

# **Beyond risk and return modeling - How humans perceive risk**

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## **Abstract**

The intention of this paper is to show that the statistical approach to risk is not enough to explain the behavior of investors. It furthermore proposes ideas and alternative approaches on how to deal with risk. Psychological findings are of particular interest as they might enhance our understanding of risk perception and assessment.

The chapter “From the normal distribution to fat tails” starts with the rejection of the normal distribution as a simplifying basis for risk and return. This rejection is supported by several empirical observations like clustering of volatility and fat tails. This leads to a two-step approach for modeling risk and return based on the distinction of conditional and unconditional changes. Conditional time series models (ARMA, ARCH, GARCH) and alternative distributions are presented (Stable Paretian, Student’s T, EVT) as a way to improve the art of risk and return modeling beyond the normal distribution assumption. The chapter ends with the conclusion that each model is only a statistical approximation and never encompasses the unpredictability of black swans and the nature of human behavior in the financial markets.

After having discussed the limitations of the purely statistical approach to risk and return this paper goes beyond the standard theory of finance for two purposes. Firstly, behavioral finance provides some arguments for the limitation of statistics in assessing risk. Secondly, an alternative approach to risk perception is presented. This alternative is called Prospect Theory, a rather psychology-based approach using preferences to explain investors’ actions by human behavior in decision making processes. Starting point is the utility function and the value function followed by a description of the two phases: framing and evaluation. The value function is then clearly distinguished from the utility function by elaborating certain effects like reference points, loss aversion or the weighting function. In this section the paper enters the arena of human risk perception which is far from being monetarily rational in the sense of the homo oeconomicus. With Cumulative Prospect Theory there exists an extension to multiple outcome scenarios where risk does not necessarily

have to be known. In such a situation, besides risk, there also exists immeasurable uncertainty. Current research confirms and rejects parts of (Cumulative) Prospect Theory which is not necessarily a bad sign as human behavior is rarely exactly replicable and the complexity does not really allow generalizations. Therefore, even if the theory is not completely correct it still enhances our understanding of risk perception and human decision making which can be a very valuable input for agent-based models.

The next chapter analyses in more detail possible distortions from psychological biases in the assessment of risk. In this context the law of small numbers, overconfidence and feelings/experience are discussed. Knowing these biases complicates the idea of developing a risk model even further. However, this is again another step to better understand the underlying processes and motives of decision making in the context of financial markets.

The last chapter is an attempt to link the different aspects to get a holistic view on risk behavior. Two possibilities are discussed: Hedonic psychology, with the distinction between blow up and bleeding strategy, and heuristic-based explanations for real observations like clustering of expectations and trust in experts.

This leaves space for further research as we do not have a tool that is based on current findings and can actually help us in explaining and predicting behavior in financial markets. One possibility would be to link all these aspects in the approach of computational finance to develop agent-based models in which market observations, psychological findings and the situational context can be integrated.

## 1. Introduction

The financial crisis of 2008 has demonstrated again that our current understanding of risk is not sufficient in order to measure risk properly, and hence prevent large losses in the financial market.

Literature offers several perspectives on how to measure risk and on how individuals perceive risk. On the one hand there is an ongoing discussion about the distribution of returns and risk in financial markets. Academics try to model these distributions in order to gain a better understanding how returns and risks are distributed and can be measured. These efforts can be seen as an extension to the neoclassical linear capital market models, which did provide a measure of risk based on the normal distribution but which have been rejected by empirical observations. On the other hand, behavioral finance offers a range of explanatory aspects on how individuals perceive risk and act upon it. These two approaches have been coexisting for the past decades but there has been little effort to integrate them in order to offer a more holistic picture of risk perception and measurability.

This paper starts with the chapter “from the normal distribution to fat tails”. In this chapter, we first present some empirical observations invalidating the normal distribution as a suitable assumption to model risk and returns. Next, we will introduce better measures of risk and returns from research developments in the field of risk and return modeling, namely the combination of time-series models and fat-tailed distributions. Chapter 2 concludes that the normal distribution is not suitable for risk and return measurement and that current research in this field is hitting some borders, where more complex and complicated extensions and models are hardly providing any new evidence or explanatory power to our current understanding of risk and returns. Therefore we present some emerging research fields, which might be valuable for future developments. One of these emerging research fields is the work with artificial agents to simulate market movements. If one wants to simulate individual behavior in a market setting by using agents, it must be ensured that these agents exhibit observable human behaviors.

Human behavior is at the core of behavioral psychology and behavioral finance. Behavioral psychology and finance are not only concerned with human behavior but also with the underlying human decision making. In order to better understand human decision making, chapter 3 first explores the roots of decision making theories by presenting past developments in the field of utility functions, as utility functions are in neoclassic theory the basis of any decision making processes. This subchapter ends with the presentation of Kahneman and Tversky's Prospect Theory's value-function. Afterwards, Prospect Theory will be introduced in some more details, whereas we present the value and the weighing function and related biases. Having explained Prospect Theory, we will continue with Cumulative Prospect Theory, focusing on the additional explanatory power by Cumulative Prospect Theory over original Prospect Theory. Subsequently, we provide a summary of some critiques to the (Cumulative) Prospect Theory and conclude the chapter with the idea that Cumulative Prospect Theory might still have some drawbacks, however that it is the most explanatory theory in decision making research so far, as it is general enough, still adheres to many empirical observations, has at least some predictive power and is yet not too complex. By making the theory more complex, some other empirical observations might be able to be included, however, we question if this extra complexity is worth the effort, especially because the theory might lose some of its predictive power.

In Prospect Theory, Kahneman and Tversky distinguish between effects attributed to Prospect Theory and other cognitive biases. We follow their differentiation and present cognitive biases such as the law of small numbers, overconfidence as well as feelings in the perception of risks in our chapter 4 "probability distortions". Cognitive biases have an important influence on the assessment of probabilities and therefore on the assessment of risk. If the ultimate goal is to model human decision making and risk perception, cognitive biases are a very important component to this.

In our last chapter 5 we will focus on linking and combining the aforementioned fields to provide a more holistic picture on risk perception and measurability.

## 2. From the normal distribution to fat tails

In the past decades, there has been a growing number of alternatives to the former assumption that risk and returns follow a normal distribution (e.g. Mandelbrot, 1963a; Stoyanov et al., 2011). Nowadays, there seems to be universal agreement on risk and returns following fat-tailed distributions (Rachev et al., 2010). In the following, critiques to the normal distribution are presented in chapter 2.1 first. Second, chapter 2.2 presents how risk and returns are recently modeled. The chapter ends with a discussion and an outlook based on recent developments in risk modeling in chapter 2.3.

### 2.1 The Normality Assumption and Empirical Phenomena

The idea that risk and returns follow a normal distribution originates from neoclassical theory<sup>1</sup>. The advantage of using the normal distribution is that it offers convenient mathematical properties, as risk is defined as variance of the expected returns. However, this convenience leads to a simplifying assumption, as the normal distribution does not reflect real returns (Ortobelli et al., 2010).

