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Comment on 'Geomagnetic Depth Sounding by Induction Arrow Representation: A Review' by G. P. Gregori and L. J. Lanzerotti

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In a recent review of the concept of induction vectors as an indicator of lateral variations of subsurface conductivity structure, *Gregori and Lanzerotti* [1980] attempted to work out a unifying concept which the authors interpreted as demonstrating the equivalence of the various induction vectors in use, thereby making differences between them merely formal and without physical significance. This approach certainly cannot be approved in general, because due consideration of the precise meaning of the different induction vectors, according to the original publications, will show that each of them is a physically well-distinguished quantity, their equivalence being limited to rather restrictive assumptions.

In order to demonstrate these differences between the Parkinson vector $\mathbf{v}_p$, the Wiese vector $\mathbf{v}_w$, and the Schmucker vector $\mathbf{v}_s$, transfer functions will be introduced: Let

$$\mathbf{B}(t) = [H(t), D(t), Z(t)]$$

designate the total magnetic variation field, as observed at a particular point of the earth's surface at a particular time t, H, D, and Z being its components along magnetic north, along magnetic east, and vertically downward, respectively. For the purpose of showing physical differences between the three induction vectors we will mostly consider harmonic events, that is, disturbances of the particular form

$$F(t) = F_0 \cos(\omega t + \phi_F)$$

which is a real function in the time domain and refers to an arbitrary field component. In the frequency domain we introduce the complex function $\hat{F}(\omega) = F_0 e^{i\phi_F}$ and thus have the conversions

$$\text{Re}\,[\hat{F}(\omega)e^{i\omega t}] = F(t) \qquad \text{Im}\,[\hat{F}(\omega)e^{i\omega t}] = -F(t + T/4)$$

where $\omega = 2\pi/T$.

Schmucker [1964, 1970] considered normal and anomalous variations and elaborated on the physical significance of both. According to him an arbitrary field component F may be decomposed into a normal part F_n and an anomalous part F_a. Henceforth it will be assumed that $Z_n \ll Z_a$, which is usually closely met in mid-latitudes and is equivalent to a quasi-homogeneous normal field. From the linearity of Maxwell's equations it then follows that we may connect the components

$$H(t) = H_0 \cos(\omega t + \phi_H)$$

$$D(t) = D_0 \cos(\omega t + \phi_D)$$

$$Z(t) = Z_0 \cos(\omega t + \phi_Z)$$

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in the frequency domain by

$$\hat{Z}_a = h\,\hat{H}_n + d\,\hat{D}_n \tag{1}$$

where h and d are complex transfer functions. Techniques for determining them for various (not necessarily harmonic) events have been discussed in detail by *Schmucker* [1970].

For the following discussion it will be appropriate to assume that the subsurface conductivity structure is well represented by a two-dimensional distribution. If the positive direction tangential to its strike is oriented at an angle $\theta \leq \pi$ toward south from magnetic east, then the positive direction perpendicular to strike is oriented at the same angle θ toward east from magnetic north. This corresponds to a clockwise rotation of the original geomagnetic system by the angle θ, and we may write for H' perpendicular and D' tangential to strike,

$$\hat{H}' = \hat{H}\cos\theta + \hat{D}\sin\theta \tag{2}$$

$$\hat{D}' = -\hat{H}\sin\theta + \hat{D}\cos\theta \tag{3}$$

Equation (1) may then be replaced by

$$\hat{Z}_a = h'\,\hat{H}_n' + d'\,\hat{D}_n' \tag{4}$$

with rotated transfer functions h' and d'. But $d' = 0$, as we have E polarization [*Untiedt*, 1964; *Jones and Price*, 1970]. Thus (4) reduces to

$$\hat{Z}_a = h'\,\hat{H}_n' \tag{5}$$

This equation suggests to refer all phases to the phase of $\hat{H}_n'$. We therefore set $\phi_{H_n'} = 0$ and transform (1), (2), and (5) into the time domain: Inserting

$$H_{n0}' = H_n(0)\cos\theta + D_n(0)\sin\theta \tag{6}$$

into

$$Z_a(0) = \text{Re}\,h'\,H_{n0}' \tag{7}$$

and comparing with

$$Z_a(0) = \text{Re}\,h\,H_n(0) + \text{Re}\,d\,D_n(0) + \text{Im}\,h\,H_n(T/4) + \text{Im}\,d\,D_n(T/4) \tag{8}$$

