59th ILMENAU SCIENTIFIC COLLOQUIUM
Technische Universität Ilmenau, 11 – 15 September 2017
URN: urn:nbn:de:gbv:ilm1-2017iwk-148:6

STIFFNESS INVESTIGATION OF PNEUMATIC CYLINDERS

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ABSTRACT

Pneumatic cylinders are widespread used in industrial machines these days. They are invented in various mechanisms like manipulators, and in tasks with higher force effort because of their reliability and robust design. The low level of stiffness is a significant disadvantage of pneumatic cylinders which arises from the compressibility of the air in the chambers. This property is typical for all pneumatic equipment, but using a servopneumatic system this is a significant disadvantage. Servo-pneumatic systems are unique in that they can position the piston at any point in the stroke length. With a sufficiently large disturbance, the piston does not remain in the desired position, but will displace, and returns to the starting point after the end of the disturbance. Of course the displacement can be controlled by the controller of servopneumatic systems, but the phenomenon of recovering from air compressibility characteristically remains. The displacement itself, compared with mechanical constructions, can be interpreted as a kind of flexible deformation, and the flexibility is caused by compressed air in the cylinder chambers.
The main contribution of the paper is to determine the resulting stiffness, and the restoring force of the air in the pneumatic cylinder chambers using laws of mechanical engineering.

1. INTRODUCTION

The use of technical devices is affected by their stiffness. In comparison with the commonly used technical equipment like hydraulics and electric drives the low stiffness of the pneumatic devices is well known. This phenomenon is also characterizing the double acting cylinder.
Therefore, in most cases, the pneumatic actuators are positioned at the two endpoints of their stroke. However, by servopneumatic systems, the piston can be set in any position of the stroke length. These systems have usually a rodless double acting cylinder. The construction of a rodless cylinder, for which the force lead can be achieved by a rip along the cylinder pipe and by specially designing the piston, is shown on Figure 1.

Fig. 1. Structure of a rodless pneumatic cylinder

If a double-acting cylinder piston is pushed, from its resting position, there appear a restoring force against the displacement. This force is intended to keep the pneumatic cylinder in the resting position, which depends on several factors. In case of small forces within the stiction force range of the piston, the displacement of the piston is affected by the elastic deformation of the lip seal between the cylinder and the piston (Fig. 2). For the common used cylinder size and pressure range a restoring force is in order of magnitude 10N, and produce relative displacement less than one tenth of a millimetre. This is the so called pre-sliding region.

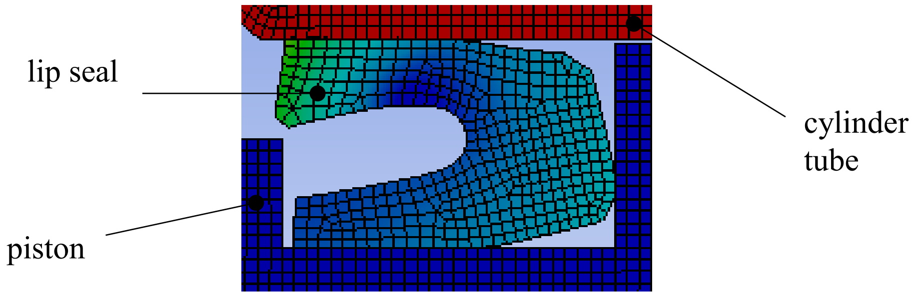

Fig. 2. Deformation of the lip seal between the cylinder and the piston.

For larger displacement force the adhesion disappears between the seal and the cylinder tube, the piston begins to move. However, this movement will slow down after a while, in spite of the existing displacement force. This is due to the compressibility of the air. In this paper, we look for the correlations of this behaviour.