Starting with the works of Mandelbrot (1963a, 1963b) and Fama (1963) it is well documented that returns are not only determined by their mean and variance (Ortobelli et al., 2010). Since then, empirical research suggested numerous phenomena which are not addressed by normal distribution models (Stoyanov et al., 2011):

- 1) Clustering of volatility – *“large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes”* (Mandelbrot, 1963a)
- 2) Autoregressive behavior - changes in price depend on changes in the past
- 3) Skewness – the distribution of returns is asymmetric around the mean return (Stoyanov et al., 2011).

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<sup>1</sup> Based on assumptions like: Rational behavior, perfect information, no transaction costs, no tax, no dividends (Sloman, 2006)



4) Fat tails - when modeling cotton prices, Mandelbrot demonstrated that returns were not normally distributed but exhibited fat tails (Mandelbrot, 1963a), meaning that there is a higher probability of large and extreme losses or profits than predicted by normal distribution. Later, this observation was supported by further empirical research. However, tail thickness varies depending on the observed asset (Stoyanov et al., 2011).

5) Temporal behavior of tail thickness – tail thickness can vary over time. In regular markets, tail thickness is smaller than in turbulent markets (Stoyanov et al., 2011).

6) Tail thickness varies across frequencies – *“high-frequency data tends to be more fat-tailed than lower-frequency data”* (Stoyanov et al., 2011).

These phenomena have been empirically researched and suggest that models implying the normal distribution are not sufficient to address these empirical observations. The sole use of risk management tools like Value at Risk (VaR) is inadequate and should be complemented by additional tools like Extreme Value Theory (EVT) (Stoyanov et al., 2011). This helps dealing with very rare events that are not covered by tools like VaR and thus balancing off weaknesses and combining strengths of different tools.

## **2.2 Modeling Risk and Returns**

Risk and returns can either be modeled on the assumption of conditional or unconditional changes.

Unconditional models assume that changes are statistically independent and occur only on the basis of some underlying distribution. E.g. random walk theory assumes that the normal distribution is the underlying distribution of risk and returns, leading to further theories such as Optimal Portfolio Theory (Markowitz, 1952a), Capital Asset Pricing Model (CAPM) (Sharpe, 1964) as well as option pricing theory (Black and Scholes, 1973; Merton, 1973).

However, as mentioned in the chapter before, there is empirical evidence that prices also exhibit conditional behavior, meaning that changes in the past might affect future changes.

In order to model these two aspects, practitioners apply a two-step process. First, a conditional time series model is applied to explain conditional phenomena. The outcome is a residual, to which then an assumed unconditional underlying distribution is applied.

In the following, conditional time series models will be explained. Afterwards, alternative distributions to the normal distribution will be presented.

### 2.2.1 Conditional Time Series Models

The first conditional models were based on ARMA (autoregressive moving average). These models allow some conditionality, however, they cannot model volatility clustering effects, which are observed in empirical data (Curto et al., 2009).

In 1982, Engle introduced the first ARCH model (autoregressive conditional heteroskedasticity), which can also address volatility clustering effects. It was later generalized to the GARCH (generalized autoregressive conditional heteroskedasticity) by Bollerslev in 1986.

In combination, ARMA-GARCH models (Curto et al., 2009) are applied to explain the effect of volatility clustering and auto-correlation. In reality these models can be applied to stock indices (Curto et al., 2009), currencies (Mittnik et al., 2000) and credit spreads (Manzoni, 2002).

However, these models can in its pure form only be applied to data where the distribution (1) is known or (2) can be assumed. This is not always applicable to financial markets data (Jalal and Rockinger, 2008). In order to overcome this disadvantage, time series models are applied to a data set, which has not necessarily to be specifically distributed (McNeil, 2000). The outcome is a residual to which then an unconditional distribution can be applied. Possible distributions include Stable Paretian distributions, Student's  $t$  distributions and Extreme Value Theory

distributions. They are described in some more detail in the following subchapter.

### 2.2.2 Methods to Incorporate Fat Tails

In the following, we will present some alternatives to the normal distribution, which better address unconditional distribution phenomena like fat tails.

There is a broad variety of possible distributions in use. Stable Paretian distribution, Student's  $t$  distribution and Extreme Value Theory (EVT) are the most popular ones and will therefore be described in more detail. All of them have been studied for a long time outside the financial market context (Rachev et al., 2010). Stable Paretian distribution and Student's  $t$  distribution include the normal distribution as a special case and can therefore be regarded as an extension to traditional normal distribution approaches (Rachev et al., 2010).

#### *2.2.2.1 Stable Paretian Distributions*

The idea of using Stable Paretian distributions to model returns originates from the works of Mandelbrot (1963a) and Fama (1963, 1965) who suggested them. When modeling cotton prices, Mandelbrot empirically found that the applied Stable Paretian distribution describes the observed returns better than the normal distribution does. However, further research indicates, that they do not describe all of the above mentioned phenomena, e.g. varying tail thickness across frequencies (Stoyanov et al., 2011).

#### *2.2.2.2 Student's $t$ Model*

Student's  $t$  distribution was suggested by Blattberg and Gonedes (1974). Similar to the normal distribution, student's  $t$  distributions are symmetric around the mean and have one single peak. However they are more peaked and have fatter tails than the normal distribution (Rachev et al., 2010). From a practitioner's perspective it is the most commonly used alternative to the normal distribution for modeling asset returns, as it is relatively easy to use and to implement (Rachev et al., 2010). However, Student's  $t$  models, too, cannot describe all above mentioned phenomena

and have some limitations in the modeling of the tail. This is largely due to the assumption that the residual follows a Student's t distribution and a fixed value for the degrees of freedom. As a consequence, the risk of almost normally distributed return data will be significantly overestimated (Rachev et al., 2010).

### 2.2.2.3 *Extreme Value Theory (EVT)*

The two above mentioned distributions classes are used for models describing the full distribution of returns. Extreme Value Theory (EVT) is an alternative to this, which originates from the modeling of extreme events in nature, such as floods, earthquakes, etc. providing a model describing only the tail of the distribution (Stoyanov et al., 2011). EVT faces two major challenges: 1. sample size has to be very large to ensure sufficient observations of the tail and 2. there is no standard approach to identify where the body of the distribution ends and the tail begins. Goldberg, Miller and Weinstein (2008) suggest indications for the sample size and a method to determine the tail's size (so called Kuipers test). However, these challenges make EVT modeling more an art than a science (Rachev et al., 2010). Nevertheless, EVT is used in practice, e.g. Embrechts, Klüppelberg and Mikosch (1997) provide an application of EVT in finance. Based on experiences and real-life cases the authors try to build a bridge between statistical and probabilistic theories on the one side and their application in the financial and insurance sector on the other side.

