Helmut Meister

On a Bypass Centrality Concept for Networks drawing on the Shapley Value

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Helmut Meister
email: meisterhelmut1@t-online.de

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Abstract

First, we analyse game-theoretic solution concepts for the assessment of members of a network drawing on the Shapley Value. Our approach starts with the concept of Betweenness Centrality as known from Social Network Analysis. We will also be interested in Centrality Concepts, which satisfy the conditions of Core Allocations of the value of the whole network and their relationship to Shapley Value based concepts. It turns out that a centrality concept derived from the membership of vertices in global shortest paths within the network provides a Core Allocation and is therefore in some sense agreeable by the members of the network. We will also consider relative shortest paths within coalitions of vertices. For this approach, which leads to a concept of Bypass Centrality, we get a different assessment method, which better reflects the local connectivity of the network and respects the capability of vertices to form bypasses for connections potentially blocked for some reason. For this type of allocations it seems to be an open problem, whether the Shapley Value satisfies the conditions of a Core Allocation in general. Apart from these game-theoretical questions, our focus concerns the computational complexity for the calculation of the Shapley Value, which is in general considered to be a NP-complete problem. For the computation of the Shapley Value based on global shortest paths an efficient algorithm has already be found. We can show that also for the concept based on relative shortest paths an algorithm exists, which solves the problem in pseudo-polynomial time, depending on a limitation of the number of connecting paths considered for each pair of vertices. This shows that in our situation the approach reduces the calculation to a weakly NP-complete problem.

Keywords: *Social Network Analysis, Centrality Measures, Game Theory, Shapley Value, Core, Algortithm, Complexity*

1 Motivation

We consider networks consisting of nodes and edges. The network model serves as an appropriate abstraction for transportation systems, informal networks or social structures. A special field of interest for Mathematical analysis of networks is the identification as well as the numerical assessment of important vertices and edges based on the structural properties of the network. We will focus on this aspect and discuss some approaches from a game-theoretic view. One of the most intensively used concepts of assessment of actors within the set of all possible coalitions from a certain set of actors is the Shapley Value. On the individual level this value can be interpreted as the average marginal contribution of a given actor to every possibly existing coalition. The basic idea behind the Shapley Value strongly suggests to apply it in the context of Social Network Analysis (SNA). The list of contributors to SNA is very long. An overview can for instance be found in [3]. The marginal contribution of an actor to an existing coalition of vertices may be a new path of information flow or flow of objects which becomes accessible through the inclusion of the actor. The practical disadvantage of this approach results from the definition of the Shapley Value as an expectation value over all permutations of the set of actors. The calculation of this expectation value turns out to be a NP-hard problem in the general case and is often impractical even for networks with few vertices. Therefore it is remarkable that in some cases there exist relatively simple calculation schemes and efficient algorithms to overcome these difficulties. One of these approaches focuses on Betweenness Centrality measures and is described in the paper of P. L. Szczepanski, T. Michalak and T. Rahwan ([4]). The authors consider the case where, with respect to a permutation of the set of all actors, a certain actor is the first, who resides on a shortest path connecting two different actors. We will concentrate on another interesting interpretation of the same resulting centrality measure. In fact, this value of an actor is identical for the case, where the actor completes a shortest path within an existing coalition in the network. The latter interpretation seems to be more intuitive, because the actor functions as the missing link to improve the connectivity within the coalition.

Apart from these insights, we propose to take also the concept of the Core of a coalition game into account for the assignment of centrality values to the actors of a network. It seems to be reasonable to assign values to the actors in a way that no coalition can oppose to it. In this sense the concept of the Core satisfies the requirements of fairness and acceptance by the actors concerning the assignment of values. This property of Core Allocations may for instance be important in the case where centrality measures serve as basis for a prioritized list of nodes according to which financial resources have to be distributed. In the underlying situation it is not very difficult to show that the given game is a convex game. As known from game theory for a game of this type the Core is not empty and the Shapley Value is the center of the Core. This result may be seen as a justification for the chosen approach.