2. FLEXIBILITY OF AIR IN CYLINDER CHAMBERS

Double acting pneumatic cylinders have two compressed air chambers. The compressed air of both chambers is a kind of flexible intermediate element. This pneumatic-mechanical analogy was recognized in several places [1], [2] where the compressibility of the air was replaced by an air-spring (Fig. 3.) Due to analogy the moving velocity points of the outer ends of the springs can be interpret to the charging and discharging mass flow of the chambers.

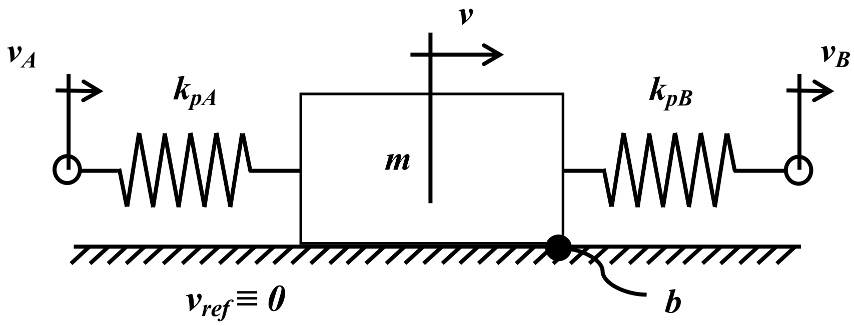

Fig. 3. Pneumatic cylinder analogue mechanical model

3. SIMILARITIES TO STRENGTH OF MATERIALS

At a given force induced to a material or structure it is the stiffer the smaller its deformation. The elasticity modulus or the Young modulus E is used to characterize the elasticity and stiffness in the theory of strength. This is the descriptive correlation of the specific deformation ε due to the material's load σ:

$$E = \frac{\partial \sigma}{\partial \varepsilon} \tag{3.1}$$

The Hook's law, which is well-known, states, that the stress of a body to the strain
in the elastic range is load independent within the linear elastic range of most materials (e.g. a significant part of metals) to describe the relationship between load and deformation. However, it is known as bulk modulus *B* instead of Young's-modulus *E*. In case of gases as in case of rubbers the modulus of elasticity depends not only on the material properties but also on the load, therefore further analysis is needed.
The transformation of the equation **(3.1)** allows a derivation of valid connection to pneumatic systems. For the calculation of tensile or compressive stress, in the nominator of the correlation is the load force F in the denominator the cross-section *A*:

$$\sigma = \frac{F}{A} \tag{3.2}$$

In the relation **(3.2)** the stress σ corresponds with piston cross section area *A* and the piston acting force *F* to pressure *p* in case of a pneumatic cylinder.
The denominator of equation **(3.1)** is the fractional extension or strain:

$$\varepsilon = \frac{\Delta l}{l_0} \tag{3.3}$$

For the pneumatic cylinder, ε corresponds to the specific length change of the chamber with length *x* and displacement Δx, so the equation **(3.1)** can be written:

$$B = x \cdot \frac{\partial p}{\partial x} \tag{3.4}$$

The relation describes the change of the state of the gases is also suitable for describing the mixed state between adiabatic and isothermal condition depending on the polytropic exponent n. It is important to mention this because adiabatic behaviour only applies to very short-term changes that goes over time to an isothermal change in the state of the ambient temperature. In the polytropic state of the gas, the volume of the chamber *V* is written by piston cross section *A* and chamber length *x*:

$$p \cdot V^n = p \cdot A^n \cdot x^n = const. \tag{3.5}$$

The relationship **(3.5)** derivate with displacement, i.e. changes in the pressure *p* of the chamber by position *x*, the bulk modulus *B* can be expressed:

$$B = n \cdot p \tag{3.6}$$

Equation **(3.6)** shows that the bulk modulus of the pneumatic system (chamber) is not constant, but is proportional to the pressure.

