Discussion Paper 98-10

Market Depth and Order Size

- An Analysis of Permanent Price Effects of DAX Futures´ Trades -

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Revised version: February 1998

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Abstract

In this paper we empirically analyze the permanent price impact of trades by investigating the relation between unexpected net order flow and price changes. We use intraday data on German index futures. Our analysis based on a neural network model suggests that the assumption of a linear impact of orders on prices (which is often used in theoretical papers) is highly questionable. Therefore, empirical studies, comparing the depth of different markets, should be based on the whole price impact function instead of a simple ratio. To allow the market depth to depend on trade volume could open promising avenues for further theoretical research. This could lead to quite different trading strategies as in traditional models.

Acknowledgements

We are grateful for helpful comments of David Brown, Herbert Buscher, Frank deJong, Bruce Lehmann, Jonas Niemayer, Dirk Schiereck, participants of the 1997 European Finance Association Meeting, the 1997 CBOT European Futures Research Symposium, and an anonymous referee.

Non-Technical Summary

In this paper we analyze the permanent price impact of trades in financial markets by investigating the relation between unexpected net order flow and price changes. We mainly focus on four questions. Does large order flow convey more information than small order flow? Does net buy and net sell volume convey the same amount of information? Does the information content of order flow increase linearly with its size? Are there alternative measures of trading activity which convey more information than order flow?

We use intraday data on German index futures (DAX futures). Our analysis based on a neural network model provides us with the following results. Firstly, the information content of order flow increases with its size. Secondly, we find that buyer initiated trades and seller initiated trades do not differ with respect to their information content. Thirdly, the relation between net order flow and price changes is strongly non-linear. Large orders lead to relatively small price changes whereas small orders lead to relatively large price changes. Finally, net order flow measured as contracts traded offers the best explanation for price changes. Net number of trades explains price changes almost as well. However, the relative net order flow, i.e. net order flow divided by volume, does not provide the same level of explanation. The results are found to be quite robust with respect to the estimation procedure.

Overall, the results of our paper suggest that the assumption of a linear impact of orders on prices (which is often used in theoretical papers) is highly questionable. Thus, market depth cannot be described sufficiently by a single number. Therefore, empirical studies, comparing the depth of different markets, should be based on the whole price impact function instead of a simple ratio. To allow the market depth to depend on trade volume could open promising avenues for further theoretical research. This could lead to quite different trading strategies as in traditional models.

1. Introduction

Trades move security prices for two reasons. Firstly, trades convey information causing other investors to revise their beliefs. Secondly, trades move prices due to market frictions. Whereas the first price effect is permanent, the second one is transient. This paper concentrates on the first component. It analyzes the empirical relationship between the size of order flow and its permanent price impact. The particular form of this relation is important for market comparisons and for the development of trading strategies.

Kyle´s (1985) well known paper is a seminal theoretical contribution to determine the permanent price impact of trades. Kyle assumes that a monopolistic insider, a market maker, and noise traders interact. The market maker observes the aggregate net order flow of insider and noise traders, X_t. This order flow provides a signal about the liquidation value of the asset to the market maker. Based on this signal, she revises her beliefs and sets the price such that it equals the expected liquidation value given the observed order flow. The resulting equilibrium price change, $\Delta P_t \equiv P_t - P_{t-1}$, takes the form

(1) $$\Delta P_t = \lambda \, X_t ,$$

where $X_t > 0$ indicates a net buy and $X_t < 0$ a net sell order flow. The positive parameter λ is a measure of market depth. The smaller the value of λ, the deeper a market. It determines how much the market maker adjusts the price in response to net order flow.

Equation (1) implies that price changes are linear in order flow. This linear structure crucially relies upon two assumptions. Firstly, orders of the insider depend, in a linear way, on private information. Secondly, order flow and equilibrium price are continuous variables, i.e. order and price discreteness are negligible. It is questionable whether both assumptions provide good description of actual markets. As O`Hara (1995, p.96) states, "... a linear pricing rule is, at best, an approximation of the actual price behavior." It is an empirical task to estimate the actual form of price impact. In our paper we empirically address four main questions related to (1):

- Does large order flow convey more information than small order flow?

- Does net buy and net sell volume of the same size convey the same amount of information?
- Does the information content of order flow increase linearly with its size?
- Are there alternative measures of trading activity which convey more information than order flow does?

In the model of Kyle (1985) price changes are completely information induced. However, as noted above, prices might also change due to market frictions. This dichotomy [Hasbrouck (1996)] needs to be considered when empirically studying the information content of order flow. Empirical work on the information content of trades has focused mainly on block trades [see for example Kraus/Stoll (1972), Holthausen/Leftwich/Mayers (1987, 1990), Chan/Lakonishok (1995), Keim/Madhavan (1996), LaPlante/Muscarella (1997)]. Block studies normally partition the total price impact of a block into a permanent [information induced] and a temporary [market friction induced] component and analyze both components for different block sizes. Therefore, block studies determine (among others) the depth of markets, but only at time when block trading occurs. A second line of research avoids this concentration on block events and measures general market depth [see for example Hasbrouck (1991), Algert (1990)]. They assume that (unexpected) midquote changes are information induced so that the permanent price impact of trades (λ) can be estimated by relating changes of midquotes to net order flow. In this paper we follow that approach to answer the questions raised above.

Several aspects of the four main questions (stated above) have been addressed in earlier studies. Does large net order flow convey more information than small order flow? The empirical results of Hasbrouck (1988, 1991), Algert (1990), Madhavan/Smidt (1991), and Easley/Kiefer/O'Hara (1997) suggest a positive answer.

Does net buy and net sell volume of the same size convey the same amount of information? Karpoff (1988) argues that short selling restrictions might prevent insiders from exploiting negative information in the stock market. This should lead to a higher information content of buy orders as compared to sell orders. Empirical support in favor of this short selling hypothesis is provided for example by Karpoff (1988), Madhavan/Smidt (1991), and Chan/Lakonishok (1993).

Does the information content of order flow increase linearly with its size? Empirical work on this issue is provided by Hasbrouck (1988, 1991) and Algert (1990). They both find evidence for a non-linear, concave relation between permanent price change and order flow. Large order flow seems to convey more information than small order flow, but the marginal information content seems to decrease. This result is consistent with the findings of several studies which do not concentrate on the permanent component of price changes, but study transaction price changes [Marsh/Rock (1986), Hausmann/Lo/MacKinlay (1992), deJong/Nijman/Röell (1995) and Plexus/Group (1996)] and the shape of the order book [Biais/Hillion/Spatt (1995)].