The concepts of Betweenness in Social Network Analysis draw on shortest paths between two nodes within the whole network. We will modify this concept and allow bypassing through the assignment of a characteristic value to each coalition of nodes which reflects the internal structure of the coalition as well as their capability to interact, even though some external connections are not available. In detail, we consider relative shortest paths formed by the members of the coalition themselves rather than including other members. This concept emphasizes the local capabilities of the network to contribute to the connectivity of subnetworks. Unfortunately, the according game is not convex as in the case of global shortest paths. Therefore, there is no guaranty that the Shapley Value is a Core Allocation. However, as many examples show, it seems that even for these games there is a chance for the Shapley Value to satisfy the conditions of a Core Allocation. We will identify some special characteristic functions and networks for which the questionable property can be verified.

2 Notations and Preliminaries

We assume all networks $G(N, E)$ of nodes N and edges E to be embedded in a metric space and to satisfy some regularity conditions, i.e. different vertices have a positive distance. This implies the validity of the triangle inequality and the possibility to calculate the length of a path within the network. Paths are defined as sequences of at least two pairwise different adjacent vertices. The set of all shortest paths connecting two not identical nodes s and t within the network will be denoted by Ψ_{st}[1]. Additionally, we will make use of the set of all shortest paths, which will be addressed by

$$\Psi := \Psi^N := \bigcup_{s \neq t} \Psi_{st}. \tag{1}$$

We will also consider shortest paths relative to a given set $S \subset N$. The shortest paths connecting two not identical nodes s and t by vertices from S, is denoted by Ψ^S_{st}. The set of all shortest paths within S will be addressed by Ψ^S. We have to note that a shortest connection relative to a subset S of N may be different from the (overall) shortest connection relative to N. But, generally, the length of paths from Ψ^S_{st} is greater than that of paths in Ψ^T_{st}, whenever $S \subset T$. To keep notation simple, we will not distinguish between the path $\psi = (v_1, \ldots, v_r)$ as a sequence of points and the set of points $\{v_1, \ldots, v_n\}$. The length of a path ψ in N will be denoted by $\lambda(\psi)$. We will further assume that a utility function $u : \Phi \to \mathbf{R}_+$ exists, which is defined on the set Φ of all paths in N, and is monotone decreasing in the length of paths. For instance, $u := \frac{1}{\lambda}$ would be a candidate for this. Another candidate is $u(\psi) := \frac{1}{\#\psi}$, which seems to be an appropriate measure for networks with equal length for all edges. [2]

For each path $\psi_{st} \in \Phi$ we denote the interior points of the path by $\nabla\psi_{st}$, i.e. $\nabla\psi_{st} := \psi_{st} \setminus \{s, t\}$ and the set of endpoints of the path by $\partial\psi_{st} := \{s, t\}$.

Our analysis focuses on two different characteristic functions. The first will be based on the connectivity of coalitions $S \subset N$ by the set of shortest paths

[1] This set may contain zero, one or more elements. Nevertheless, applying a suitable perturbation on the edge lengths ensures that between each pair of nodes there exists at most a single shortest path.

[2] The cardinality of $S \subset N$ will be denoted by $\#S$.

within N and aggregates the utilities of these paths:

$$v_{global}(S) := \sum_{s,t \in S, s \neq t} \sum_{\psi \in \Psi_{st}} u(\psi) \qquad (\forall S \subset N). \tag{2}$$

This measure of the connectivity of the set S is known as the concept of **Betweenness** of points from the subsets S within N and is well-known and analyzed in Social Network Analysis.

We will therefore call v_{global} the **Global Connectivity Function**. The first important insight concerns the convexity of v_{global}.

2.1 Lemma: *The game with v_{global} as characteristic function is convex.*

Proof: To prove this statement, we only need to have a look at marginal contributions of players i. In doing so, we conclude that

$$v_{global}(S \cup \{i\}) - v_{global}(S) = \sum_{s,t \in S \cup \{i\}} \sum_{\psi \in \Psi_{s,t}, i \in \psi} u(\psi) \qquad (\forall S \subset N, i \notin S). \tag{3}$$

Since the right side of this equation is monotone increasing in S, we conclude that v_{global} is convex. □

The second characteristic function is similarly defined, but measures the set of shortest paths relative to a coalition S. Formally, we define the function v_{bypass} by

$$v_{bypass}(S) := \sum_{s,t \in S, s \neq t} \sum_{\psi \in \Psi^S_{st}} u(\psi) \qquad (\forall S \subset N) \tag{4}$$

and call it the **Bypass Connectivity Function**.