## 2.3 Conclusion and Outlook

In practical application, modeling the described phenomena remains a two-step process: First, a time-series model is applied to explain the clustering of volatility and the auto-correlation (e.g. ARCH or GARCH see Bollerslev, 1986). Second, one of the described fat-tailed models is employed for the residual (Rachev et al., 2010).

However, there is no all-explanatory theory suggesting a distributional model for financial returns. From a modeling point of view, this is largely a statistical problem. (Stoyanov et al., 2011).

Further research is required to gain a better understanding of the origin of temporally restricted observations like volatility clustering and excess curtosis. Up to now some explanatory theories relate observations to e.g. retail attention (Barber et al., 2009) and attention shocks (Engelberg, Sasseville and Williams, 2009). Behavioral aspects are applied to explain some observations; however, current research has not yet focused on relating temporally restricted observations specifically to Prospect Theory. We will have a closer look at Prospect Theory in the following chapter.

Recently, the term of econophysics arose, which describes the application of mathematical models from physics to explain financial and economic behavior. It combines non-linear models, scaling laws, statistical mechanics and Cauchy and Levy distributions to offer more robust explanations than traditional linear models and normal distributions (Ray, 2011). This leads to new opportunities for the application of power-law distributions, e.g. for financial market fluctuations such as fluctuations in stock prices, trading volumes and the number of trades (Gabaix et al., 2003). Furthermore, econophysicists expanded their area of interest to other topics in economics like the distribution of income and wealth or the size of cities and firms (Gabaix, 2009).

Agent-based modeling (or agency based Computational Economics) is another emerging field of research. It tries to gain insights into economic processes by studying the dynamics of many interacting agents. (Barr et al., 2011) Based on stochastic processes and some basic definitions for the behavior of agents computer simulations are run. The dynamic interactions create effects that sometimes look similar to observations in reality (Ausloos et al., 2004).

Finally, Taleb (2009) argues, that some events (black swans) cannot be predicted, no matter what model is applied. He introduces a map consisting of four quadrants:

	<b>Simple payoffs / decisions</b>	<b>Complex payoffs / decisions</b>
<b>Distribution 1</b>	Extremely safe	Safe

(“thin tailed”)		
<b>Distribution 2</b> (no or unknown characteristic scale)	Safe	Dangers <sup>a</sup>

<sup>a</sup>The dangers are limited to exposures in the negative domain (i.e., adverse payoffs). Some exposures, we will see, can only be “positive”.

Thereby he distinguishes between thin- and fat-tailed distributions as well as single and complex payoffs. In the simple payoff quadrants, models can be applied and might have some predictive power. In an environment of complex payoffs, models can still be applied for thin tailed distributions, however, losing some of their predictive power over the first quadrant models. However, in the fourth quadrant with complex payoffs and no known or unknown distribution characteristics, predictive models cannot be applied or are bound to fail. This quadrant is the domain of black swans (Taleb, 2009). Taleb’s critique is primarily directed at practitioners who base their decision making solely and blindly on models in an environment of unknown distribution characteristics and complex payoffs, because this is dangerous and might cause enormous damage, as we could observe during the last crisis.

There is much effort put in modeling financial risk and returns. However, every model is based on certain assumptions and has its limitations as mentioned above. At the moment, mathematical models become more and more complex and complicated. However, at this point they do not seem to further improve our current understanding of risk and returns and still leave many questions unanswered.

In order to pursue the idea of creating artificial agents to simulate their behavior and interactions, it is necessary to equip these agents with “true”, observable human behavior and decision making skills. Human decision making in this sense is a core aspect of behavioral psychology and behavioral finance, which will be investigated in more detail in the following chapters.

### 3. Prospect Theory

In neoclassical theory, humans are *homines oeconomici*, behaving completely rational in order to maximize their expected utility. However,

this assumption does not adhere to many empirical observations. In the following chapter the concept of utility functions and their implications shall be presented to gain a better understanding of the theoretical developments in decision making research and of the origins of Prospect Theory. Subsequently, Prospect Theory and Cumulative Prospect Theory will be introduced, representing the latest and most popular theory in decision making research. Afterwards some critiques to (Cumulative) Prospect Theory will be presented and discussed.

### **3.1 The Utility Concept in Standard Neoclassical Theory**

According to the rationality assumption, there is a positive relationship between individual's wealth and utility. In reality, a variety of observations can be made, offering explanations for different forms of utility functions.

Expected utility theory by von Neumann and Morgenstern (1944) was the first theory introducing a utility function. According to von Neumann and Morgenstern (1944) the utility function is concave, which represents the individual's decreasing marginal utility of wealth. The utility function implies risk aversion; hence individuals would not accept fair bets (Levy and Levy, 2002).

Friedman and Savage (1948) claim that the utility function cannot be strictly concave, because investors buy insurances and lotteries as well as both simultaneously. The purchase of insurances means that the investor prefers certainty over uncertainty. Buying lotteries is the exact opposite, preferring uncertainty over certainty. A utility function with only one concave region cannot explain the behavior of investors buying insurances and lotteries at the same time. Friedman and Savage conclude that the utility function must have at least one concave and one convex region, potentially followed by another concave region.

Later, Markowitz (1952b) introduced the idea that decisions are taken based on change from current wealth. The Markowitz utility function can therefore be interpreted as a value function as it focuses on changes from current status (Levy and Levy, 2002). Markowitz further suggests that

individuals are risk averse for losses and risk seeking for gains if the outcomes are not extreme. If outcomes become extreme he argues that individuals become risk averse for gains and risk seeking for losses. Therefore, the Markowitz's utility function implies three inflection points and has a reversed S-shape (Markowitz, 1952b; Levy and Levy, 2002).