Are there alternative measures of trading activity which convey more information than order flow does? Barclay/Warners (1993) find empirical evidence that informed investors tend to use medium size orders. This suggests that insiders exploit signals by trading frequently. Consequently, information can be drawn from the frequency and not from the size of orders. This finding is consistent with Jones/Kaul/Lipson (1994), who report that it is the occurrence of transactions per se, and not their size, that generates volatility. Therefore, the number of orders might provide superior information in comparison with order flow.

This study extends previous work in several respects. Firstly, we use neural networks to estimate the relation between order flow and midquote changes. Neural networks combine the advantages of parametric methods (simple parameter inference) and nonparametric methods (functional flexibility) used in previous studies by Hasbrouk (1991) and Algert (1990). Secondly, we employ a non-linear forecast model in order to extract unexpected order flow and unexpected midquote changes. As for example Madhavan/Smidt (1991) and Madhavan/Richardson/Roomans (1997) point out, the revision in beliefs does not depend on order flow but on unexpected order flow. The results of Bessembinder/Seguin (1993) point in the same direction, as unexpected volume is found to determine volatility. Thirdly, we test whether the information content of buy orders is identical to that of sell orders. Such an asymmetry in price responses to buy and sell orders might create an opportunity for profitable price manipulation [Allen/Gorton (1992)]. Finally, we study the robustness of our results with respect to different measures of order imbalance. This allows us to analyze the information content of different trade variables. For example, Easley/O'Hara (1992) argue that the number of trades provides information whereas in Easley/O'Hara (1987) trade size is informative.

The organization of the paper is as follows. In Section 2 we formalize our hypotheses about the price response of net order flow and describe the econometric framework which is used to test them. In this context neural networks are introduced and briefly contrasted with other approaches found in the literature. Section 3 includes the data description. In Section 4 we present our empirical results. This includes some robustness checks with respect to other non-linear regression techniques and different measures of order flow. Section 5 concludes. Since this paper focuses entirely on permanent price impact, we use price change and permanent price change interchangeable. The size of order flow within a given interval is also called order size.

2. Hypotheses and Econometric Methodology

It is the main objective of this study to characterize the information content of trades through a function λ, which might depend on the net order flow X. Important aspects of λ´s functional form can be expressed as the following hypotheses: (i) Net demand ($X > 0$) leads to a price increase. (ii) Net supply ($X < 0$) leads to a price decrease. (iii) The information content is identical for net demand and net supply of the same size. (iv) The information content of order flow increases linearly with its size.

As a starting point, it is instructive to look at the hypotheses (i) to (iv) in the context of a linear model.

(2) $$\Delta P_t = \alpha_0 + \alpha_1 X_t + \varepsilon_t \ .$$

Equation (2) is a slightly more general formulation of (1), which includes a possibly non zero intercept and a zero mean stochastic error term ε_t. To allow for different slope coefficients for positive and negative order flow, we define X_t^+ as

(3) $$X_t^+ \equiv \begin{cases} X_t, \text{when } X_t > 0 \\ 0, \text{ when } X_t \leq 0 \end{cases} ,$$

i.e. X_t^+ equals the net order flow whenever it is positive and takes a value of zero otherwise. Using this definition, the model[1]

(4) $$\Delta P_t = \alpha_0 + \alpha_1 X_t + \alpha_2 X_t^+ + \varepsilon_t$$

allows us to state hypotheses (i) to (iii) as the following parameter restrictions. If $\alpha_1 + \alpha_2 > 0$, then net demand leads to a price increase (hypothesis i). If $\alpha_1 > 0$, then net supply leads to a price decrease (hypothesis ii). If $\alpha_2 = 0$, then the information content is identical for net demand and net supply of the same size (hypothesis iii). A crucial limitation of model (4) is the assumption of a (piecewise) linear relationship between net order flow and price change. Consequently, model (4) can't be used to test for non-linearity of the relationship.

To accomplish this, a more flexible functional form is needed, which should still allow for statistical tests of the hypotheses (i) to (iii). Neural networks are ideally suited for this purpose. Our hypotheses can still be formalized as simple parameter restrictions and checked by statistical test procedures. In addition, neural networks are flexible enough to approximate virtually any (measurable) function up to an arbitrary degree of accuracy [see for example Hornik/Stinchcombe/White (1989)]. Furthermore, Chen/White (1997) extended the results of Barron (1993) by showing an improved rate of approximation to an unknown target function. As a consequence, we might achieve quite accurate estimates of the underlying function even with quite parsimonious networks.

The term neural network is not uniquely defined and comprises a variety of different network types and models. In this study we use the most common type, a single layer perceptron network. A linear regression model can easily be nested in such a network. For our application this leads to the following specification:

(5) $$\Delta P_t = \alpha_0 + \alpha_1 X_t + \alpha_2 X_t^+ + \sum_{h=1}^{H} \beta_h \cdot g(\gamma_{1h} X_t + \gamma_{2h} X_t^+) + \varepsilon_t \ .$$

[1] The parameters and error terms in (4) can clearly be different from those of equation (2), but for ease of notation we use the same symbols throughout the paper.

Model (5) encompasses model (4) and adds a non-linear network part. The function λ resulting from model (5) is given as $\alpha_1 + \frac{\alpha_2 X_t^+ + \sum_{h=1}^{H} \beta_h \cdot g(\gamma_{1h} X_t + \gamma_{2h} X_t^+)}{X_t}$. The network part of (5) consists of H hidden units, where $\beta_1, \ldots, \beta_H, \gamma_{11}, \ldots, \gamma_{1H}$, and $\gamma_{21}, \ldots, \gamma_{2H}$ are unknown parameters and g is a non-linear transfer function. The number H of hidden units is as yet unspecified. In general, the more complex the relationship between order flow and midquote change, the more hidden units will be needed to adequately approximate it. Most commonly, the transfer function g is chosen to be either the logistic or the hyperbolic tangent function. The latter is used here. As $\tanh(-x) = -\tanh(x)$ and $\tanh(0) = 0$, this choice simplifies the analysis of the model. In the framework of model (5) the hypotheses (i) to (iv) can be examined as follows:

(i) If $\alpha_1 + \alpha_2 > 0$ and $\beta_h \cdot (\gamma_{h1} + \gamma_{h2}) > 0, \forall h = 1, \ldots, H$, then net demand leads to a price increase, whose magnitude strictly increases with net demand.[2]

(ii) If $\alpha_1 > 0$ and $\beta_h \cdot \gamma_{h1} > 0, \forall h = 1, \ldots, H$, then net supply leads to a price decrease, whose magnitude strictly increases with net supply.