The measure v_{bypass} seems to be very similar to v_{global} in the concept of Betweenness, but requires that the elements of subsets S of N are able to form connections by their own not being dependent on the other nodes of N. We will therefore address this concept by **Bypass Centrality** rather than Betweenness par excellence. For the complete set N the concepts of Bypass Centrality and Betweenness in the classical sense coincide, that means

$$v_{bypass}(N) = v_{global}(N). \tag{5}$$

It does not conform with the concept of Betweenness to assign values also to the end points of paths, which can better be characterized as dispatchers for connections. With respect to this inconsistency we can consider dispatching points to be more central for the concept of Closeness than for Betweenness. An illustrative example is the set of angle points of a block graph. Nevertheless, no path is complete without the end points. Therefore, they will be taken into account in the subsequent calculations. The convexity of v_{bypass} is not assured in general. But we can make a note on a weaker property.

2.2 Lemma: *The characteristic function v_{bypass} is super-additive.*

Proof: Let $S, T \subset N$ be two arbitrary disjoint sets. Then

$$\begin{aligned} & v_{bypass}(S) + v_{bypass}(T) = && (6)\\ & \quad = \sum_{\psi \in \Psi^S} u(\psi) + \sum_{\psi \in \Psi^T} u(\psi) \leq && (7)\\ & \quad \leq \sum_{\psi \in \Psi^{S \cup T}} u(\psi) = && (8)\\ & \quad = v_{bypass}(S \cup T). && (9) \end{aligned}$$

□

This result implies that there exists an individual rational imputation of the game with characteristic function v_{bypass}, which is of course also evident from the fact

$$v_{bypass}(\{s\}) = 0 \quad \forall s \in N. \tag{10}$$

Therefore, the value of the whole network can be distributed among the members anyway as long as each member gets a non-negative share.

For bypassing we will consider a suitable set of paths connecting a given pair of vertices s and t in the network. To make this approach more precise we define an algorithm which identifies the paths of interest.

2.3 Algorithm: Given an arbitrary pair of vertices s and t in the network, we start with a global shortest path ψ_{st}^0 connecting both vertices. This path may be the result of any efficient algorithm like the well-known technique of Dijkstra [1]. Given any path ψ_{st}^i connecting the vertices s and t, we remove the set $\nabla\psi_{st}^i$ of internal vertices of the path from the network and apply the algorithm to the remaining network. The algorithm will give us a new path ψ_{st}^{i+1} or tell us that there exists no further connection. This procedure will identify a series $\psi_{st}^0, \ldots, \psi_{st}^m$ of paths with non-decreasing length. If we assume that there are no connecting paths for s and t with the same length, the given series will be unique. The obtained series of paths will be called the **Bypass Bundle** of s and t.

We have to note that this algorithm will stop immediately, if we start with a path for which s and t are connected by an edge. If we want to avoid this effect, we can also block the edges of the iterated paths instead of removing the interior points. But, this question is not important for the further investigations.

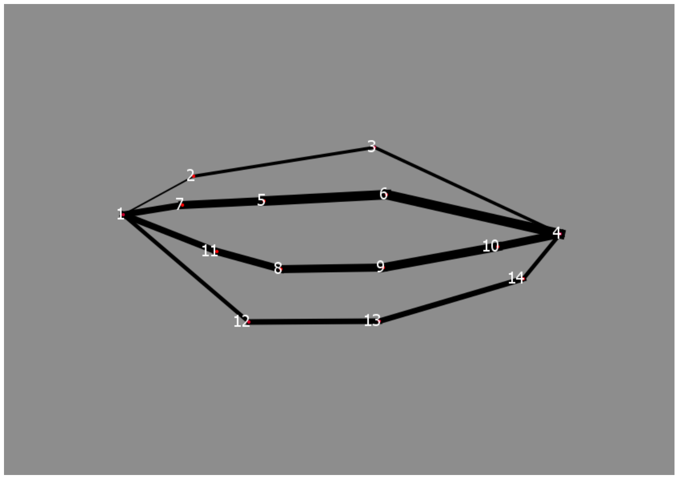

Figure 1: Bypass Bundle obtained by algorithm 2.3

Normally, we will not be interested in the whole set of bypasses. We will rather consider a limited number of such paths depending on the structure of the network. Some bypasses may be too long in comparison with the shortest connection and will therefore be of minor interest. If the connectivity of the network is weak enough, there may also be a natural limitation of the number of bypasses.