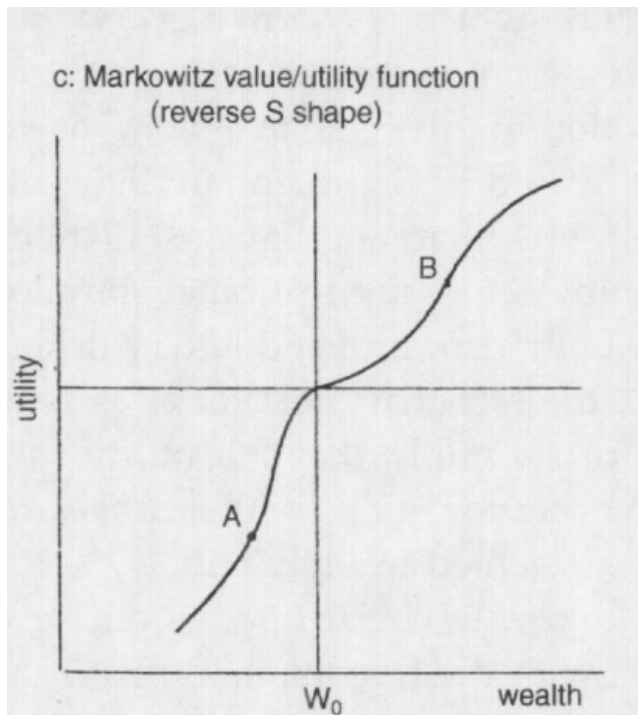
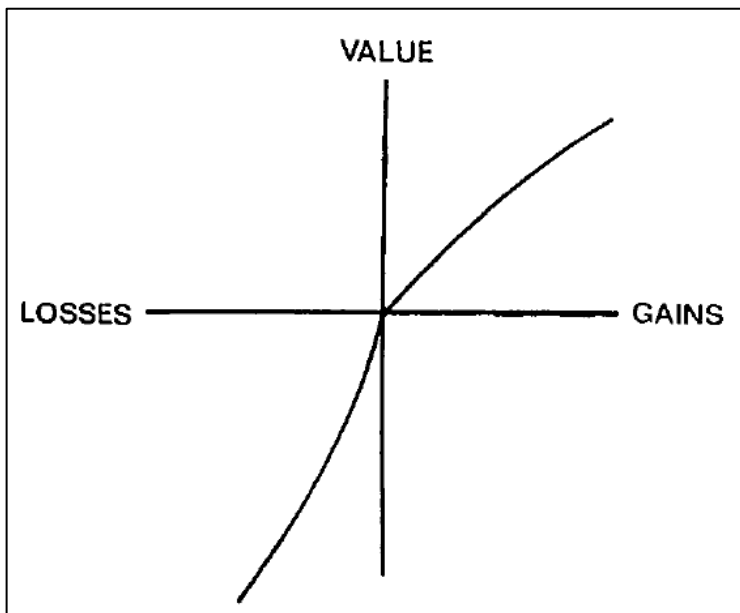


Figure 1: Markowitz's utility function with three inflection points (M. Levy and H. Levy, 2002)

The development in neoclassical theory reveals that over time, new utility functions have been suggested, in order to explain more empirical observations. The latest step was to introduce relative value rather than absolute utility functions, implying that individuals perceive changes from current status, rather than absolute changes. Until now, the Prospect Theory's S-shaped value function as suggested by Kahneman and Tversky (1979 and 1992) is the most well-known and most investigated value function (Levy and Levy, 2002).





Hypothetical value function as described by Kahneman and Tversky (1979).

In chapter 3.2.1, it will be introduced in more detail.

### 3.2 Introduction to Prospect Theory

Kahneman and Tversky's Prospect Theory (1979) claims that a value  $V$  of a prospect that pays  $\$x$  with a probability  $p$  is given by

$$V(x, p) = v(x) w(p)$$

Thereby  $v$  represents the measure of the subjective value of the consequence  $x$ , and  $w$  measures the impact of probability  $p$  on the prospect's attractiveness (Kahneman and Tversky, 1979; Trepel et al. 2005).

Prospect Theory distinguishes two phases: 1. framing and editing and 2. evaluation phase.

1. In the framing and editing phase, an individual performs a preliminary analysis of the offered prospect, which often results in a simpler representation of this prospect (Kahneman and Tversky, 1979). According to Kahneman and Tversky (1979) several operations are potentially performed in the framing and editing phase, e.g. coding: individuals perceive outcomes as gains and losses rather than final states. Therefore, prospects are perceived relative to some neutral reference point; the combination of probabilities; segregation of riskless

components of prospect from risky components of the prospect; cancellation: discarding of components that are shared by both offered prospects; simplification by rounding probabilities and outcomes or discarding extremely unlikely outcomes; detection of dominance: scanning of offered prospects to detect dominated alternatives, which are rejected without further evaluation.

2. In the evaluation phase the prospects from the framing phase are evaluated and the prospect with the highest value is chosen. Tversky and Kahneman (1986) suggest that “*the theory distinguishes two ways of choosing between prospects: by detecting that one dominates another or by comparing their values.*”

According to Trepel et al. (2005), Prospect Theory differs from expected utility function in three main ways:

1. Utility function over states of wealth is replaced by a value function over gains and losses, relative to a reference point, with  $v(0) = 0$ .
2. The subjective value function is not weighted by outcome probabilities but by a normalized decision weight  $w$ , so that  $w(0) = 0$  and  $w(1) = 1$ , representing the impact of the relevant probability on the valuation of the prospect.
3. Prospect Theory takes into account principles like framing and editing, which is not the case in the former utility functions.

Prospect Theory incorporates several empirical findings, rejecting expected utility theory as a model of decision making under risk. Hence, according to Prospect Theory, people do not always act in the purpose of rationally maximizing their expected utility as suggested by expected utility theory (Kahneman and Tversky, 1979).

### 3.2.1 The Value Function

Based on several experiments, Kahneman and Tversky (1979) suggest that in summary a value function is (1) defined on deviations from a reference point, (2) generally concave for gains and convex for losses, and (3) steeper for losses than for gains. Its steepest point is the reference

point, whereas the utility function suggested by Markowitz (1952b) is shallower in that point.

However, due to the introduction of decision weights (see the following chapter weighting function), the scaling of the value function is complicated (Kahneman and Tversky, 1979).

Based on these characteristics, several effects can be observed impacting the value function and therefore decision making under risk (known probabilities) and - as we will see in the next subchapter “Cumulative Prospect Theory” - on decision making under uncertainty (unknown probabilities).

#### *3.2.1.1 Reference Point Effect*

The existence of a reference point is central to Prospect Theory. It implies that people take decisions based on changes rather than on final states. Hence, the reference point can be described as the individual’s point of comparison, for comparing the status quo to alternative scenarios (Taleb, 2004). Due to this feature, one evaluates identical facts differently depending on if the prospect is formulated as negative or positive in relation to the reference point (Kahneman and Tversky, 1979; Rajeev Gowda, 1999). Also the saying “*every day is a new trading day*” can be attributed to the reference point effect. Traders evaluate losses and gains from their trading strategy and not from the absolute levels of wealth (Taleb, 2004).

#### *3.2.1.2 Loss Aversion*

As the value function is generally concave for gains and convex for losses, but at the same time steeper for losses than for gains, losses appear larger than gains for the same objective value. In other words: a loss would be felt stronger than a gain of the same amount (Kahneman and Tversky, 1979; Rajeev Gowda, 1999).

Several observed effects can be explained by considering this characteristic. The effects are commonly summarized under the name loss aversion. Brooks and Zank (2005) give an overview of examples attributed for loss aversion, e.g. the endowment effect (Loewenstein and

Adler, 1995; Thaler, 1980), the status quo bias (Samuelson and Zeckhauser, 1988), the disparity between the willingness to pay and the willingness to accept (Bateman et al., 1997; Kahneman et al., 1990), and the disposition effect (Heath et al., 1999; Odean, 1998; Weber and Camerer, 1998). All of these examples support the S-shaped value function as suggested by Kahneman and Tversky (1992 and 1979).

