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NURBS-BASED ELEMENTS AS A BASIS FOR INTEGRATING ENGINEERING APPLICATIONS

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Abstract. *Building information modeling offers a huge potential for increasing both the productivity and the quality of construction planning processes. However, this approach has not found widespread use. One of the reasons is the insufficient coupling of the structural models with the general building model. Instead, structural engineers usually set up a structural model that is independent from the building model and consists of mechanical models of reduced dimension. An automatic generation of a structural model from a building model is therefore not possible. However, especially this would be highly valuable considering the large number of plan revisions in planning processes.*
This can be overcome by a volumetric formulation of the problem. A recent approach employed the p-version of the finite element method to this problem. This method, in conjunction with a volumetric formulation is suited to simulate the structural behaviour of both 'thick' solid bodies and thin-walled structures.
Besides other disadvantages of this combination, there remains a notable discretization error in the numerical models. This paper therefore proposes a new approach for overcoming this situation. That approach combines the Isogeometric analysis together with the volumetric models. The concept of the isogeometric analysis is, in short, the application of NURBS functions for representing the geometry as well as for the shape functions of finite elements. These functions possess some beneficial properties regarding numerical simulation. Their use, however, leads to some intricacies related to the setup of the stiffness matrix. The key factor is, though, that it allows the exact *transfer of the geometry to the FE mesh, thereby increasing the quality of the numerical models.*
This paper highlights some effects that result from the application of NURBS functions in the FEM and some necessary steps for integrating building models and structural design.

1 INTRODUCTION

Planning and construction of buildings can be considered as a distributed process: due to the increasing complexity of construction projects the work has to be distributed among several consultants. Each of these consultants is concerned with certain aspects of the building process, for example with structural design, energy management, HVAC and cost planning. They all set up their own particular models of the future building. Hence, these models are reproduce only the field of the respective consultant.
All construction processes have in commmon that they are subject to a tremendous cost and time pressure. Therefore, the participants have to work concurrently on their tasks. Some of them even have to base their models on incomplete or preliminary plans. This necessitates large efforts for coordinating different plan versions since some of the partial models affect each other. As a result, the integration of the results from the different fields into the overall plan will lead to inconsistencies. A well-known example for such conflicts is the intersection of the load bearing structure with components of HVAC. In light of the aforementioned time pressure the removal of such errors must be considered a waste of resources.

These problems can be lessened to a certain degree by *Building Information Modeling* (BIM). This technology is a translation of the 'Product Data Models' of mechanical engineering to the field of building construction. The idea behind these models is to incorporate all plans related to a project into a central database. This is done by enriching a purely geometrical model with additional semantic data. These reflect, in a sense, the different views of the consultants on the building. The key factor is that these additional data are *linked* with the underlying geometric model. This allows the automatic derivation of certain properties of the building (for instance floor spaces) directly from the geometry. Furthermore, it allows a reduction of inconsistencies for two reasons. Firstly, the central database provides every involved party at any time with the most current plans of the project. As the building model contains the plans of *all* other fields as well, possible conflicts can be determined easier. Secondly, changes of the geometry of the building will lead to fewer revisions in the specific plans. As the data therein is mostly linked to the underlying geometry it will be automatically updated during changes of the overall model.

The implementation of BIM is provided by the so-called *Industry Foundation Classes* (IFC) that have been established as a standard (ISO 16739) by the *Industry Alliance for Interoperability* (IAI). This standard provides a framework for representation and exchange of data regarding buildings. Despite their promising concept, these classes have only been applied to single major projects rather than having found widespread use. An overview of the reasons for this (with puts the focus on product data models in general but also considers the IFC) is given in [1]: on one hand this is due to a low 'motivation' of the industry to adopt such general models and on the other hand there are legal issues regarding the evidential value of digital documents. Concerning the IFC in particular, one can identify another, more specific reason. It is the insufficient coupling of the structural model with the building model. This fact is related to two features of structural models and finite element discretizations:

1. Structural engineers decompose the actual structure into a set of structural elements like beams, plates and shells. These sub-models are coupled by forces and displacement variables that are transmitted between them. This allows an independent treatment of the structural elements. In addition, these models are often simplified by neglecting certain physical effects or by assuming a particular displacement 'behaviour' [2]. An example for the former is the neglection of time-dependent parameters from an otherwise stationary model.

