first into account that the optimal effort g_i^1 is a function of $w_i^1 - rU_i^1$, i.e.:

$$w_i^1 - rU_i^1 = k(g_i^1) - \frac{k'(g_i^1)}{\delta'(g_i^1)}(r + \delta(g_i^1))$$

Thus, by differentiating this equation w.r.t. g_i^1 taking U_i^1 parametrically, the inverse of $\frac{\partial g_i^1}{\partial(w_i^1 - rU_i^1)}$ can be found:

$$\frac{\partial(w_i^1 - rU_i^1)}{\partial g_i^1} = -\frac{(k''(g_i^1)\delta'(g_i^1) - \delta''(g_i^1)k'(g_i^1))}{(\delta'(g_i^1))^2}(r + \delta(g_i^1))$$

Thus, the first order conditions to the problem (4.18) can be written as follows:

$$\frac{-(r + \delta(g_i^1)) + (y - w_i^1)\delta'(g_i^1) \frac{(\delta'(g_i^1))^2}{(r + \delta(g_i^1))(k''(g_i^1)\delta'(g_i^1) - \delta''(g_i^1)k'(g_i^1))}}{(r + \delta(g_i^1))^2} = 0$$

From this equation, the expression for the optimal wage w_i^1 can be obtained:

$$w_i^1 = y - \frac{(r + \delta(g_i^1))^2(k''(g_i^1)\delta'(g_i^1) - \delta''(g_i^1)k'(g_i^1))}{(\delta'(g_i^1))^3}$$

Appendix 4.10.2. Proof of Proposition 4.2: Taking into account the derivatives of g_L^0 and g_H^0 , which can be found as in the proof of Proposition 4.1, the first order condition for the

problem (4.25) can be then written as follows:

$$\begin{aligned} & \beta^0 \cdot \frac{-(r + \delta(g_L^0)) + (y - w^0)\delta'(g_L^0) \frac{(\delta'(g_L^0))^2}{(r + \delta(g_L^0))(k''(g_L^0)\delta'(g_L^0) - \delta''(g_L^0)k'(g_L^0))}}{(r + \delta(g_L^0))^2} + \\ & + (1 - \beta^0) \cdot \frac{-(r + \delta(g_H^0)) + (y - w^0)\delta'(g_H^0) \frac{(\delta'(g_H^0))^2}{(r + \delta(g_H^0))(k''(g_H^0)\delta'(g_H^0) - \delta''(g_H^0)k'(g_H^0))}}{(r + \delta(g_H^0))^2} = 0 \end{aligned}$$

Rewriting this equation leads to:

$$\begin{aligned} & \beta^0(r + \delta(g_L^0))^2 \cdot \left[-(r + \delta(g_L^0)) + (y - w^0) \frac{(\delta'(g_L^0))^3}{(r + \delta(g_L^0))(k''(g_L^0)\delta'(g_L^0) - \delta''(g_L^0)k'(g_L^0))} \right] + \\ & + (1 - \beta^0)(r + \delta(g_H^0))^2 \cdot \left[-(r + \delta(g_H^0)) + (y - w^0) \frac{(\delta'(g_H^0))^3}{(r + \delta(g_H^0))(k''(g_H^0)\delta'(g_H^0) - \delta''(g_H^0)k'(g_H^0))} \right] = 0 \end{aligned}$$

Simplifying this equation further gives the following expression:

$$\begin{aligned} & \beta^0(r + \delta(g_L^0))^3(k''(g_L^0)\delta'(g_L^0) - \delta''(g_L^0)k'(g_L^0)) \cdot \\ & \cdot [-(r + \delta(g_L^0))^2(k''(g_L^0)\delta'(g_L^0) - \delta''(g_L^0)k'(g_L^0)) + (y - w^0)(\delta'(g_L^0))^3] + \\ & + (1 - \beta^0)(r + \delta(g_H^0))^3(k''(g_H^0)\delta'(g_H^0) - \delta''(g_H^0)k'(g_H^0)) \cdot \\ & \cdot [-(r + \delta(g_H^0))^2(k''(g_H^0)\delta'(g_H^0) - \delta''(g_H^0)k'(g_H^0)) + (y - w^0)(\delta'(g_H^0))^3] = 0 \end{aligned}$$