The above defined algorithm does of course not end up in all connections between vertices s and t. This will be illustrated by the following example.

2.4 Example: Figure 2 shows how bypassing works.

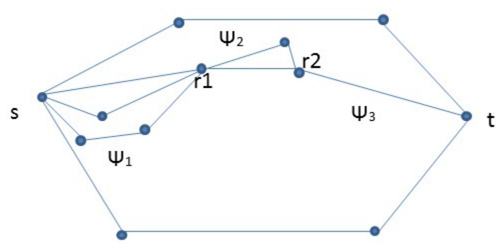

Figure 2: Bypassing by minor valued connections

The paths ψ_1 and ψ_2 are bypasses for ψ_{s,r_1} and ψ_{r_1,r_2}, respectively. But the combined path $\psi_1 \oplus \psi_2 \oplus \psi_3$ is not considered as bypass for $\psi_{s,t} = \psi_{s,r_1} \oplus \psi_{r_1,r_2} \oplus \psi_{r_2,t}$, because there exist common intermediate points.

In the sequel, we assume that different paths between two vertices have different length. We will call this kind of network a $non-degenerate\ network$. In this case, $\#\Psi_{st}^S$ is a boolean function with values 0 or 1 indicating, whether a connection of s and t exists in S or not. The first result is an immediate consequence of the initial remarks about the length of shortest paths relative to a subset.

2.5 Lemma: *In a non-degenerate network we have*

$$v_{bypass} \leq v_{global} \tag{11}$$

and

$$v_{bypass}(N) = v_{global}(N) \tag{12}$$

This result shows that v_{bypass} is dominated by the convex characteristic function v_{global}, and on account of this, that the corresponding game has a nonempty Core. More precisely, the Core of the game with v_{bypass} includes the Core of the game with v_{global}. Hence, the Shapley Value for v_{global} is a member of the Core for v_{bypass}. It is still an open question, in which cases the Shapley Value for v_{bypass} is a Core Allocation.

Some examples show that v_{bypass} is generally not convex. Nevertheless, there exist many examples, where the Shapley Value for v_{bypass} is a Core Allocation even though not the center of the Core.

The question whether the Shapley Value is generally part of the Core, seems to be a bit difficult. In a first, but very simple case the open question can be answered positively. The following analysis focuses on the characteristic function v_{bypass} and the relationship between Core Allocations and Shapley Values.

3 Acyclic Networks

For a special case, we can answer the open question of group rationality of the Shapley Value positively.

3.1 Lemma: *Let N be an acyclic graph, then v_{bypass} is a convex characteristic function and therefore, the Shapley Value $SV_{v_{bypass}}$ is an element of the Core* **Core**(v_{bypass})*.*

Proof: To prove this statement, we only need to have a look at marginal contributions of players i. Since N is an acyclic graph, the connecting path between two different points in N is unique, if it exists. Thus, we conclude that

$$\psi \in \Psi^S \Leftrightarrow \psi \in \Psi^N, \psi \subset S \qquad \forall S \subset N, \tag{13}$$

which yields us the equality $v_{bypass} = v_{global}$. The assertion is now a consequence of Lemma 2.1. □

4 Block Graphs

Block graphs have some nice properties in the context of Shapley Values and Core of vertex coalitions. A game theoretic analysis of this type of graphs is given in the bachelor thesis of a degree holder in mathematics of the German Feruniversität [2]. We will only sum up a few results of this analysis.