The advantages of this approach are obvious: the simplified models need less effort for evaluation and their results are easier to interpret. The dimensioning of structural parts is usually based on such simplified models as well. However, this has the effect that some of the structural elements cannot be combined without the definition of additional coupling conditions. This definition must - to the knowledge of the author - still be done manually as it requires insight on the actual problem.

2. The setup of the structural models is nowadays usually done with the FEM. The discretization that is part of this method is mostly done with linear elements. Apart from simple geometries this leads to an approximation of the problem geometry. That is, the numerical model is only *neighbour* to the original problem. This constitutes an bound on the accuracy of the solution that can be obtained from the numerical model.
   Furthermore, the approximation by elements does not allow the reconstruction of the original geometry from the element mesh. This mesh does not convey any information of the underlying, potentially curved geometry. Should adaptions of the FE mesh become necessary, one would have to (at least partially) remesh the model. This includes the definition of the coupling conditions.

A key feature of building processes is the iterative development of the plans. As a result, a large number of revisions are common and large efforts have to be put into revising the structural model. Hence it is understandable that structural engineers rely on simpler models which can be adapted more easily.

1.1 Related Work

Two PhD theses have already been dealing with the integration of the structural planning processes. They have in commmon that they employed the p-version of the FEM together with a volumetric formulation of the mechanical model. The first of these theses focused on the derivation of a suitable numerical models from the building model. Building models are at first a *semantic description* of a building. Therefore they have to be converted into a geometric model. In [3] the IFC model has been converted into a *Boundary-representation* model (BRep). This model then had to be corrected, i.e. it contained gaps as well as intersections that had to be removed. The BRep model was then decomposed into a so-called 'connection model' which consisted of *difference objects* connected by *connection objects*. These objects have a hexahedral shape or can be created by sweeping a planar polygonal domain [4]. Hence, they can then be meshed easily using hexahedral elements.
In order to overcome the numerical problems that resulted from distorted elements, solid elements with higher order ansatz functions were used. This combination is not only suitable for simulating 'thick' solids but also for the representation of thin-walled structures [2]. It is obvious, though, that the use of a fully volumetric formulation of the problem leads to a higher computational effort for the solution of the numerical model.
This problem has been treated in [5] by exploiting the connection model that is part of the meshing stage. This connection model is actually a graph that represents the structural interdependency of the components. It can also be used for controlling a recursive substructuring of the model. That idea eventually allowed the evaluation of the stiffness properties on the component level. These 'partial stiffnessses' were then combined into the system stiffness matrix by using the hierarchical dependence between the components. This approach allowed for an efficient treatment of local modifications of the structure. Instead of recomputing the stiffness matrices of all parts, only the stiffness of the modified components had to be considered. The system

stiffness matrix could then be assembled by reusing most of its values which lead to tremendous time savings.
Furthermore, this hierarchical system of components has also been used to setup a framework for concurrent work on a central building model. Using principles of relational databases as well as a client-server-architecture, the author demonstrates the feasibility of concurrent access to the overall model. These ideas are exemplified by two applications. The first application is a building process simulation with an embedded numerical simulation. The second example is an application that bases dimensioning of reinforced concrete parts on the results of a volumetric analysis.
This proves the feasibility of a fully volumetric approach to building models. Still, these models contain a notable discretization error that must be reduced by suitably refining the element mesh. The sole use of hexahedral elements based on the p-version of the FEM seems not to be suffient as well [5].