### 3.2.2 The Weighting Function

As described by the initial Prospect Theory formulae  $V(x, p) = v(x) w(p)$ , the outcome of the value function  $v(x)$  is multiplied with a decision weight  $w$ . Decision weights represent subjective probabilities; however,

*“[...] they do not obey the probability axioms and should not be interpreted as measure of degree of belief. [...] **Decision weights measure the impact of events on the desirability of prospects, and not merely the perceived likelihood of these events. The two scales coincide if the expectation principle holds true, but not otherwise.**” (Kahneman and Tversky, 1979).*

Tversky and Fox (1995) and Tversky and Wakker (1995) suggest that the weighting function exaggerates small probabilities but underestimates large ones. In this context Kahneman and Tversky (1979) differentiate between overweighting, which is a property of decision weights, and overestimation, which applies in the context of probability assessment of rare events (Kahneman and Tversky, 1979 and see chapter 4 “Probability Distortions” for over- and underestimation effects in the assessment of probabilities).

There are several empirical observations linked to the weighting function, which demonstrate violations of the theorem of individuals rationally maximizing expected utility from a neoclassical perspective. The most important observations are risk seeking in the domain of losses and the certainty effect, which is also referred to as zero risk bias.

Risk seeking in the domain of losses suggests that individuals with reference points (see chapter 3.2.1 for reference point) are risk averse in

the domain of gains. However, in negative or loss situations they become more likely to choose risky options in order to restore gains (Daniel Kahneman and Tversky, 1979; Quattrone and Tversky, 1988). However, this behavior is irrational as it will ultimately lead to failure if another loss strikes (see begging also for resurrection).

Another observation first mentioned by Allais (1953) is the certainty or zero risk bias, which implies that people rather prefer small but certain benefits over larger uncertain benefits (see Baron, 1994 for a discussion). This goes along with people tending to value the reduction of a negative outcome from e.g. 5 percent to zero percent higher than the reduction of a more negative outcome e.g. 30 percent to 20 percent (Baron, 1994; Rajeev Gowda, 1999). This might induce people to pay large irrational premiums for “certainty”, e.g. when renting a car it is quite common to buy a damage waiver to minimize the risk of any additional payments to the rental firm (Shapira and Venezia, 2008).

These effects attributed to the weighting function demonstrate that people do not always behave rational in the sense of maximizing neoclassical expected utility. However, people might still obtain some sort of utility, as they exhibit such behaviors. Risk seeking in the domain of losses for example gives people the assurance that they tried everything to prevent this failure. This behavior might cost additional losses but gains certainty which in total could be a maximization of subjective utility.

### 3.3 Cumulative Prospect Theory

Initial Prospect Theory (Kahneman and Tversky, 1979) has some drawbacks, which were partially resolved by extending the theory. Quiggin (1982) and Schmeidler (1989) solved the problems with non-additive probabilities and Gilboa and Schmeidler (1989) found a solution to the problem of unknown probabilities. This was a necessary step to come closer to reality where exact risks and probabilities are usually unknown. They also extended the theory to cover situations different from the original setting with two non-zero outcomes although any proposed extension to more than two nonzero outcomes has other drawbacks (Kothiyal et al., 2011). These improvements were incorporated in Cumulative Prospect Theory by Tversky and Kahneman (1992) offering a comprehensive theory adhering to empirical observations (Kothiyal et al., 2011). Further extensions were developed by Tversky and Fox (1995) by introducing the principle of bounded subadditivity, which suggests that an event turning impossibility into possibility or possibility into certainty has a greater impact than making a possibility more or less likely to happen (see also Tversky and Wakker 1995).

### 3.4 Limitations of (Cumulative) Prospect Theory

(Cumulative) Prospect Theory received some scientific acknowledgement and underwent several empirical tests. Some of these supported, others rejected (Cumulative) Prospect Theory or aspects of it.

Typically, researchers tried to identify the best fitting form for the value and weighting function. Other works investigate in more detail some aspects of prospect theory, e.g. Levy and Benita (2009) who state that according to Prospect Theory, equally likely outcomes should be perceived as equally likely. Levy and Benita (2009) argue that in their research individuals significantly overweighed moderate outcomes relative to extreme outcomes. Also they argue that in their findings the value function has a reversed S-shape, compared to the value function as suggested by Tversky and Kahneman (1979 and 1992).

Levy and Levy (2002) argue that

*“[...] the value function is mainly justified by experimental investigation of the certainty equivalents of prospects confined either to the negative or to the positive domain, but not of mixed prospects, which characterize most actual investments”*

They reject the S-shaped value function, because in their research at least 62%-76% of the subjects could not be characterized by such preference (Levy and Levy, 2002). They suspect that former results supporting the S-shaped value function are due to the applied laboratory test design in which subjects had to choose between two investments by answering a questionnaire. However, the participants could only choose between either positive or negative outcomes and not mixed cases (as required if reference dependence shall be investigated). Hence, these test designs do not conform to real life situations e.g. as found in financial markets (Levy and Levy, 2002). Brooks and Zank, (2005) state that

*“theoretical analyses of loss aversion have focused on properties of the utility function in the spirit of Markowitz (1952) and Kahneman and Tversky (1979), and consequently view loss aversion as model specific (e.g., Fishburn, 1977; Holthausen, 1981; Wakker and Tversky, 1993; Schmidt and Traub, 2002; Neilson, 2002; Köbberling and Wakker, 2005). In Tversky and Kahneman (1992) both probability distortions and utility were capturing sensitivity towards the sign of outcomes, a finding which has also been confirmed by other experimental studies (Edwards, 1955; Currim and Sarin, 1989; Abdellaoui, 2000; Abdellaoui, Bleichrodt and Paraschiv, 2004) and mentioned in Camerer and Ho (1994, note 18).[...]However, the parsimonious way of accommodating loss aversion through a single parameter is problematic. Loss averse behavior is more complex, and, as we argued, a model free approach to loss aversion gives more flexibility in uncovering its determinants and subsequently analyze its implications.” (Brooks and Zank, 2005)*

Furthermore, Diecidue et al. (2007) summarize that many other authors found violations to Prospect Theory, e.g. Barron and Erev (2003), Birnbaum (2006), Goeree et al. (2002), Gonzalez-Vallejo et al. (2003), Harbaugh et al. (2002), Lopes and Oden (1999), Neilson and Stowe (2001).

A more general discussion about Cumulative Prospect Theory is offered by Tversky and Kahneman (1992) themselves, who state, that Cumulative Prospect Theory is more general than its predecessor theories. However, they suspect that decision weights are influenced by more factors than implied by the theory, e.g. by the formulation of the prospect or the spacing of outcomes, e.g. Camerer (1992) suggests that the weighting function exhibits more curvature if outcomes are more widely spaced. Tversky and Kahneman (1992) argue that any adaption should be evaluated if the potentially gained insights are worth the increase in complexity and loss of predictive power of the theory.