4.1 Definition: *A **connected graph** $G(N, E)$ is a graph, for which each pair of distinct vertices is connected by a path. A graph $G(N, E)$ is said to be a **Block Graph** if and only if for each pair of connected sub-graphs, the intersection is a connected sub-graph or empty. A **complete subset** of a network is a graph for which all vertices are mutually connected.*

Within a block graph each closed not self-crossing cycle must be a complete sub-graph. This property is essential for the proof of the following fact:

$$\Psi^S \cap \Psi^T = \Psi^{S \cap T} \quad \forall S, T \subset N, S \cap T \neq \emptyset. \tag{14}$$

A detailed proof is given in [2]. Applying this result to a given set $S \subset N$, we conclude

$$\Psi^S \subset \Psi^N \quad \forall S \subset N. \tag{15}$$

Therefore, the local shortest paths are all global shortest paths within block graphs. As a consequence of this result, we obtain an important property of block graphs.

4.2 Theorem: *For each block graph $G(N, E)$ we have*

$$v_{bypass} = v_{global} \tag{16}$$

Moreover, this leads us to the following game theoretic insight.

4.3 Corollary: *Let $G(N, E)$ be a block graph. Then v_{bypass} is a convex characteristic function and therefore, the Shapley Value $SV_{v_{bypass}}$ is an element of the Core* **Core**(v_{bypass}).

Proof: The proof is an immediate consequence of the previous theorem together with Lemma 2.1. □

5 Probabilistic Analysis

The following result serves as preparation for a probabilistic analysis of the Shapley Values of members in the network.

5.1 Lemma: *We denote the set of all permutations of N by* $\mathbf{Perm}(N)$*. For each subset S of N and $k \in N$, we define*

$$M_S(k) := \{\pi \in \mathbf{Perm}(N) | \pi(i) \leq \pi(k)(i \in S)\}. \tag{17}$$

Then, the probability of each $k \in S$ to be the last entry of S is given by

$$\mathbf{Prob}(M_S(k)) = \frac{1}{\#S} \quad \forall S \subset N, k \in S, \tag{18}$$

Moreover, each pair of subsets L, R of N satisfies

$$M_L(k) \cap M_R(k) = M_{L \cup R}(k). \tag{19}$$

In addition, the algebra of the sets $M_L(k)$ provides a scheme to calculate the probabilities for finite unions of sets.

5.2 Remark: Let be given an arbitrary finite sequence $K_1, \ldots, K_m$ of subsets of N. Then, by the well-known formula of Poincaré and Sylvester, we obtain the expression

$$\begin{aligned} \mathbf{Prob}\left(\bigcup_{i=1}^{m} M_{K_i}(k)\right) &= \sum_{i=1}^{m}\left((-1)^{i+1} \sum_{I \subset \{1,\ldots,m\}, \#I = i} \mathbf{Prob}\left(\bigcap_{i \in I} M_{K_i}(k)\right)\right) = \\ &= \sum_{i=1}^{m}\left((-1)^{i+1} \sum_{I \subset \{1,\ldots,m\}, \#I = i} \mathbf{Prob}\left(M_{\cup_{i \in I} K_i}(k)\right)\right). \end{aligned} \tag{20}$$

The calculation steps for this expression need still exponential time depending on m. But keeping in mind that we want to apply these results to Bypass Bundles, we can restrain the number of bypasses artificially by a certain limit or apply it to networks which have per se a limited number of bypasses. For instance, closed circles will have at most two members in each Bypass Bundle independent from the number of vertices.

6 Analysis for Multiple Connecting Paths

We consider the impact of multiple connections between each pair of points. A stochastic analysis concerning permutations which assign certain values to vertices will be given. As described in the algorithm 2.3, we can identify a uniquely defined Bypass Bundle, if we assume the network to satisfy some non-degeneration conditions. In this way, we will analyse the situation of two vertices s, t for which the connecting paths are given by a sequence of paths $\psi_1, ..., \psi_h$ in strictly increasing order of their utility such that the interiors of the paths are pairwise disjoint. Suppose now, the vertex k is an element of ψ_l for some $l \in \{1, \ldots, h\}$.