(iii) If $\alpha_2 = 0$ and $\gamma_{2h} = 0, \forall h = 1, \ldots, H$, then the information content is identical for net demand and net supply of the same size.[3]

(iv) If $\beta_1 = \beta_2 = \ldots = \beta_H = 0$, then price changes depend (piecewise) linearly on order flow.

We test the linear model against the alternative of a non-linear neural network (hypothesis iv) using tests suggested by White (1989b) and Teräsvirta/Lin/Granger (1993). The first method was compared to other linearity tests in the study of Lee/White/Granger (1993) and found to be very powerful against a variety of non-linear alternatives. The second test had even better results than the first one in the simulation study of Teräsvirta/Lin/Granger. Once non-linearity

2 This follows from the fact that the hyperbolic tangent is a strictly increasing function.
3 This follows from the symmetry of the hyperbolic tangent function around the origin.

is checked and the number of relevant hidden units is specified, hypotheses (i) to (iii) can be tested by standard Wald-tests as proposed by White (1989a).

In the literature on the price - order flow relationship alternative models have been used to capture non-linearities. For example, Hasbrouck (1991) includes squared trade size as an additional regressor, and deJong/Nijman/Röell (1995) add the reciprocal of trade size to the regression equation. The advantage of these specifications is obvious. The model can be readily estimated and allows for straightforward statistical inference and significance tests. However, a drawback lies in the restrictive assumption that the relationship is characterized by a quadratic or reciprocal.

Algert (1990) uses the locally weighted regression of Cleveland/Devlin (1988) to analyze the impact of order imbalances on price revisions. Due to the local fitting, this method is capable to describe many kinds of functional forms. This flexibility however comes at a cost. In the context of this study the main disadvantage lies in the difficulty to perform inference, for example about symmetric effects of positive and negative order imbalances. Moreover, computational costs are quite high, since a separate regression has to be run for every point of the target function. In addition, the choice of adequate data windows and weighting functions is somehow arbitrary and may have an impact on the results. Similar pros and cons apply to other local fitting procedures like kernel regression.

Hausmann/Lo/MacKinlay (1992) focus on the discreteness of price changes due to a minimum tick size and use an ordered probit model to analyze (among other things) the price impact of orders. The ordered probit model assumes a linear relationship between the regressors (for example order imbalance) and a latent variable which in turn governs price changes. The relation between this latent variable and the discrete price changes is determined by the breakpoints of the ordered probit model. Depending on the breakpoints, different non-linear relations can be obtained. A difficulty occurs, however, when discreteness effects change with shifts in price level. To avoid this difficulty we concentrate on returns instead of price changes. Moreover, since we try to forecast expected price changes by means of a neural network model in a preliminary step, the unexpected changes, i.e. the residuals of the forecast model, no longer take a few discrete values.

In our view neural networks combine the advantages of simple parametric models with those of nonparametric local fitting procedures. Similar to linear regression models neural networks are still based on parameters, which facilitates statistical inference. Like nonparametric methods, neural networks do not require restrictive assumptions about the form of the target function. From a computational point of view, numerical optimization methods are needed to estimate neural networks. However, they are usually less costly than alternative nonparametric methods. Furthermore, once the network is estimated it provides a closed functional form which is readily available for further analysis. An example of which is the calculation of derivatives.

3. Data Description

Our empirical study is based on data of German Stock Index futures (DAX futures) from September 17th 1993 to September 15th 1994. DAX futures are screen traded on the fully computerized German Futures and Options Exchange (DTB) between 9.30 a.m. and 4.00 p.m.. Liquidity is provided by traders and voluntary market makers who place limit orders into the centralized electronic order book. This order book is open to all market participants. All orders (market and limit orders) are submitted electronically to the market via a trading terminal where orders are automatically matched, based on price and time priority. The minimum transaction size is one futures contract, which amounts to DM 100 per index point, and the minimum tick size is half an index point.[4]

Our data set consists of all time stamped best bid quotes, best ask quotes, transaction prices, and transaction quantities for DAX futures. We concentrate on the nearest to deliver contracts. There is no information in the data set on whether a trade is initiated by a buyer or a seller. However, this information is needed to estimate models (4) and (5). Therefore, we classify trades as buyer or seller initiated using an algorithm similar to Lee and Ready (1991). A trade is classified as buyer-initiated if the transaction price is equal to or higher than the current best ask price. If the transaction price is equal to or lower than the current best bid price, the trade is classified as seller-initiated. The classification procedure leads to a time series which includes bid and ask quotes, transaction prices, transaction sizes and information on whether the trade is buyer or seller initiated.

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4 See for example Bühler/Kempf (1995) for a more detailed description of DAX futures.

In the next step one minute intervals are formed. The pooling of observations has the advantage that we achieve a higher variation in net order flow than would be obtained for single trades. This allows us to study price effects for a wider range of net order flows.[5] However, the pooling comes at the cost that causality might run not only from order flow to prices, but also from price changes to order flow. Investors might react to price changes by adjusting their trading quantity. Examples include program trading, momentum trading and index arbitrage. When estimating our models using pooled data, we implicitly assume that within the same interval the causality runs only from order flow to prices. By choosing a short interval of one minute we try to keep the effects of price changes on net order flow small, while still preserving a reasonable range of values for the net order flow.

Table 1: Descriptive Statistics of Logarithmic Price Changes and Net Order Flow

	Logarithmic Midquote Changes in Percentage Points (Δp)	**Net Order Flow in Number of Contracts (X)**
Mean	-0.000063	0.0048
Standard Deviation	0.0403	35.87
Minimum	-0.779	-1817
1%-Quantile	-0.109	-104
25%-Quantile	-0.023	-12
50%-Quantile	-0.0	0
75%-Quantile	0.023	12
99%-Quantile	0.104	105
Maximum	0.735	1824
Autocorrelations:		
Lag 1	0.055	0.185
Lag 2	0.021	0.081
Lag 3	0.003	0.055
Lag 4	0.006	0.043
Lag 5	-0.002	0.034

5 Some quantiles of the pooled net order flow are shown in Table 1. On a trade-by-trade basis, trade size is within a much smaller range. For example, less than 0.2% of all buyer initiated trades exceeds 50 contracts. The same is true for seller initiated trades.