4. PRESSURE CHANGE AS RESTORING FORCE

Holding the required position of the pneumatic cylinder could mean that no intervention of the cylinder valves is needed they can be stayed in closed position. Based on this, the chambers of the cylinder can be considered as a closed system. To determine the resulting spring stiffness of the chambers in the stroke length of the cylinder, it is advisable to prescribe the pressure variations for each chamber separately for small displacement Δx. It is known that there are dead volumes outside the stroke length of the working cylinder, caused by the inner volumes of the cylindrical caps, connectors and tubes. The volume of the compressed air of the chambers should also be taken into account. The state change of the compressed air of the chambers these small volumes should also be taken into account, even if the piston is not able to reach these theoretical points. This range is indicated by a dashed line drawn on the cylinder cups as it shown on Fig 4. The thickness of the cylinder piston must also be taken into account. These considerations are taken together by the extended cylinder stroke length l_k.

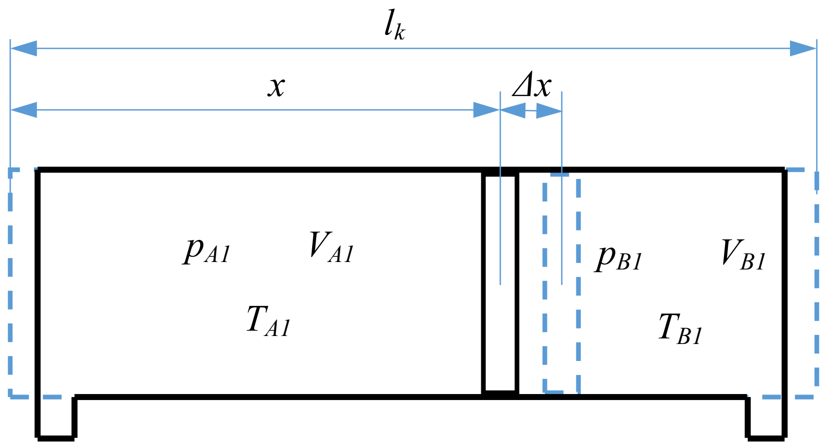

Fig. 4. Principal abbreviation of pneumatic cylinder with state variables and extended cylinder stroke length

Holding the cylinder piston in position with compressed air, it has a little resistance against small displacement, but the restoring force due to increasing displacement of the chambers increases too. The stiffness of the chambers is written for the polytropic equation **(3.5)**, assuming small displacement Δx.

$$p_{A1} \cdot A_A^n \cdot x^n = p_{A2} \cdot A_A^n \cdot (x + \Delta x)^n \qquad (4.1)$$

$$p_{B1} \cdot A_B^n \cdot (l_k - x)^n = p_{A2} \cdot A_B^n \cdot (l_k - x - \Delta x)^n \qquad (4.2)$$

In the case of the prescribed relations, the piston cross-section of the chambers does not change, so it can be simplified. In addition to simplification, it is important to note that the chamber pressures themselves do not depend on the piston cross-section. Accordingly, the correlation of the change in chamber pressures is valid for both with piston rod and for rodless cylinders. In contrast, the restoring force is not independent of the effective cross-section, but is proportional to the pressure and the piston cross-section. The goal is to express the pressure change of the chambers as a function of the displacement of the piston, derived from **(4.1)** and **(4.2)**, where the pressure at the equilibrium point for chamber "A" is p_{A1}, and after the displacement p_{A2}, respectively p_{B1}, and p_{B2} for chamber "B":

$$\Delta p_A = p_{A2} - p_{A1} = p_{A1} \cdot \left[\frac{x^n}{(x+\Delta x)^n} - 1\right] \tag{4.3}$$

$$\Delta p_B = p_{B2} - p_{B1} = p_{B1} \cdot \left[\frac{(l_k - x)^n}{(l_k - x - \Delta x)^n} - 1\right] \tag{4.4}$$