Kothiyal et al. (2011) argue that Tversky and Kahneman (1992) did not analyze their Cumulative Prospect Theory for continuous distributions (Kothiyal et al., 2011). Kothiyal et al. offer a generalization of the aforementioned theory for continuous distributions, based on the works of Gilboa and Schmeidler (1989), Quiggin (1982) and Wakker (1993). The extension to apply Cumulative Prospect Theory to continuous distributions allows its application in all sorts of lognormally distributed functions in finance, e.g. stock prices (Hull, 2006), continua of investment options and possible returns.

There are many limitations of (Cumulative) Prospect Theory or aspects of it. However, most authors recognize the importance of the theory. Diecidue et al., (2007) state

*“[...] however, to date there is no more successful and tractable theory available for decision under risk or uncertainty. Many phenomena remain unpredictable in this domain, with only post-hoc heuristic explanations conceivable”.*



### 3.5 Conclusion and Outlook

Empirical research used to reveal observations that were considered irrational from the neoclassical perspective (Kothiyal et al., 2011). Kahneman and Tversky (1979 and 1992) suggested a theory that explained most of these “irrationalities” and which was furthermore the first model for decision making under risk to allow theoretical analyses and predictive applications. Kothiyal et al.’s (2011) generalization to apply Cumulative Prospect Theory to continuous distributions might allow the integration into asset pricing functions as a next step. However, as Tversky and Kahneman stated, and we share their opinion, that

*“Theories of choice are at best approximate and incomplete. One reason for this pessimistic assessment is that choice is a constructive and contingent process. When faced with a complex problem, people employ a variety of heuristic procedures in order to simplify the representation and the evaluation of prospects. These procedures include computational shortcuts and editing operations, such as eliminating common components and discarding nonessential differences (Tversky, 1969). The heuristics of choice do not readily lend themselves to formal analysis because their application depends on the formulation of the problem, the method of elicitation, and the context of choice [...] [...] evidence indicates that human choices are orderly, although not always rational in the traditional sense of this word” (Tversky and Kahneman, 1992).*

Hence, theories of decision making help us to better understand people’s behavior, although complete and correct predictive explanations cannot be achieved because of the individuality of each person. *“Many phenomena remain unpredictable in this domain, with only post-hoc heuristic explanations conceivable” (Diecidue et al., 2007).*

Nevertheless, advances in decision making research help us to gain a better understanding of the implied decision taking “processes”. Financial market behavior is based in some way on the behavior of

individuals and accordingly on their decision making. The distributions of chapter 2 are only descriptions of the results of these behaviors. In order to fully understand the underlying foundation of these distributions we need to understand how individuals behave and make decisions under various different circumstances. The insights gained from the investigation of human decision making might be transferable to agent-based modeling as proposed in the conclusion of chapter 2 so that agents can be equipped more realistically. Simulations with these agents might enable us in turn to gain a better understanding of financial markets. Also some current observations in financial markets might be explainable by aforementioned effects, as will be discussed in chapter 5.

#### 4. Heuristics and Biases in Assessing Risks

In the previous chapter Prospect Theory and its impact on decision making was presented. In this chapter distortions in the perceived probability of events will be presented. In order to clearly differentiate between these two chapters, recall that the preceding chapter presented some effects related to the weighting function, whereas the weighting function or “[...] *decision weights measure the impact of events on the desirability of prospects, and not merely the perceived likelihood of these event. The two scales coincide if the expectation principle holds true, but not otherwise.*” (Kahneman and Tversky, 1979). In the following chapter we will demonstrate heuristics, which affect the perceived likelihood of events, such as the belief in the law of small numbers and overconfidence. Also, we will discuss how feelings might affect return perception. This is again one more step to understand the behavior of individuals which leads finally to realistic return distributions and a better understanding of the behavior of markets.

##### 4.1 The Law of Small Numbers

According to the law of large numbers, large random samples closely represent the population from which they are drawn (Rabin, 2000). In contrast, the law of small numbers, first described by Tversky and Kahneman (1971), is the effect that people think that small random samples are highly representative for their underlying population (Tversky and Kahneman, 1971, 1974, 1992). Hence, subjects have a truncated, narrower distribution in their minds than actually given in the population (Taleb, 2004). Tversky and Kahneman (1974) suggest that “*people tend to believe in the law of small numbers, but not in the law of large numbers*”. This means that they overestimate the representative of small random samples, however underestimate the representativeness of large random samples (Rabin 2000; Tversky and Kahneman, 1971; Tversky and Kahneman, 1974).

According to Tversky and Kahneman (1971) the law of small numbers can be explained by the representative heuristic, which is usually

employed when people evaluate the probability of an object or event A, belonging to a class or process B (Tversky and Kahneman, 1971; Tversky and Kahneman, 1974).

The law of small numbers states that outliers are often underestimated (due to the narrower distribution in subjects' minds) (Taleb, 2004). This might contradict the observation that the probability weighting function overestimates the odds of rare events (Tversky and Fox, 1995; Tversky and Wakker, 1995). If rare events are considered as outliers then the two ideas suggest opposite tendencies: underestimation (the law of small numbers) and overestimation (probability weighting function). The law of small numbers is thought of to be closely related to other biases, such as the gambler's fallacy (widely researched), over-inference (hardly researched), regression errors and others (Rabin, 2000).

The gambler's fallacy describes the effect when people think that early draws from a random sample influence the probability of the next draws (Rabin, 2000). There exists a broad literature on the existence and impact of gambler's fallacy, e.g. Tversky and Kahneman, 1971; Tversky and Kahneman, 1974; Rabin, 2000; Croson and Sundali, 2005. Rabin (2000) offers an application to financial markets, suggesting that *“due to the gambler's fallacy – investors underpredict repetition of short strings of performance while – due to over-inference, they over-predict repetition of longer strings.”* This might be a psychological explanation to the observation of short-term underreaction but medium-term overreaction to announcements by firms in financial markets. (Rabin, 2000). In practice this leads to a time-lag in the incorporation of information into prices (Barberis et al., 1998). In the long-run this creates overpriced assets as investors *“classify some stocks as growth stocks based on a history of consistent earnings growth, ignoring the likelihood that there are very few companies that just keep growing”* (Barberis et al., 1998). This reasoning is transferable to the explanation of investors' trust in successful fund managers. After one successful year a fund manager is assumed to have a less successful year, however, if the fund manager has

several successful years in a row investors consider this manager to have outstanding skills (Barberis et al., 1998).

A similar effect to the gambler's fallacy is the hot hand fallacy. This effect describes that "*people expect to see more switching among signals than they actually will, they mistake true i.i.d. randomness for streakiness*" (Rabin, 2000).

#### **4.2 Overconfidence**

Overconfidence is the empirical observation that subjects' estimation of their accuracy (confidence) exceed the actual accuracy, e.g. when subjects are asked to express an answer to a question at a certain confidence level, they may be correct only at a lower confidence level (Kahneman and Tversky, 1996; Keren, 1991; Yates, 1990). However, overconfidence is not universal, as it is often eliminated or reversed for easy items or questions. This effect is referred to as difficulty effect (Kahneman and Tversky, 1996).