The last expression can be rewritten also as:

$$\begin{aligned} & \beta^0 x^0(g_H^0) \left[-\frac{x^0(g_L^0)}{r + \delta(g_L^0)} + (y - w^0)(\delta'(g_L^0))^3 \right] + \\ & + (1 - \beta^0) x^0(g_L^0) \left[-\frac{x^0(g_H^0)}{r + \delta(g_H^0)} + (y - w^0)(\delta'(g_H^0))^3 \right] = 0 \end{aligned}$$

From this equation it is easy to receive the optimal w^0 :

$$w^0 = y - \frac{x^0(g_L^0)x^0(g_H^0) \cdot \left[\frac{\beta^0}{r + \delta(g_L^0)} + \frac{(1 - \beta^0)}{r + \delta(g_H^0)} \right]}{\beta^0(\delta'(g_L^0))^3 x^0(g_H^0) + (1 - \beta^0)(\delta'(g_H^0))^3 x^0(g_L^0)}$$

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Summary

This dissertation studies the impact of social networks on social welfare and wage inequality in a labour market with search and matching frictions. Social contacts are considered in the sense of information transmission about vacancies from employed to unemployed workers (social capital). The main chapters of this thesis are based on three independent articles. Chapters 2 and 3 are joint works with J.-Prof. Dr. Anna Zaharieva.

Chapter 2 is a revised version of the IMW working paper No. 491 with the same title. This paper develops a search model with heterogeneous workers and social networks. High ability workers are more productive and have a larger number of professional contacts. Firms can choose between a high cost vacancy in the regular market and a low cost job opening in the referral market. The model predicts that a larger number of social contacts is associated with a larger wage gap between high and low ability workers and a larger difference in the equilibrium unemployment rates. The net welfare gain of referrals is estimated at 1.2%. Next we demonstrate that the decentralized equilibrium with referrals is inefficient for any value of the bargaining power. There are two reasons for the inefficiency. First, the private gain from creating a job in the referral market is always below the social gain, so the equilibrium unemployment of high ability workers is above its optimum. Moreover, high ability workers congest the market for low ability workers, so the equilibrium wage inequality is inefficiently large. This is in contrast to the result of Blazquez and Jansen (2008) showing that the distribution of wages is compressed in a search model with heterogeneous workers. Finally, we show that a combination of transfers and subsidies can restore the optimal allocation. If this policy is implemented the net welfare gain of referrals rises up to 1.8%.

Chapter 3 is a paper published in the *Journal of Economic Behavior and Organization* 119 (2015), pp. 211-233 (doi:10.1016/j.jebo.2015.08.012) with the same title. This paper presents a search model with heterogeneous workers, social networks and endogenous search intensity. There are three job search channels available to the unemployed: costly formal applications and two costless informal channels - through family and professional networks. Low productivity workers expect low wages implying low incentives for preparing formal job applications. Hence low productivity workers rely on family referrals as a method of last resort. In contrast, professional referrals are used by firms to hire high productivity employees. Formal hiring is then a most frequent employment channel for workers in the middle range of the productivity distribution. This explains a U-shape referral hiring pattern observed in empirical studies and a strong selection of workers on productivity across the three channels. Moreover, combining family and professional referrals into one informal channel may generate a spurious result of wage premiums (penalties) if high (low) productivity workers are dominating in the empirical data and their productivity is not fully observable to the econometrician.

Chapter 4 is the IMW working paper No. 548 with the same title. In this paper, the search model is proposed, in which homogeneous firms are uncertain about the job seekers' number of friends, who can help them in the job search (social capital). All workers have the same productivity and differ only in the social capital. A firm offers a take-it-or-leave-it wage contract to a worker after checking the worker's profile and her public number of non-fictitious

social contacts in the Social Network System in the Internet. This number serves as a noisy signal of the social capital for firms and cannot be influenced by the worker only for signalling purpose. The model generates a positive relationship between the number of contacts in the Social Network System and the wage offered by firms in the equilibrium. In addition, the presence of firm's uncertainty with respect to workers' possibilities to find jobs through social contacts increases overall social welfare.

Keywords: social capital, social networks, referrals, wage dispersion, wage compression, family contacts, professional networks, U-shape, referral puzzle, wage premiums, wage penalties, asymmetric information, uncertainty, reservation wage, Social Network System, Facebook, LinkedIn, wage contract, social welfare