We consider the pairwise disjoint sets $L := \bigcup_{i\in H} \psi_i$ with $H \subset \{1,\dots,h\}, l \in H$ and $\nabla\psi_i, i \in K$ with $K \subset H^c$ of paths not containing k in their interior. We are interested in permutations, for which the vertices of set L have all entry times less or equal to k and the sets $\nabla\psi_i, i \in K$ do all have at least one vertex with later entry time. First, we can put the following fact on the record.

6.1 Lemma: *A permutation π of all vertices puts k as last of the vertices of L and keeps all sets $\nabla\psi_i, i \in K$ incomplete at the entry time of k, if and only if*

$$\pi \in M_L(k) \setminus \bigcup_{i\in K} M_{L+\nabla\psi_i}(k). \tag{21}$$

Proof: The first statement of the assertion is equivalent to

$$\pi \in M_L(k) \quad \text{and} \quad \pi \notin M_{\nabla\psi_i}(k) \quad (i \in K), \tag{22}$$

which is in turn equivalent to

$$\pi \in M_L(k) \cap \bigcap_{i\in K} M^c_{\nabla\psi_i}(k). \tag{23}$$

From this fact, the assertion follows by a simple transformation. □

We will now give a probabilistic investigation of the case, where the entry time of k is the latest entry time of all members of paths with index in H and none of the paths $\psi_i (i \in K)$ is completed at the entry time of k. Clearly, the missing points of the paths $\psi_i (i \in K)$ must be interior points. We are interested in the set of permutations for which this event happens. This set of permutations is given by $D(k,H,K) := M_L(k) \setminus \bigcup_{i\in K} M_{L+\nabla\psi_i}(k)$ with $L := \left(\bigcup_{i\in H} \psi_i\right)$. The next result provides a simple scheme for the calculation of the probability of the incidence in question.

6.2 Corollary: *The probability for the event of k to be the last entry of $L := \left(\bigcup_{i\in H} \psi_i\right)$ and all paths $\psi_i, i \in K$ remain incomplete, is given by*

$$\begin{aligned}\mathbf{Prob}(D(k,H,K)) &= \sum_{I\subset K} (-1)^{\#I} \mathbf{Prob}\left(M_{L+\sum_{i\in I}\nabla\psi_i}(k)\right) = \\ &= \sum_{I\subset K} (-1)^{\#I} \frac{1}{\#L + \sum_{i\in I} \#\nabla\psi_i}.\end{aligned} \tag{24}$$

Proof: The calculation of the desired probability is a consequence of Remark 5.2 together with Lemma 5.1. □

Next, we consider the case, where ψ_l is the path with maximal utility of the paths $\psi_i (i \in H)$. Due to the increasing order of utilities, this situation implies that the marginal contribution of k is the utility of ψ_l minus the maximum of the utilities of paths with index in $H \setminus \{l\}$. In all cases where H contains paths with higher utilities than the utility of ψ_l, the contribution of k is zero. Therefore, these cases are irrelevant. We first put the following fact on the record.

6.3 Remark: We denote the utilities of the paths ψ_i by $u_i := u(\psi_i)$ for $i = 1, \ldots, h$ and set $u_0 := 0$. We define $H_j := \{j, l\}$ for $j = 1, \ldots, l$, $H_0 := \{l\}$ and $K_j := \{j+1, \ldots, h\} \setminus \{l\}$ for $j = 0, \ldots, l$. First, we assume $k \neq s, t$. Then, vertex k contributes a positive value for a permutation π, if and only if there exists a $j \in \{0, \ldots, l-1\}$ such that $\pi \in D(k, H_j, K_j)$. Moreover, all the sets $D(k, H_j, K_j)$ are pairwise disjoint for $j = 0, \ldots, l-1$. The vertex k contributes the value $u_l - u_j$ in each case. For $k = s$ or $k = t$, vertex k contributes u_l for a permutation π , if and only if $\pi \in D(k, H_l, K_l)$.

Keeping this remark in mind, we are now in a position to specify an algorithm, which determines the Shapley Value of k for the bypass characteristic function.