For each one minute interval we calculate the net order flow, X, as the difference between the buyer and seller initiated trading volume. Trading volume is measured as the number of futures contracts traded. The price change during each interval, Δp, is calculated as the difference of the log midquotes prevailing at the end of successive intervals.[6] These logarithmic changes in midquotes are used as proxies for information induced price effects. To avoid possible biases at the beginning of a trading session, observations within the first 15 minutes after the opening of the DTB are excluded. This procedure leaves us with 92491 observations. Descriptive statistics concerning midquote changes and net order flow are provided in Table 1.

For both, midquote changes and net order flow, the mean values are very small and not significantly different from zero. They exhibit a positive autocorrelation at the first few lags, while the effect is stronger for net order flow. As can be seen from the quantiles, both variables are almost symmetric around zero. Very few extremely large observations of absolute net order flow occur. The estimation results are not sensitive to the inclusion or exclusion of these values. Thus, all estimation and test results reported in the next section are based on the whole data set.

4. Results

4.1 Forecast Model

In order to distinguish between expected and unexpected changes in midquotes and order flow we use a forecast model based on past observations of both variables. The forecast procedure consists of three steps. In the first step the following linear vector autoregression is used:

$$\Delta p_t = b_{10} + \sum_{i=1}^{I} \left[b_{1i} \Delta p_{t-i} + c_{1i} X_{t-i} \right] + \varepsilon_{1t} \tag{6}$$

$$X_t = b_{20} + \sum_{j=1}^{J} \left[b_{2j} \Delta p_{t-j} + c_{2j} X_{t-j} \right] + \varepsilon_{2t} \, . \tag{7}$$

6 Note that Δp is used instead of ΔP to indicate a change in logarithmic prices.

The number of lags (I and J) are chosen independently for both equations by means of the information criterion of Schwarz (1978). It turns out that observations of up to four periods in the past have noticeable explanatory power for net order flow. The price change is best explained with two lags only. Table 2 provides the OLS regression results for equations (6) and (7).

Both variables enter significantly into either equation indicating feedback effects. A look at the explanatory power shows that price changes can be explained by past values only to a very small extent. This is consistent with the idea of informational efficient markets. As already seen by the autocorrelations in Table 1, net order flow shows a stronger persistence. Although there are eight significant terms in equation (7), the adjusted R-squared of about five percent is small.

Table 2: Results of the Linear Forecast Model

	Independent Variable: Δp [Equation (6)]		
	Estimated Parameters	**Standard Errors**	**p-value**
b_{10}	-0.000073	0.000132	0.577
b_{11}	0.01977	0.00915	0.031
b_{12}	0.01738	0.00601	0.004
c_{11}	0.000068	0.000009	0.000
c_{12}	-0.000016	0.000006	0.005
	Independent Variable: X [Equation (7)]		
	Estimated Parameters	**Standard Errors**	**p-value**
b_{20}	0.0329	0.115	0.775
b_{21}	120.65	5.42	0.000
b_{22}	43.12	4.21	0.000
b_{23}	18.71	4.10	0.000
b_{24}	16.47	3.97	0.000
c_{21}	0.0888	0.0081	0.000
c_{22}	0.0165	0.0047	0.000
c_{23}	0.0211	0.0048	0.000
c_{24}	0.0201	0.0045	0.000
$\overline{R}^2 = 0.0058$ (Equation 6)	Standard errors are obtained by the heteroskedasticity consistent		

$\overline{R}^2 = 0.0515$ (Equation 7)	estimator of White (1980).

In the second step it is checked whether past information has a non-linear impact on order flow or price change. The test of Teräsvirta/Lin/Granger (1993) indicates some non-linear structure in the residuals of models (6) and (7). Several neural networks were fitted to the residual series and compared by help of the Schwarz Information Criterion (SIC). For both variables, the most adequate network turned out to be a specification with two hidden units and the first two lags of both Δp and X as explanatory variables. The explanatory power of the networks is small, however. In either model the adjusted R-squared improves by less than one percent.

In the final step, we calculate the residuals of the networks estimated in the second step. These residuals are unpredictable (even by a non-linear model) and can serve as proxies for unexpected price change and net order flow. They are denoted by Δp^u and X^u, respectively.

4.2 Unexpected Price Changes and Order Flow

Using the variables, Δp^u and X^u, obtained in 4.1, we focus on the functional relationship between unexpected net order flow and price changes. Table 3 provides the results of the (piecewise) linear regression model of equation (4).

Table 3: Regression Results for the Piecewise Linear Model (4)

	Estimated Parameters	**Standard Errors**	**p-value**
α_0	0.000619	0.000957	0.517
α_1	0.000696	0.000055	0.000
α_2	-0.000058	0.000092	0.528
$\overline{R}^2 = 0.338$	Estimation is carried out by Ordinary Least Squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980).		

The slope coefficient α_1 is significantly positive as expected. With an estimated value of about 0.0007 a net supply (demand) of 100 units will on average cause a price decrease (increase) of 0.07 percentage points. If one assumes a DAX futures level of 2100 index points, which is about the average index level for the time period under study, the corresponding decrease in the futures price is around 1.5 index points. The intercept α_0 is insignificant indicating that on average prices do not change without order flow. Furthermore, α_2 is insignificant. This suggests that the information content of buyer and seller initiated trades does not differ significantly. This result is consistent with Karpoff (1988), since our study is based on data of a futures market where short selling is not restricted. An adjusted R-squared value of roughly 0.33 indicates that one third of the variation in the price changes can be attributed to net order flow.

Thus, the results provided so far are based on the linear model. Since earlier studies question the assumption of linearity, we proceed by examining whether a non-linear relation between unexpected price changes and net order flow exists in our data. For this purpose, we test for neglected non-linearity using the neural network tests of White (1989b) and Teräsvirta/Lin/Granger (1993). Both tests strongly reject the null hypothesis of a linear model against the alternative of some neglected hidden units. The corresponding test statistics are shown in Table 4. Thus, hypothesis (iv) is rejected, and a non-linear model such as (5) describes the relationship between net order flow and price changes more adequately than model (4).