There is an alternative approach that promises to overcome these problems. It is based on the *Isogeometric analysis* [6, 7]. The idea behind this method is to base both the geometric description and the shape functions of the FEM on *Non-uniform rational B-splines* (NURBS)[1]. They are an ubiquitous tool in CAD and computer graphics and provide a general notation for representing a broad range of geometries. This includes straight lines, curves and complex curved surfaces and, most importantly, even allow the representation of solids. The key feature of the isogeometric analysis is the *exact* transfer of an objects geometry to a finite element. A numerical model based on NURBS would represent the actual problem geometry much better that the piecewise linear or quadratic approximations that are ususal for finite element discretizations - leading in principle to better results.

An outline of the properties of NURBS relevant to isogeometric analysis will be given in section 2. For further details the reader is referred to the two standard books [8, 9] which deal with NURBS from the view of computer graphics. A treatment of NURBS from the numerical analysts view has so far only been done in a growing number of papers. A monograph on isogeometric analysis is scheduled to appear in autumn of this year.
Some of the intricacies that are implicated by the use NURBS as shape functions for the FEM will be described in section 3. This includes for example the numerical integration of the element stiffness matrices as well as the computation of the partial derivatives of these functions, which are important for setting up the strain-displacement relationships for the elements.
Section 4 then concludes the article with a summary of the steps that are necessary for a successful coupling of building models with structural design.

[1]This is not quite true: NURBS functions provide but one possibility. [6] mentions alternative functions that seem also suitable for the isogeometric concept - NURBS were chosen because they represent the most mature of these concepts

2 NURBS-BASED DESCRIPTION OF GEOMETRIES

An geometric description based on NURBS consists of a set of basis functions and corresponding *control points*. The actual geometry of a curve or surface then results from the interpolation of the control points' coordinates. In the one-dimensional case, these points form a so-called control polygon, whereas in the two- and three-dimensional case they constitute (in terms of topology) a grid of points.
The elements that determine this interpolation are the NURBS functions, which are piecewise rational functions defined on a parametric domain Ω_N. By iterating over the domain, one determines for each parametric coordinate the contribution of all control points to the curve. NURBS-based geometry can therefore be interpreted as some sort of mapping $G : \Omega_N \rightarrow \mathbb{R}^n$.

A NURBS curve $\mathcal{C}$ is defined by

$$\mathcal{C}(\xi) = \sum_{i=0}^{n} R_{i,p}(\xi)\, \mathbf{P}_i \tag{1}$$

where ξ is the current parametric coordinate in Ω_N, $\mathbf{P}_i$ are the $(n+1)$ control points and $R_{i,p}$ are the corresponding basis functions of degree p. A NURBS-based surface - a *patch* - is defined in a similar way by the mapping from a two-dimensional parametric domain into $\mathbb{R}^n$:

$$\mathcal{P}(\xi,\eta) = \sum_{i=0}^{n}\sum_{j=0}^{m} R_{i,p}(\xi)\, R_{j,q}(\eta)\, \mathbf{P}_{ij} = \sum_{i=0}^{n}\sum_{j=0}^{m} R_{ij,pq}(\xi,\eta)\, \mathbf{P}_{ij} \tag{2}$$

with (ξ,η) being the coordinates of a point in Ω_N. The bivariate basis functions $R_{ij,pq} = R_{i,p}R_{j,q}$ are computed by the product of the two univariate basis functions. This allows for a different number of control points in each parametric direction as well as for different degrees p, q of the basis functions.
A solid is defined accordingly as

$$\mathcal{S}(\xi,\eta,\chi) = \sum_{i=0}^{n}\sum_{j=0}^{m}\sum_{k=0}^{l} R_{ijk,pqr}(\xi,\eta,\chi)\, \mathbf{P}_{ijk} \tag{3}$$