6.4 Algorithm: For all vertices s, t we construct the Bypass Bundle as described in algorithm 2.3, and calculate $\mathbf{Prob}(D(k, H_j, K_j))(u_l - u_j)$ for the notations given by remark 6.3. Then the mean marginal contribution of $k \neq s, t$ to the underlying partial network of the Bypass Bundle is given by

$$SV_{v_{bypass}}(k) = \sum_{j=0}^{l-1} \mathbf{Prob}(D(k, H_j, K_j))(u_l - u_j). \tag{25}$$

In the case $k = s$ or $k = t$ we get

$$SV_{v_{bypass}}(k) = \mathbf{Prob}(D(k, H_l, K_l))u_l. \tag{26}$$

The given algorithm for the calculation of the Shapley Values is weakly NP-complete, since the algorithm 2.3 takes polynomially increasing time and the steps of algorithm 6.4 have also polynomial dependence on the number of vertices and the number of considered bypasses in the network. By these means, we have proven the following theorem.

6.5 Theorem: *In the given situation, the Shapley Value can be calculated by the algorithms 2.3 and 6.4 in pseudo-polynomial time. The problem is therefore weakly NP-complete.*

7 Comparison of the Results with Classical Concepts

We will illustrate the results of the assessment scheme of Bypass Centrality in comparison to the concept of Global Betweenness Centrality using Shapley's Value as well as the classical concept of Betweenness Centrality.

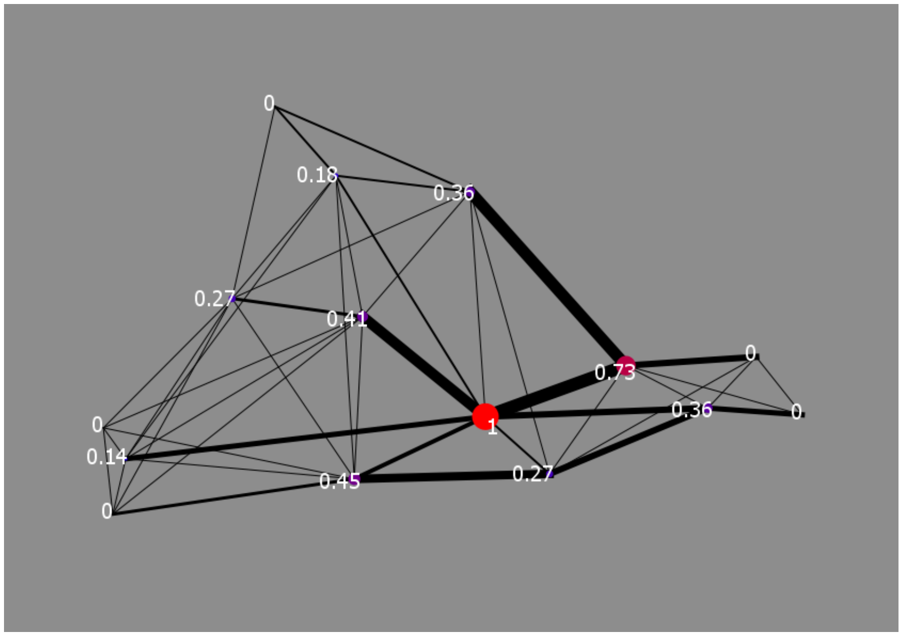

Figure 3: Classical Betweenness Centrality

Some of the vertices have zero value, because they never appear on shortest paths between other vertices.