Other studies of overconfidence examine the difference between the perception of unlike events and their actual occurrence as well as the inability to calibrate from past errors (Alpert and Raiffa, 1982; Hilton, 2003; Kahneman and Lovallo, 1993; Taleb, 2004). Barron and Erev (2003) conducted experiments which led them to the conclusion that subjects underweight small probabilities in sequential experiments when they derive the probabilities themselves. Slovic, Fischhoff, Lichtenstein, Corrigan and Combs (1977) present the intuition to this effect in "*preference for insuring against probable small losses*" (Taleb, 2004).

Glaser and Weber (2007) suggest that overconfidence appears in three forms. 1) *Miscalibration* describes the observation that people's probability distributions are narrower than in reality. If asked to state a confidence interval for a set of questions or estimations people tend to overestimate their accuracy. 2) *Too tight volatility estimates* are closely linked to miscalibration. If asked to state the volatility of stock prices or risk premiums CFOs, professional stock traders and students underestimate the historic volatilities. 3) *Better than average* simply

means that people overestimate their abilities. Glaser and Weber (2007) state the example of Svenson (1981) that 82% of a group of students think that they belong to the best 30% of drivers with respect to driving safety.

### **4.3 Feelings and Experience in Risk Perception**

There is evidence that people do not only judge probabilities of events or actions based on thinking, which is influenced by the aforementioned heuristics, but also based on feelings. Slovic and Peters (2006) argue that *“people judge a risk not only by what they think about it but also by how they feel about it. If their feelings toward an activity are favorable, they tend to judge the risks as low and the benefits as high; if their feelings towards the activity are unfavorable, they tend to make the opposite judgment—high risk and low benefit”*(Slovic and Peters, 2006). This effect is also referred to as the affect heuristic (Finucane et al., 2000).

Another example for how feelings affect people’s perception of risk is the so called competence hypothesis by Heath and Tversky (1991). They show that people who were knowledgeable about one discipline preferred to bet on this discipline rather than on chance events from another discipline, even if the other event was judged by them as equally likely to happen. This observation is consistent with the idea that people prefer to bet on a fair and not a biased coin.

More recent research by Cohen et al. (2008) suggests a model, in which risk perception depends on past experience. This allows them to gain explanations for changes in the insurance demand behavior after loss events in the form of catastrophes.

#### **4.4 Conclusion and Outlook**

As we have seen, there is no comprehensive integrated theory on probability distortions. Many heuristics and biases are very well investigated; however, they are only loosely related to one another. Hence, there is no comprehensive predictive model of how people assess the likelihood of events.

Slovic (2000) points out that some psychological studies highlight the fact that risk perception may also strongly depend on the context in which individuals took a decision. Possible contexts might correspond to:

1) Past experience

Insurance decisions are at least partially based on experience. People who never experienced a flood might underestimate the probability that they will suffer from flood damages (Browne and Hoyt, 2000; Kunreuther, 1996).

2) Anticipatory feelings about future states

The future is uncertain and if the decision making takes time anticipatory feelings change and as a consequence the preferences change. Hopefulness, anxiety, and suspense are examples for feelings that might influence the perception of risk (Caplin and Leahy, 2001).

3) Presentation of decision outcomes

The theory of choice is based on the principle of invariance: Preferences should not vary with the description or presentation of alternatives. Two characterizations should be seen as alternative descriptions for the same problem. However, this normative idea might not hold for some situations in reality. (Tversky and Kahneman, 1986).

The differentiation between weighting function effects and effects attributed to the perception of the likelihood of events is subtle and sometimes hardly distinguishable, especially if the abovementioned statement of context dependent decision-making is taken into account.

## 5. Linking and Combining Theories and Heuristics

The preceding chapters contribute to a more holistic picture and understanding of risk perception. However, the described heuristics and theories are only loosely connected. The challenge is to integrate and link them in order to offer a more comprehensive theory on risk perception.

Some implications of risk perceptions are discussed in the literature. Taleb (2004), Rabin (2000) and others discuss the effects and implications of risk perception from different angles and to different applications. The discussed effects are usually brought forward in a narrative or explanatory way and do not include models or are derived from models (e.g. Rabin, 2000, derived his explanation from a model). Nevertheless, these examples help to gain an easier access to the problems and issues discussed and they provide the grounds for the application of theoretical models to real life situations and thereby investigate their explanatory power.

In respect to practical application it is most important to identify if observed phenomena and their consequences are of temporary nature or sustainable. If they occur only temporarily, they fall into the domain of algo- and high frequency trading to be exploited. If they are sustainable, they would influence decision making on a longer run, e.g. in private wealth, asset or pension fund management.

In the following, some examples will be presented, which go one step further and connect aspects of preceding chapters to their underlying reasons or applications in reality.

### 5.1 Hedonic Psychology and the Preference for Asymmetric Payoffs

*“Hedonic psychology (...) is the study of what makes experiences and life pleasant or unpleasant.” (Kahneman et al., 1999)*

Based on this definition, hedonic psychology can also be seen as an intermediate step towards the study of what drives and motivates people and thus influences their behavior.



In his article “Bleed or Blow up?” Taleb (2004) suggests a relation between the non-normal distribution of returns, skewed payoffs of financial instruments, overconfidence and Prospect Theory. He suggests that due to immanent hedonic behavior, most people exhibit a preference for skewed payoffs. In consequence, most people follow a “blow up” investment strategy. This implies that they might realize steady and constant low wins over a certain time. However, once in a while they will “blow up” when events take place, which were not considered by them. People who prefer the “bleeding” investment strategy bet on these rare events. This implies that they might incur low but steady losses over a while. Only when a rare event takes place, they realize large gains. Taleb (2004) argues that the bleeding strategy might be more successful from a return on investment view; however, it is very hard for individuals to apply this strategy, due to their immanent hedonic behavior. People prefer to go to work and be happy. While incurring constant losses, people suffer from increased stress, while there are no successes motivating them. Consequently, they will be less happy overall. In contrast if people follow the “blow up” strategy, they incur constant “wins”, which motivates and stimulates them. These people will be happy most days, only when bad luck hits, they will be unhappy. Nevertheless, regarded over a longer period of time, they will be happy most of the time, leaving them more happy than the subjects following a “bleed” strategy (Taleb, 2004).

In this article, Taleb demonstrates that a more holistic approach on risk/return functions as well as risk perception and behavior can be taken, by linking several of the abovementioned aspects. Especially, the relation to hedonic psychology might lead to new insights on why people behave in one way or another. Ideas like reference point effect, loss aversion, certainty effect, the law of small numbers and overconfidence give a reason for the deviation of people’s behavior from the rational utility maximization in neoclassical theory. However, they do not give a specific reason for the deviating behavior. Hedonic psychology goes one

step back to look at the motives of people's behavior. Potentially, this might create an additional layer of explanatory power.