The right side of the equations is brought to common denominator:

$$\Delta p_A = p_{A2} - p_{A1} = p_{A1} \cdot \frac{x^n - (x+\Delta x)^n}{(x+\Delta x)^n} \tag{4.5}$$

$$\Delta p_B = p_{B2} - p_{B1} = p_{B1} \cdot \frac{(l_k - x)^n - (l_k - x - \Delta x)^n}{(l_k - x - \Delta x)^n} \tag{4.6}$$

Combining the counters of the relationship $(x + \Delta x)^n$ can be approximated by using its Taylor series:

$$f(x) = (x+\Delta x)^n = x^n + n \cdot x^{n-1} \cdot \Delta x + \frac{n \cdot (n-1) \cdot x^{n-1}}{2} \cdot \Delta x^2 + \cdots \tag{4.7}$$

Using the first and second members of the series (the error of the line can be estimated by the rest terms of the series):

$$\Delta p_A = p_{A1} \cdot \frac{x^n - (x^n + n \cdot x^{n-1} \cdot \Delta x)}{(x+\Delta x)^n} = -p_{A1} \cdot \frac{n \cdot x^{n-1} \cdot \Delta x}{(x+\Delta x)^n} \tag{4.8}$$

$$\begin{aligned} \Delta p_B = p_{B1} \cdot \frac{(l_k - x)^n - [(l_k - x)^n + n \cdot (l_k - x)^{n-1} \cdot \Delta x]}{(l_k - x - \Delta x)^n} = \\ = p_{B1} \cdot \frac{n \cdot (l_k - x)^{n-1} \cdot \Delta x}{(l_k - x - \Delta x)^n} \end{aligned} \tag{4.9}$$

In the denominator, Δx is small for both x and l_k-x, so in the denominator$(x + \Delta x)^n = x^n$, respectively$(l_k - x - \Delta x)^n = (l_k - x)^n$.

$$\Delta p_A = -p_{A1} \cdot \frac{n \cdot x^{n-1} \cdot \Delta x}{x^n} = -p_{A1} \cdot n \cdot \frac{\Delta x}{x} \tag{4.10}$$

$$\Delta p_B = p_{B1} \cdot \frac{n \cdot (l_k - x)^{n-1} \cdot \Delta x}{(l_k - x)^n} = p_{B1} \cdot n \cdot \frac{\Delta x}{(l_k - x)} \qquad (4.11)$$

As expected from the equations **(4.10)** and **(4.11)** can be shown that the displacement of the piston results in one chamber decreasing pressure in the other chamber increasing pressure.

$$\Delta p_F = \Delta p_B - \Delta p_A = p_{B1} \cdot n \cdot \frac{\Delta x}{(l_k - x)} + p_{A1} \cdot n \cdot \frac{\Delta x}{x} \qquad (4.12)$$

After a mathematical transformation of **(4.12)**:

$$\Delta p_F = \Delta p_B - \Delta p_A = n \cdot \Delta x \cdot \frac{p_{A1} \cdot (l_k - x) + p_{B1} \cdot x}{x \cdot (l_k - x)} \qquad (4.13)$$

If both chambers have the same initial pressure ($p_{A1}=p_{B1}$):

$$\Delta p_F = n \cdot \Delta x \cdot \frac{p_{A1} \cdot (l_k - x) + p_{A1} \cdot x}{x \cdot (l_k - x)} = p_{A1} \cdot n \cdot \Delta x \cdot \frac{l}{x \cdot (l_k - x)} \qquad (4.14)$$

The recovering pressure relative to the initial pressure of the chambers can be expressed:

$$\frac{\Delta p_F}{p_{A1}} = n \cdot \Delta x \cdot \frac{l}{x \cdot (l_k - x)} \qquad (4.15)$$

The equation **(4.15)** gives information on the relative recovering pressure for small displacements.