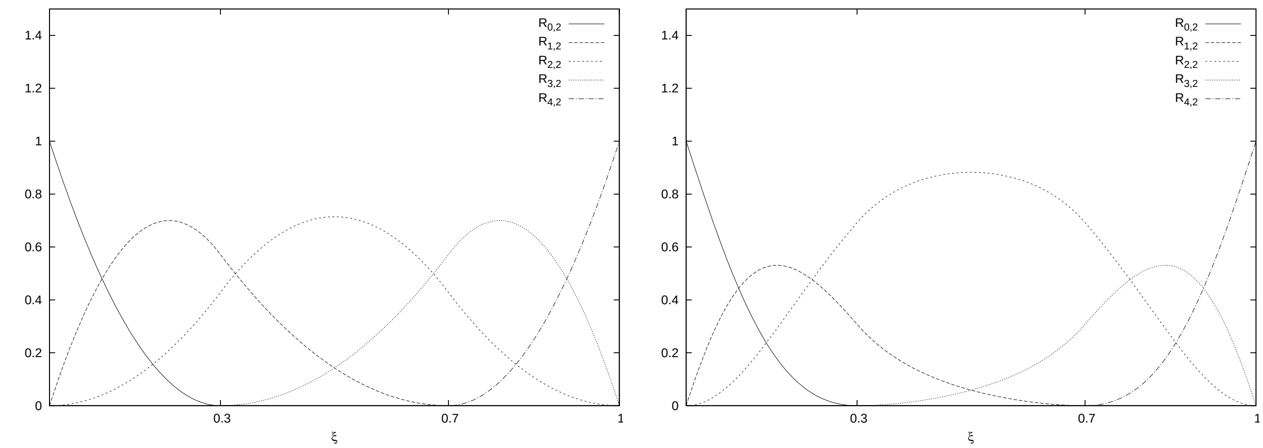
with the basis functions $R_{ijk,pqr}$ resulting from the triple product of univariate bases. Again, this formulation allows both different numbers of control points and degrees of the bases. The disadvantage of this representation is the severe overhead that is induced for displaying simple bodies, for example bodies having straight edges and/or planar surfaces. In order to describe, for instance, a simple cube by basis functions of higher degree, one has to provide a *full spatial grid of control points* instead of eight vertices for the corners of the cube.

2.1 Basis functions

The univariate basis functions $R_{i,p}$ that are needed for the interpolations of the control points are built from the combination of B-spline functions $N_{i,p}$ with a set of weighting factors:

$$R_{i,p}(\xi) = \frac{N_{i,p}(\xi)\, w_i}{\sum_{j=0}^{n} N_{j,p}(\xi)\, w_j} \quad , \; w_i > 0 \; \forall i \tag{4}$$

The weights w_i affect the influence of the control points on the curve as can be seen in figure 1. The geometrical motivation for the introduction of these weights as well as for the rational form of the basis functions can be taken from [8, 9].



$\mathbf{W} = \{1.0, 1.0, 1.0, 1.0, 1.0\}$ $\mathbf{W} = \{1.0, 1.0, 3.0, 1.0, 1.0\}$

Figure 1: The influence of the weights on the NURBS basis functions. The images show the quadratic basis functions that result from the knot vector $\Xi = \{0, 0, 0, 0.3, 0.7, 1, 1, 1\}$. For the image on the right-hand side, the weight w_2 that is associated with $R_{2,2}$ has been increased

The B-Spline functions $N_{i,p}$ can then be evaluated very efficiently by employing the following recurrence relation by Cox/de Boor

$$
\begin{aligned}
N_{i,0}(\xi) &= \begin{cases} 1 & \text{for} \quad \xi \in [\xi_i, \xi_{i+1}) \\ 0 & \text{else} \end{cases} \\
N_{i,p}(\xi) &= \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)
\end{aligned} \tag{5}
$$