we realize that (8) reduces to

$$Z_a(0) = \text{Re}\,h\,H_n(0) + \text{Re}\,d\,D_n(0) \tag{9}$$

where

$$\text{Re}\,h = \text{Re}\,h'\cos\theta \tag{10}$$

$$\text{Re}\,d = \text{Re}\,h'\sin\theta \tag{11}$$

The transformation properties of Re h and Re d suggest that these quantities be identified as the components of a vector quantity, which may be defined as

$$\mathbf{v}_s = -(\mathrm{Re}\ h,\ \mathrm{Re}\ d) \tag{12}$$

This is the in-phase induction vector, or Schmucker vector, as used by *Schmucker* [1964, 1970]. It is obviously directed perpendicular to the strike of the conductivity distribution and points toward regions of enhanced conductivity. Two significant differences between the definitions of $\mathbf{v}_s$ after *Schmucker* [1970] and *Gregori and Lanzerotti* [1980] must be emphasized:

1. Schmucker expresses anomalous vertical fields in terms of normal horizontal fields, whereas Gregori and Lanzerotti consider only total field components.
2. Schmucker considers components of vertical fields in phase with horizontal fields, whereas Gregori and Lanzerotti do not take possible phase lags into account.

In (9), normal fields in Schmucker's sense are assumed to be known for any location considered. This may be related to some difficulties in practice and at least requires simultaneous observations at different locations. *Banks* [1973] stated idealizing assumptions which allow this problem to be overcome: As $Z_n \ll Z_a$ has been assumed before, his requirements simply reduce to $H_a \ll H_n$ and $D_a \ll D_n$ in the present context. Hence instead of (9) we just have

$$Z(0) = \mathrm{Re}\ h\ H(0) + \mathrm{Re}\ d\ D(0) \tag{13}$$

But (12) is still valid; if we invert the sign, we may also write

$$\mathbf{v}_w^* = -\mathbf{v}_s = (\mathrm{Re}\ h,\ \mathrm{Re}\ d) \tag{14}$$

Equation (13) obviously suggests another method of determining the components of the induction vector $\mathbf{v}_w^*$: If different harmonic events of the same frequency are considered at exactly the time of an extremum of H', then Re h and Re d can be determined. However, (13) is obviously not suitable for evaluating actual events, since it requires the knowledge of the component H' perpendicular to the initially unknown strike direction. Consequently, *Wiese* [1962, 1965] referred all phases to the observable field component Z. Setting $\phi_Z = 0$ and considering instantaneous field components at $t = 0$, that is at the time of an extremum of Z, he obtained

$$Z_0 = \frac{\mathrm{Re}\ h}{\cos \phi_{H'}} H(0) + \frac{\mathrm{Re}\ d}{\cos \phi_{H'}} D(0) \tag{15}$$

$$\mathbf{v}_w = \left(\frac{\mathrm{Re}\ h}{\cos \phi_{H'}}, \frac{\mathrm{Re}\ d}{\cos \phi_{H'}}\right) = -\frac{\mathbf{v}_s}{\cos \phi_{H'}} \tag{16}$$

where $\mathbf{v}_w$ is the Wiese vector proper. This was also shown by *Untiedt* [1970] and *Schmucker* [1980]. Of course, other phases of Z may also be selected. A complete discussion of the dependence of the magnitude of the resulting Wiese vectors on the choice of the phase of Z at which field components are compared has been given by *Siebert* [1969]. This distinction may have some practical importance when considering Wiese vectors obtained during different surveys. But no physical significance is inherent in it, as different relations corresponding to (15) define corresponding Wiese planes, which invariantly extend tangential to the strike of the conductivity distribution and differ only by their inclination toward the horizontal plane. A corollary of this is, of course, that the respective Wiese vectors invariantly point into a direction perpendicular to strike, differing only by their magnitude.

In practice, the requirements $H_a \ll H_n$ and $D_a \ll D_n$, which are only valid for rather weak conductivity contrasts, cannot be met in general. But as *Wiese* [1962] showed, the useful properties of the Wiese planes and Wiese vectors already apply for any two-dimensional conductivity distribution, as long as the normal field is horizontal. However, instead of (15) and (16) we can now only state that

$$Z_0 = a\ H(0) + b\ D(0) \tag{17}$$

$$\mathbf{v}_w = (a, b) = -m\ \mathbf{v}_s \tag{18}$$

where m is not known in general. In fact, *Beamish* [1977] calculated $\mathbf{v}_w$ and $\mathbf{v}_s$ for a set of stations in Kenya and could show that $\mathbf{v}_w$ and $\mathbf{v}_s$ were not even antiparallel, which should be expected because of the three-dimensional conductivity distribution prevailing in that area.