The number of hidden units, *H*, in equation (5) is chosen based on the SIC. The SIC takes its minimal value for a network with one hidden unit. Since the choice of an appropriate number of hidden units is important for the power of our subsequent tests, we validate the network by another criterion. The network model (5) with one hidden unit is estimated, and tests for additional non-linearity in the data were carried out. The tests of White (1989b) and Teräsvirta/Lin/Granger (1993) do not detect any further non-linear structure, thus no additional hidden unit is necessary.

Table 4: Test Results for the Hypotheses (i) to (iv)

Hypothesis (H_0)	Test-Statistic	Distribution under H_0	p-value
(i): $(\alpha_1 + \alpha_2) = 0$ $\wedge \beta_1 \cdot (\gamma_{11} + \gamma_{12}) = 0$	(Wald) 178.30	$\chi^2(2)$	0.000
(ii): $\alpha_1 = 0$ $\wedge \beta_1 \cdot \gamma_{11} = 0$	(Wald) 218.59	$\chi^2(2)$	0.000

(iii): $\alpha_2 = 0 \wedge \gamma_{12} = 0$	(Wald) 4.18	$\chi^2(2)$	0.124
(iv): $\beta_1 = \beta_2 = \ldots = \beta_H = 0$	Teräsvirta/Lin/Granger, White[7] 777.78 , 143.12	$\chi^2(2)$, $\chi^2(2)$	0.000, 0.000

Once the number of hidden units is specified we can test hypotheses (i) to (iii) formulated in Section 2. Table 4 summarizes the corresponding results: The relationship between unexpected net order flow, X^u, and price change, Δp^u, is non-linear (hypothesis iv), strictly increasing with net demand (hypothesis i), strictly decreasing with net supply (hypothesis ii) and not significantly asymmetric (ii).

Up to this point, we have no information on how the linear and non-linear parts of model (5) contribute to the overall results. Such information is provided by the estimated parameters shown in Table 5. The slope coefficient α_1 takes a much smaller value than the one estimated in the linear models and it becomes insignificant. However, the positive coefficients β_1 and γ_{11} indicate a positive, but non-linear price impact of net order flow. The α_2 coefficient remains insignificant. Also, the non-linear part of the model does not induce asymmetry since γ_{12} is not significantly different from zero.

Table 5: Regression Results for the Neural Network Model (5)

	Estimated Parameters	**Standard Errors**	**p-value**
α_0	0.000355	0.00023	0.116
α_1	0.000108	0.00008	0.150
α_2	-0.000044	0.00003	0.125
β_1	0.056154	0.01044	0.000
γ_{11}	0.017712	0.00232	0.000
γ_{12}	0.000361	0.00127	0.777
$\overline{R}^2 = 0.395$	Estimation is carried out by Non-linear Least Squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980).		

[7] The test was performed using the first two principal components of the output of originally ten hidden transfer functions. These principal components account for more than 95% of the total variation.

Figure 1 depicts the relationship between unexpected net order flow and unexpected price changes. The non-linearity of the relation is apparent. A small net order flow leads to a relatively large absolute price change. A large net order flow leads to a relatively small absolute price change.

Figure 1: Relation Between Unexpected Net Order Flow and Unexpected Logarithmic Price Changes[8]