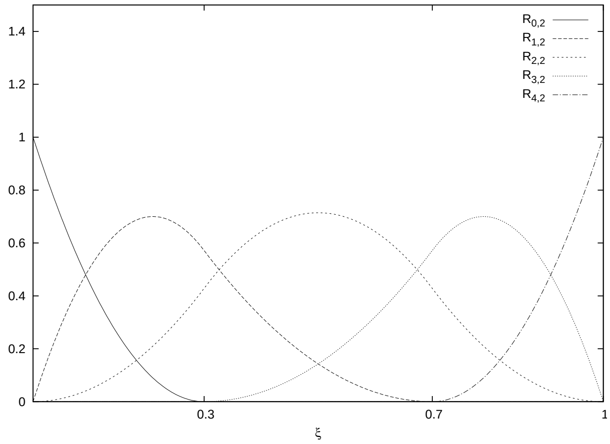
Key element for the computation is the so-called *knot vector* Ξ which is a set of parametric coordinates. These coordinates denote the limits of the *knot spans*, i.e. segments in the parametric domain that will be mapped to curve or surface segments. Depending on their degree each basis function is nonzero only over certain knot spans. The dependence of the basis functions on the length of the knot spans can be seen in figure 2.

As mentioned before, NURBS functions possess some properties being useful for an application as shape functions. Only the most important properties are listed below. For the details the reader is referred to [6, 8, 9, 10].

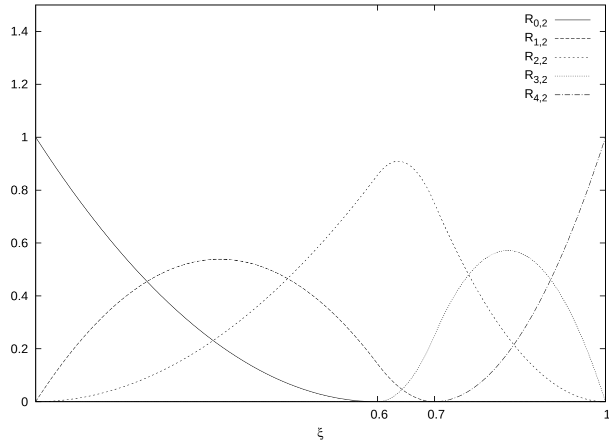
1. They are non-negative:
$$R_{i,p}(\xi) \geq 0 \quad \forall i, p, \xi \in \Omega_N \tag{6}$$

2. They constitute a *partition of unity*:
$$\sum_i R_{i,p}(\xi) = 1 \quad \forall i, p, \xi \in \Omega_N \tag{7}$$

3. The basis functions are C^0-continuous at the boundaries of the parametric space and possess, in general, higher continuity within the domain. The continuity depends on the multiplicity of the knots (i.e. the entries in the knot vector): multiple knots decrease the continuity of the basis functions at the respective knot to C^{p-k} where k is the multiplicity of the considered knot.



$\Xi = \{0, 0, 0, 0.3, 0.7, 1, 1, 1\}$



$\Xi = \{0, 0, 0, 0.6, 0.7, 1, 1, 1\}$

Figure 2: The influence of the knot spacing on the NURBS basis functions. The images show the quadratic ($p = 2$) basis functions that result from the knot vector shown below the respective image. The influence of the knot spans on the basis functions is demonstrated by the image on the right-hand side, where the knot at $\xi = 0.3$ has been shifted to $\xi = 0.6$

4. The basis functions possess a local support: $R_{i,p}(\xi) = 0 \quad \text{for} \quad \xi \notin [\xi_i, \xi_{i+p+1})$. In any given knot span at most $(p + 1)$ basis functions are nonzero. It is especially this property that enables an efficient evaluation of the basis functions. An algorithm exploiting this property can be found in [8].