## **5.2 Further Explanations of Observations Due to Heuristics and Biases**

Levy and Benita (2009) suggest that the clustering of expectations towards a consensus might be tracked back to the overweighting of moderate probable outcomes compared to extreme outcomes, because individuals perceive a systematically lower variance in the data than actually given. They further argue that this might have a *“dramatic effect on portfolio efficiency, asset pricing and the interpretation of econometric tests of asset pricing models”*.

On the basis of his model about the law of small numbers, Rabin (2000) puts forward an explanation, why people might pay for financial advice from experts who are actually not experts (see also Taleb, 2007, Chapter 10 – The Scandal of Prediction). People might over- or underestimate the ability of financial analysts by considering their past performances and falling for the law of small numbers. Financial analysts with several “good” consecutive years will rather be judged as successful, analysts with several “bad” consecutive years will be perceived as unsuccessful. However, the observed years do not necessarily provide fundamental evidence of the ability of a financial analyst. Hence, people tending to fall for the law small numbers might pay for financial advice which is not worth it (Rabin, 2000).

## **5.3 Agent-based Modeling – Computing Human Behavior**

One possible way of linking theories and combining aspects of decision making under risk and uncertainty is the already mentioned interdisciplinary approach of agent-based modeling. This approach becomes more and more applicable to economic investigations as *“technological changes in analytic and computing methods are opening up new avenues of study”* (Colander, 2005) The advantage of this approach is its flexibility, the agents can be equipped with any kind of characteristic (Chakraborti et al., 2011; Chen and Huang, 2008). This is

particularly interesting because it allows neoclassical characteristics and modern characteristics from behavioral finance to be combined in one model (Colander, 2005). This is supported by Chen and Huang (2008) who highlight the flexibility of agent-based modeling, especially with respect to heterogeneity of agents and population dynamics which are associated with such an application of various different sorts of agents. By linking different aspects and theories it offers the opportunity to create a more holistic approach to decision-making under risk: “This field aims at improving financial modeling based on the psychology and sociology of the investors.” (Chakraborti et al., 2011) Furthermore, characteristics or rather their role and weight in the decision making process can be adapted to the situation so that the model considers the context dependency described in section 4.4. Some models are already successful in replicating stylized facts discovered in real financial data and other economic applications like distributions of income or growth rates (Gatti et al., 2005). However, agent-based models are still regarded as “toy models” (Chakraborti et al., 2011). This is largely due their simplicity although even these simple models generate some interesting results and insights. At this state, there is a clear limitation to the insights gained through such a model and its complexity as Chakraborti et al. argue that one has to trade off realism against the calibration of the mechanisms and processes. Nevertheless, as computational power and technique will advance the opportunities in this field continue to grow including the possibility of moving from “toy models” to more realistic simulations of reality. Colander (2005) predicts that agent-based models will not only be used to test theories. He rather suggests that economists will create a “virtual economy” in which for example even alternative policies can be tested. However, Colander also admits that “such a movement to agent-based modeling is far in the future.”

## 6. Summary

In this paper we aimed at providing a more holistic view on risk perception. Therefore, we started to explain in chapter 2 why the normal distribution was used to measure risk in the first place, on the basis of neoclassical theory. Then we described, some empirical observations, rejecting the normal distribution as prevalent distribution to measure risk. We continued by presenting current developments in risk and return modeling, differentiating between conditional and unconditional phenomena in asset returns, requiring a two-step process to be captured in models. First, a conditional time-series model is applied to generate a residual. This residual is then modeled in an unconditional distribution to capture fat tails. We presented Stable Paretian distribution, Student's  $t$  distribution and Extreme Value Theory as methods to measure fat tails. The chapter ends with an outlook on research in the fields of econophysics, attention shock theories and agent-based modeling. Also, Taleb (2009) points out that the appliance of any mathematical model is prone to fail in an environment of complex payoffs and unknown distributions. Eventually, this causes enormous damage, especially, if managers blindly relied upon it. Hence, we come to the conclusion of chapter 2, that mathematical modeling helps us to understand the distribution of risk and returns; taking into account, however, that this purely mathematical and statistical approach has its limitations. Considering new developments in modeling risk and returns, agent-based and experimental economics, we suggested having a closer look at human decision making in order to equip artificial agents with human behavior that can be empirically observed and modeled.

Chapter 3 begins by highlighting the importance of human decision making theory for all economic and financial theories. In order to understand recent findings in decision making research, we presented the development of utility functions in neoclassical theory, ending with Kahneman and Tversky's value function suggested in Prospect Theory. In a first step we gave a short introduction to original Prospect Theory, considering both the value and the weighting function. Additionally,

major empirical observations related to both functions were presented, questioning neoclassical expected utility theory but supporting Prospect Theory, such as the reference point effect, loss aversion, risk seeking in the domain of losses and the certainty effect. In a second step we elaborated Prospect Theory to Cumulative Prospect Theory, explaining that the major drawbacks inherent in Prospect Theory resulted from the original test setting. However, these drawbacks could be eliminated with new test designs presented in Cumulative Prospect Theory, offering a more comprehensive solution, without contradicting major findings of original Prospect Theory. More complex models are developed in order to mitigate some limitations of (Cumulative) Prospect Theory and to incorporate other observed behaviors. However, the more mathematically complex the theory becomes, the more of its general predictive power is lost. Hence, even though there is a lot of critique about (Cumulative) Prospect Theory, most research suggests that it is still the most comprehensive theory of human decision making under uncertainty, offering at least some predictive power. We conclude that (Cumulative) Prospect Theory is a powerful theory, which might be of importance, when it comes to equipping agents with human decision making behaviors.

Apart from effects that can be attributed to Prospect Theory, there are more specific cognitive biases, which affect human's ability to assess the likelihood of events. Prospect Theory is referring to these biases; however states that effects attributed to Prospect Theory differ from cognitive biases. Therefore we took a closer look at some cognitive biases in chapter 4. First, we presented the law of small numbers and other heuristics also related to the representative heuristic, such as gambler's fallacy and the hot hand. Second, we described overconfidence and named some applications in financial markets. Third, we presented some research suggesting that feelings, experiences and context might have an influence on assessing the likelihood of events or risk. Finally, we conclude that cognitive biases might also have strong effects on general risk perception.

In the last chapter 6, we offer some research on how to link all before mentioned topics in order to create a new, more holistic picture of risk perception. Taleb (2004) offers a very good example of how to link these topics by referring to hedonic psychology as a major motivator for individual's preference for asymmetric payoffs and eventual blow-ups.

Finally, it should be stated that there are many aspects to risk perception. The market perspective which applies mathematics to model financial risk and returns, the "choices under uncertainty perspective" with Prospect Theory, focusing on how individual take choices and decisions under risk and uncertainty, and a "cognitive perspective" considering other probability distortions, described by empirical research. Even though we mentioned that models of choice and decision making can only be approximations, we think that it is important to advance and improve them. The next step might be to integrate them into models of financial returns. Agent-based models already try to imitate human behavior. For these investigations, advances in Prospect Theory and probability distortion research might give valuable input and might help to further integrate these different approaches.

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