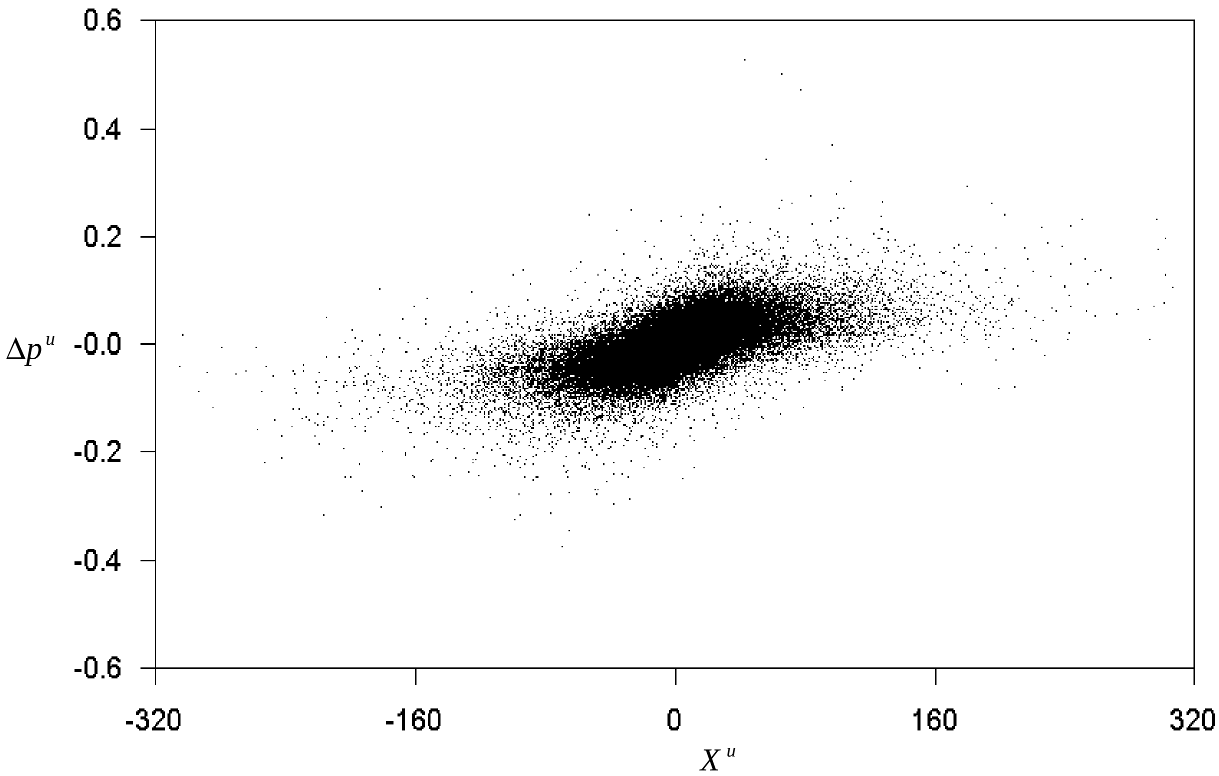
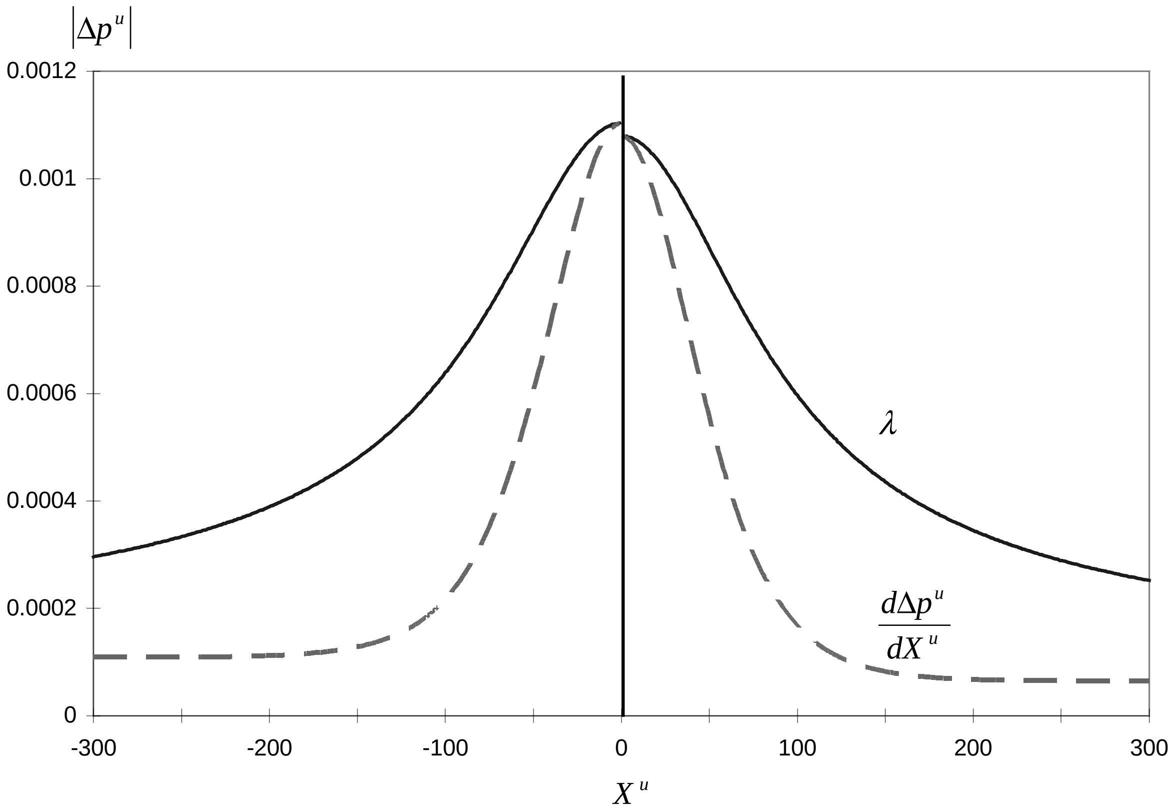


Figure 2 provides a further look at the non-linearity between unexpected net order flow and price changes. The dashed line shows the effect of a marginal change of net order flow on the (absolute) price change for different levels of order flow. It is derived from the estimated network model (5). The price change increases with net order flow almost linearly only for large positive or negative values of X^u. For absolute values of net order flow less then about 100 contracts, the non-linear part of the model clearly dominates. The impact of an additional unit of net order flow can be ten times as high for small orders than for large orders.

8 To achieve a better graphical display, Figure 1 concentrates on net order flow in the range from -300 to 300. This excludes only 30 observations.

Figure 2: Effect of a Marginal Unit of Extra Unexpected Net Order Flow on Unexpected Price Change and λ Implied by the Estimated Network Model (5)



The solid line in Figure 2 shows the function λ. It gives the average absolute unexpected price change caused by one unit of unexpected net order flow for different order sizes. The average price impact decreases monotonically with order flow. This indicates that the information content per trade unit is smaller, the larger the order size. This result is consistent with Algert (1990) and Hasbrouck (1991).

From our results, one important implication for market comparisons can be derived. Measuring market depth by a single number may lead to erroneous conclusions. Instead, a meaningful analysis should be based on the whole function $\lambda(X^u)$.

4.3 Alternative Non-linear Models

As mentioned in Section 3 above, alternative non-linear methods have been used for similar investigations in the literature, most notably in Hasbrouck (1991) and Algert (1990).

Therefore, it is instructive to check the robustness of our results with respect to the non-linear model chosen. Similar to Hasbrouck we run a quadratic regression of the following form:

(8) $$\Delta p_t^u = \alpha_0 + \alpha_1 X_t^u + \beta X_t^{u^*} + \varepsilon_t \ , \qquad \text{with } X_t^{u^*} \equiv X_t^u \cdot \left| X_t^u \right| .$$

Estimation results for model (8) are given in the following Table 6.

Table 6: Results for the Quadratic Regression Model (8)

	Estimated Parameters	**Standard Errors**	**p-value**
α_0	0.000012	0.000105	0.908
α_1	0.000772	0.0000078	0.000
β	-0.00000049	0.00000005	0.000
$\overline{R}^2 = 0.371$	Estimation is carried out by Ordinary Least Squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980).		

As expected, the quadratic term is highly significant and negative, indicating a concave relationship. Thus, we obtain qualitatively the same result as for the neural network model (5). Compared to the neural network, however, the adjusted R-squared has dropped.