3 NURBS-BASED ELEMENTS FOR ISOGEOMETRIC ANALYSIS

Finite elements based on NURBS functions possess some characteristic differences compared to 'classical' finite elements. First of all, NURBS patches do not constitute single elements but rather *subdomains*. Depending on their parametrization (as given by the knot vectors) they can include several 'ordinary' elements. The one-dimensional parametrizations shown in figure 2 would, for example, constitute three elements over the respective knot spans $[0, 0.3)$, $[0.3, 0.7)$ and $[0.7, 1)$. The higher continuity of the basis functions has a beneficial effect on the numerical integration. The direct translation of the Gaussian quadrature onto NURBS-based elements would lead to an application of the the common integration schemes *not to each patch but to each knot span*. The integration over a panel element that is based on quadratic basis functions and which has nine patch segements would need $2 \times 2 \times 9 = 36$ integration points. However, the actual number of integration points can be, as [11] suggests, much smaller. According to the findings therein the number of integraion points seems to be independent from the degree of the functions. That means that the numerical integration does not need to be done over each knot span but can be done on the patch level.

Besides the numerical integration, the nature of the basis functions has another impact on the evaluation of the stiffness matrix. The stiffness matrix $\mathbf{K}_e$ of a patch is, similar to classical finite elements, computed by the integral

$$\mathbf{K}_p = \int_{A_p} \mathbf{B}^T \mathbf{E} \mathbf{B} \, dA_p = \int_{\Omega_N} \mathbf{B}^T \mathbf{E} \mathbf{B} \, J \, d\xi \, d\eta \tag{8}$$

with A_p being the area of the patch, $\mathbf{B}$ being the strain-displacement-matrix, $\mathbf{E}$ being the material matrix and with J as the Jacobian determinant. For this article the evaluation has been done for panel elements. This leads to the following kinematic equation:

$$\boldsymbol{\epsilon} = \begin{bmatrix} u_{,x} \\ v_{,y} \\ u_{,y} + v_{,x} \end{bmatrix} = \begin{bmatrix} (\cdot)_{,x} & 0 \\ 0 & (\cdot)_{,y} \\ (\cdot)_{,y} & (\cdot)_{,x} \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{9}$$

where $[u, v]^T = \sum_i R_{i,p}(\xi, \eta) [u_i, v_i]^T$ is the vector that describes the displacements at a given point (ξ, η). The Jacobian matrix can be computed as usual by deriving the patch coordinates

$$\mathcal{P}(\xi, \eta) = \begin{bmatrix} x(\xi, \eta) \\ y(\xi, \eta) \end{bmatrix} = \sum_{i=0}^{n} \sum_{j=0}^{m} R_{ij,pq}(\xi, \eta) \, \mathbf{P}_{ij} \tag{10}$$

with respect to the parametric coordinates (ξ, η) and inverting the resulting matrix of partial derivatives. However, since NURBS are rational functions the computation of their derivatives is more complicated than for polynomial functions. Higher derivatives become increasingly unstable (in terms of numerical evaluation) due to the higher exponents in the denominator. To the knowledge of the author, there are no algorithms for computing the derivatives of the rational basis functions. There exist schemes for computing tangents to NURBS curves and surfaces (c.f. [8, 9]. Still, they seem not to be applicable for computing the derivatives of the basis functions itself. This issue demands further research. However, the computation of the derivatives using a central difference scheme appears to be a good approach since it relies on the evaluation of the basis functions which can be done very efficiently.