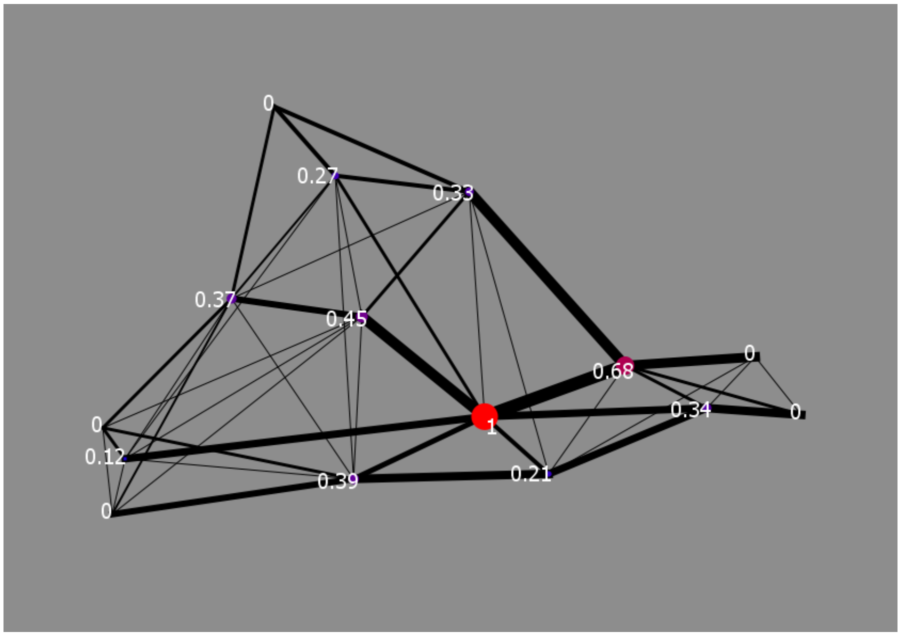

Figure 4: Global Betweenness Centrality by Shapley Value

The result is very similar to the classical Betweenness. The only difference is that paths, which appear more frequently in partial networks, assign higher values to their vertices.

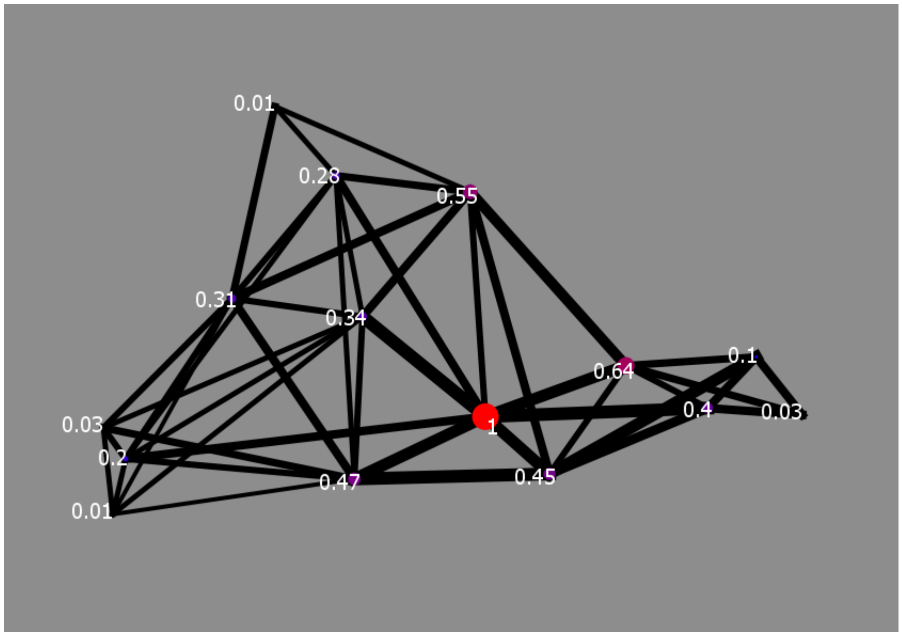

Figure 5: Bypass Centrality by Shapley Value

The assessment of the marginal area of the network is different from the previous cases. The higher assessment of these vertices results from their bypassing capabilities. The thickness of the edges reflects their use for the traffic in the network depending on the applied centrality measure. Our calculations make use of the Agent Based Simulation frame NetLogo [5] which can be downloaded from the cited website.

8 Conclusions

We have introduced a concept of Bypass Centrality, which better reflects the local structure of a network than the classical concepts. For some special cases we can show that the Shapley Values of vertices based on Bypass Centrality provide a Core Allocation. Moreover, we have found a method to calculate the Shapley Value for Bypass Centrality in polynomial time, if we assume some limitations on the number of bypassing connections. For large networks, this amount of processing time may still increase enormously. Nevertheless, we have explained an approach which opens access to much larger networks than the classical calculation of the Shapley Value running through all permutations of entry times for all vertices. The latter procedure is computationally unacceptable even for relatively small networks. This gives some justification to look for alternative approaches like in our case.

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