Algert (1990) employs the locally weighted regression procedure of Cleveland/Devlin (1988). The method estimates the value of a regression function $y = f(x)$ at a particular point x in p-dimensional space, where p is the number of regressors. At the outset, one has to specify a fraction k from a total of N observations and identify the $k \cdot N$ points with the smallest (euclidean) distance to x. This subset of observations is used to perform a weighted linear regression on y, where the weights are given by:

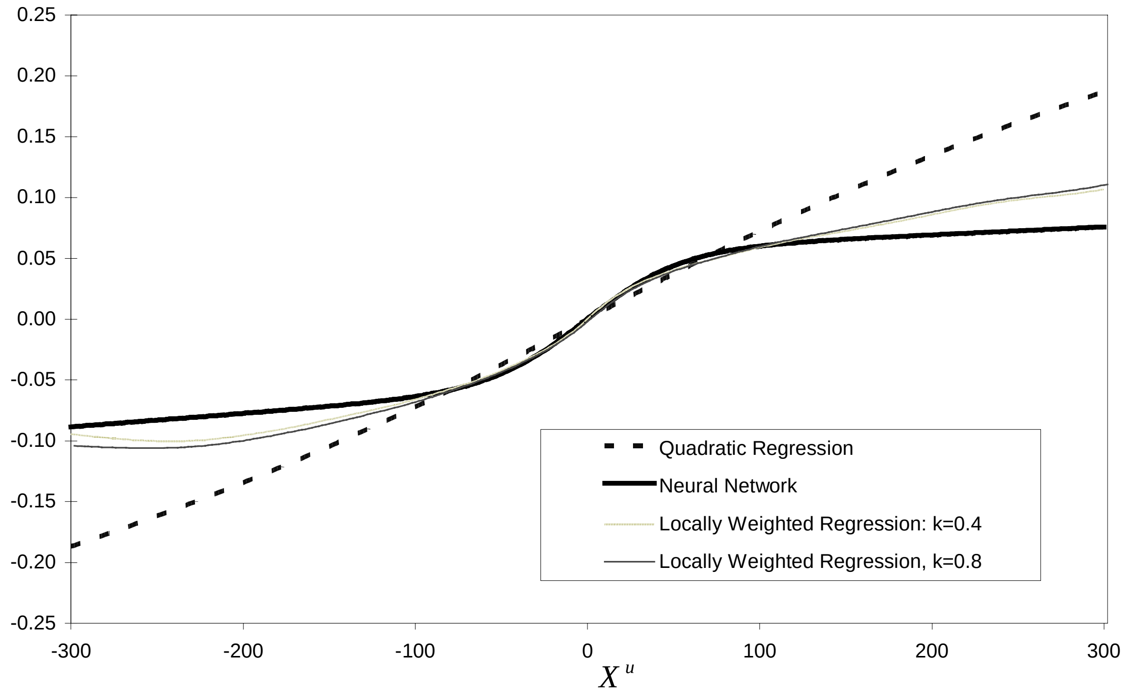
(9) $$w_i(x) = \left(1 - \left(\frac{d(x, x_i)}{d_{\max}} \right)^3 \right)^3 , \quad \text{for} \quad i = 1, \ldots, N \cdot k$$

In (9), $d(x, x_i)$ denotes the euclidean distance between x and the i´th observation x_i in the selected subset and $d_{\max}$ is the distance between x and the furthest observation in the subset.

The estimate $\hat{f}(x)$ can finally be obtained from the regression function as the fitted value $\hat{y}$ with the regressors set equal to x.

We use the method of Cleveland/Devlin (1988) to obtain an alternative estimate of the relationship between unexpected net order flow and quote changes. The regression function is estimated at 300 equally spaced points in the range -300 and 300 with k varying between 0.3 to 0.9. The results are quite insensitive to the particular choice of k. In Figure 3 the regression lines for $k = 0.4$ and $k = 0.8$, together with the fitted network and the quadratic regression function, are depicted.

Figure 3: Fitted Regression Lines for the Relation Between Unexpected Net Order Flow and Unexpected Logarithmic Price Changes. Different Non-linear Models: Neural Network, Quadratic Regression, Locally Weighted Regression



As Figure 3 shows, all estimation methods point in the same direction: The information content of trades grows less than linearly with the trade size. The methods differ, however, in the magnitude of this effect. Most notably, in comparison with the other methods, non-linearity is less pronounced for the quadratic regression. This may indicate that a quadratic term does not suffice to describe the underlying relationship.

4.4 Alternative Measures of Order Imbalance

The fourth questions raised in the introduction is whether alternative measures of order imbalance exist which convey more information than the difference between buyer initiated and seller initiated trade volume. If the net number of trades within an interval provides information, but not the size of these trades, the number of buy trades in excess to sell trades should lead to a better explanation of price changes than net volume. To investigate this, we proceeded as in Section 4.1 and estimated a non-linear forecast model for the net number of trades within a one minute interval. The residuals of this model serve as a proxy for unexpected order imbalance. As a proxy for unexpected price changes the forecast model of Section 4.1 is maintained. Alternatively, we estimated a forecast model for Δp with lagged values of the net number of trades as further explanatory variables. The residuals in either model are almost identical, thus it makes no difference which forecast model is used.

Figure 4: Relation Between Unexpected Net Number of Trades and Unexpected Logarithmic Price Change

A plot of the unexpected net number of trades against the unexpected price change is provided in Figure 4. Obviously, there is a positive relation. This is confirmed by the results of a piecewise linear regression given in Table 7. Non-linearity does not seem to exist to a noticeable extent. The neural network test of Teräsvirta/Lin/Granger (1993) for neglected non-linearity in the residuals of the piecewise linear model confirms this visual impression. The test statistic of 3.76 is not significant at a five percent level, when compared with the relevant critical values of the $\chi^2(2)$ distribution. Thus, the linear model of Table 7 can be maintained as an adequate characterization of the relationship. When compared with the most suitable model for net volume (Table 4), the explanatory power of the net number of trades is similar but slightly smaller. The adjusted R-squared decreases from to 0.395 to 0.374. In addition to the number of trades, the trade sizes seem to contain at least some information.

Table 7: Regression Results for a Piecewise Linear Model: Net Order Flow Measured by the Unexpected Net Number of Trades