Referring to the definition of the Wiese vector according to *Gregori and Lanzerotti* [1980], we thus realize the following difference:

3. Wiese considered arbitrary two-dimensional conductivity distributions, with all the consequences for the relation between $\mathbf{v}_w$ and $\mathbf{v}_s$ outlined above, whereas the relation $\mathbf{v}_w = -\mathbf{v}_s$ suggested by Gregori and Lanzerotti will only apply for weak conductivity contrasts close to the inductive limit.

For Wiese's original data, however, phase lags turned out to be small, which justified his disregard of them. Also, *Untiedt* [1970] pointed out that the case of pure self-induction seemed to apply sometimes. If it holds exactly, (17) may be written as

$$Z(t_0) = a\ H(t_0) + b\ D(t_0) \tag{19}$$

This clearly demonstrates that only in this restrictive situation the same linear relation holds for any time t_0 and independent of the frequency of the oscillation. We may also say that only in the case of the inductive limit is the Wiese plane identical with the variation plane of an arbitrary (not necessarily harmonic) disturbance. This variation plane may be identified with the Parkinson plane, or preferred plane, as introduced by *Parkinson* [1959, 1962]. The Parkinson vector $\mathbf{v}_p$ is defined as the horizontal projection of the downward unit vector perpendicular to that plane, and *Gregori and Lanzerotti* [1980] demonstrated that in this case we have

$$|\mathbf{v}_p| = \frac{|\mathbf{v}_w|}{(1 + |\mathbf{v}_w|^2)^{1/2}} \tag{20}$$

where both vectors are oriented antiparallel.

However, phase shifts may not be disregarded in every instance, and in fact Parkinson did not restrict his analysis to two-dimensional conditions close to the inductive limit but only stated that a relationship such as (19) was closely satisfied in many situations.

Untiedt [1964] could demonstrate that a three-dimensional conductivity distribution would already be sufficient to give rise to stationary variation planes, as long as the inductive limit was closely approached. On the other hand, he cautioned that a purely harmonic field variation would always occur in a plane (compare also *Siebert* [1969]): For a two-dimensional conductivity distribution the rotated system may be chosen as before. If we write

$$Z(t) = j\,H'(t) + k\,D'(t) \qquad (21)$$

for the harmonic disturbance, the coefficients are given by

$$j = \frac{Z_0 \sin(\phi_Z - \phi_{D'})}{H_0' \sin(\phi_{H'} - \phi_{D'})} \qquad (22)$$

$$k = \frac{Z_0 \sin(\phi_Z - \phi_{H'})}{D_0' \sin(\phi_{D'} - \phi_{H'})} \qquad (23)$$

If also $\phi_Z = \phi_{H'}$, we have $j = Z/H'$ and $k = 0$. Therefore $j = h'$ and $k = d'$ as expected. If, however, $\phi_Z \neq \phi_{H'}$, which implies finite conductivities, then for a two-dimensional distribution, Z of course remains independent of D' physically. But $k \neq 0$ in (23) and j and k incorporate the modulus and phase of D' tangential to strike for a given H' perpendicular to strike. Consequently, the vertical plane normal to strike will accommodate elliptically polarized fields, and the variation plane will not strike parallel to the conductivity structure any more (even though this does not imply that the inductive response of the ground has no influence on the orientation of that plane at all). The Parkinson vector $\mathbf{v}_p$ is therefore not oriented perpendicular to strike, and $\mathbf{v}_w$ (which does remain perpendicular to strike if determined as suggested by Wiese) and $\mathbf{v}_p$ are no longer antiparallel. If the polarization characteristics of the horizontal field change from event to event, which will usually be the case in midlatitudes, no stationary variation planes are compatible with this situation at all [*Untiedt*, 1964, 1970; *Porath*, 1970], and only a best plane in some sense may be defined. As *Meyer* [1980] pointed out, only for a sufficiently random distribution of polarizations and two-dimensional structure would this best plane be approximately tangential to strike. This seems to be the situation originally considered by Parkinson. But the following point must be noted:

4. Gregori and Lanzerotti clearly identified the Parkinson plane with the variation plane, which has all the consequences indicated before, that is, (20) will not be valid in general.

In conclusion we may safely state that *Gregori and Lanzerotti* [1980] oversimplified their comparison of real induction vectors considerably and completely disregarded the four points summarized here. However, careful consideration of these differences is not merely a matter of academic interest but is also indispensable when interpreting actual field data.

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