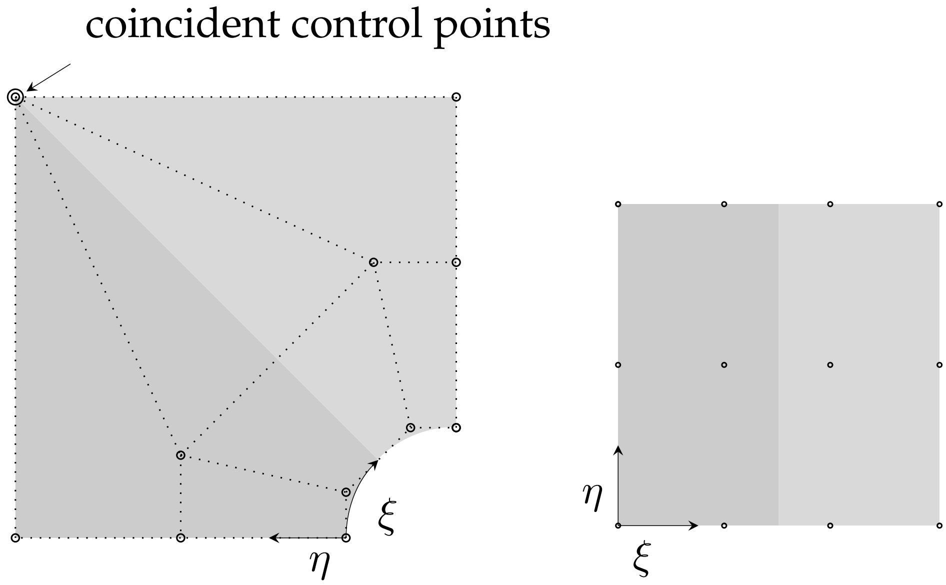


Figure 3: Panel modelled by a single NURBS patch that consists of two elements and which is constituted by a 4×3-grid of control points. The knot vectors are given as $\Xi = \{0, 0, 0, 0.5, 1, 1, 1\}$ and $\mathbf{H} = \{0, 0, 0, 1, 1, 1\}$ which leads to biquadratic basis functions. The image on the right-hand side shows the parametric domain with the (approximate) locations of the control points superimposed. This domain is then mapped onto the twodimensional panel shown on the left-hand side. In order to obtain the corner in the upper left-hand side, two control points have to be coincident. Example taken from [6]

4 CONCLUSIONS AND FUTURE WORK

The Isogeometric analysis is a relatively new concept for the unification of geometric modelling and numerical simulation. As a growing number of publications suggest, this method possesses some advantegeous numerical properties. One of these is the exact transfer of a geometrical model into a finite element mesh, thus reducing or eliminating the discretization error of the numerical models. A number of mature algorithms like knot insertion or degree elevation, that were developed for manipulating NURBS-based geometry, can be employed for refining the patch meshes [6, 8]. The application of these algorithms on NURBS-based elements constitutes the equivalents of both h- and p-refinement.
However, one point that has not been regarded is the source of the geometric models. These must be based on NURBS, and although all professional CAD applications support NURBS, they do not in general produce fully NURBS-based geometric models. The generation of a patch model is not a trivial task, since the mapping of the parametric domain onto the geometry is not always a one-to-one relation. A simple example for this is shown in figure 3. Given a grid of control points together with weights and knot vectors, it is easy to obtain the geometry and set up finite elements. The problem is the determination of the control grid along with the related parameters (control weights and knot vectors) for a desired ‘target shape’. While this can still be done manually for smaller models, it becomes increasingly difficult for larger, more complex systems.

This model generation is the core task for establishing a coupling between the IFC and structural design. The semantic descriptions of the building model must be translated into complex, patch-based models. One may argue that usual buildings are only composed of simple shapes like plates, walls and beams. The point, though, is the coupling of these structural elements. The control grids of joining NURBS patches and solids have to be aligned along

the join and their basis functions of equal degree. In order to maintain continuous joints, i.e. with no gaps or overlapping, one would need to introduce a lot of superfluous control points. This hinders the possibilities for coupling as well as for local refinement of models, as inserted control points in one patch of the model have to be regarded in the control grids of all other patches as well.
Another important aspect is the coupling of the structural model with the building model. As should have become clear now, the setup of a structural model is, in general, possible. What has not been treated so far is the following issue: which results of the structural design have to be incorporated into the building model?
It is this feedback that determines the success of the proposed approach.

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