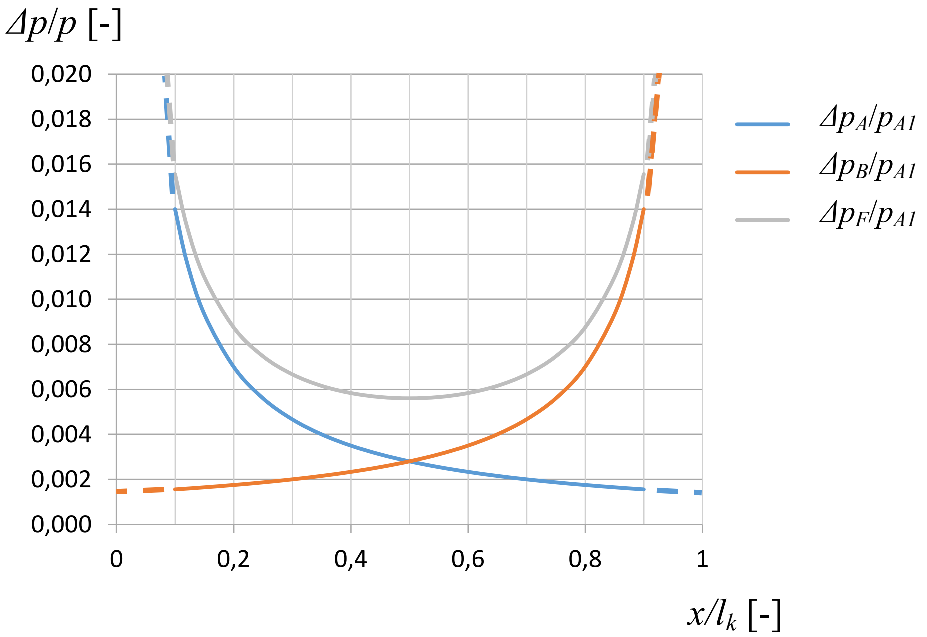

Fig. 5 The relative restoring pressure (Δp/p) depending on the relative length of the stroke (x/l) in case of a displacement relative to the stroke length of 0,1%.

Accurately reflecting the reality is not possible because depending on the heat exchange of the gas in the chambers, the state change may vary from isotherm to adiabatic (n=1.. 1,4). To describe larger movements, this approximation is not suitable. It would only be possible to calculate by the exact exponential function or by the calculation of its sums with more terms of the series. As it is shown on Fig. 5 in case of a relative displacement 0.1% offset which equal for example to 1mm displacement of a piston of a cylinder having an extended stroke length of 1m. The dashed line represents the non-existing parts of the extended stroke length caused by dead volumes.
The final relationship **(4.16)** determines the recovering force of compressed air for chambers with the same initial pressure on both sides of the piston for rodless pneumatic cylinders.

$$\Delta F = \Delta p_F \cdot A = p_{A1} \cdot A \cdot n \cdot \Delta x \cdot \frac{l_k}{x \cdot (l_k - x)} \qquad (4.16)$$

However if there is a need beside of positioning of the piston also to excite the prescribed force F, it can only be created by the pressure differential of the chambers. In this case, obviously $p_{A1} \neq p_{B1}$, and this means that the simplification in **(4.14)** not possible. The recovering force can be determined only as it is shown in **(4.13)** depending the chamber pressures.

5. CONCLUSION

From the investigation several consequences can be deduced. First the elastic modulus of the pneumatic systems is not constant, it is proportional to the pressure. This means that servopneumatic systems should be operated as high pressure as possible for increasing their stiffness. Second, the stiffness of pneumatic systems is comparable to liquid systems calculating the elastic modulus. It is known that the bulk modulus of fluids is $\sim 10^9$ Pa and the atmospheric pressure ~ 1 bar = 10^5 Pa. It follows that the pneumatic systems would have the elasticity modulus of liquids at 10.000 times the atmospheric pressure, they would be able to provide the stiffness of liquid systems. Third the equations have shown that change in chamber pressure is valid for both with piston rod and rodless cylinders. Further conclusion is that the piston recovering force does not depend on the displacement but on the relative displacement of the chamber.

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