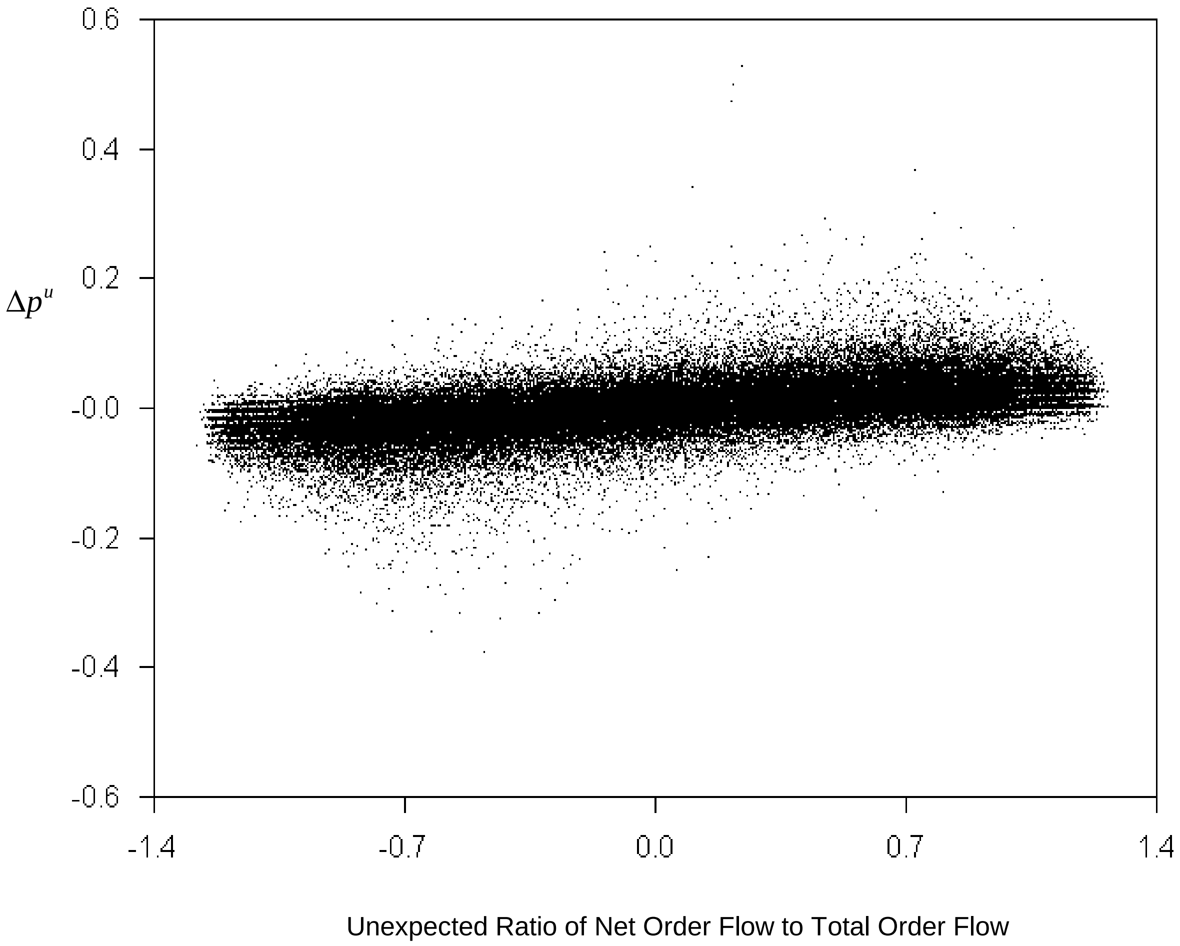
	Estimated Parameters	Standard Errors	p-value
α_0	0.000230	0.000175	0.188
α_1	0.007788	0.000081	0.000
α_2	-0.000199	0.000149	0.183
$\overline{R}^2 = 0.374$	Estimation is carried out by Ordinary Least Squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980).		

So far the analysis concentrated on some measure of net order flow which does not directly relate to overall market activity. It is possible that market participants interpret the information content of a given order flow differently in periods of large total volume and in periods with less trading activity. Therefore, more information may be contained in a relative measure of order imbalance than in an absolute one. To check this, we constructed a series of net volume divided by total volume. This variable just equals the difference between the proportions of buyer initiated and seller initiated volume.

As in Section 4.1, a non-linear time series model was built to extract unexpected changes in the order flow variable. The resulting residuals are plotted against the unexpected price changes in Figure 5.[9] The relation seems to be mainly positive, but it is difficult to judge by visual inspection whether any non-linearity is present.

[9] Note that due to the forecasting model the order flow variable does no longer fall within the range between minus one and plus one.

Figure 5: Relation Between Unexpected Ratio of Net Order Flow to Total Order Flow and Unexpected Logarithmic Price Changes



A linear model was estimated and the residuals tested for non-linearity. The test statistic is highly significant. Thus we estimated a network analogously to equation (5), whose results are given in Table 8. The form of non-linearity seems to be more complicated than for the non standardized net volume. The significant parameters α_2 and γ_{12} indicate an asymmetry. The explanatory power of the relative measure of order imbalance is comparatively weak, as can be seen from the adjusted R-squared of about 0.28. This indicates that more information is contained in the net order flow variable used in Section 4.2 than just in the proportions of buyer and seller initiated trades.

Table 8: Regression Results for the Neural Network Model: Net Order Flow Measures by the Unexpected Ratio of Net Order Flow to Total Order Flow

	Estimated Parameters	**Standard Errors**	**p-value**
α_0	-0.00305	0.00016	0.000
α_1	-0.07312	0.00319	0.000
α_2	-0.27869	0.00079	0.000
β_1	0.92941	0.00031	0.000
γ_{11}	0.10907	0.00338	0.000
γ_{12}	0.32863	0.00150	0.000
$\overline{R}^2 = 0.277$	Estimation is carried out by Non-linear Least Squares. Standard errors are obtained by the heteroskedasticity consistent estimator of White (1980).		

5. Conclusion

In this paper we analyze the permanent price impact of trades by investigating the relation between unexpected net order flow and price changes. We mainly focus on four questions. Does large order flow convey more information than small order flow? Does net buy and net sell volume convey the same amount of information? Does the information content of order flow increase linearly with its size? Are there alternative measures of trading activity which convey more information than order flow?

We use intraday data on German index futures. Our analysis based on a neural network model provides us with the following results. Firstly, the information content of order flow increases with its size. Secondly, we find that buyer initiated trades and seller initiated trades do not differ with respect to their information content. Thirdly, the relation between net order flow and price changes is strongly non-linear. Large orders lead to relatively small price changes whereas small orders lead to relatively large price changes. Finally, net order flow measured as contracts traded offers the best explanation for price changes. Net number of trades explains price changes almost as well. However, the relative net order flow, i.e. net order flow divided by volume, does not provide the same level of explanation. The results are found to be quite robust with respect to the estimation procedure.

Overall, the results of our paper suggest that the assumption of a linear impact of orders on prices (which is often used in theoretical papers) is highly questionable. Thus, market depth cannot be described sufficiently by a single number. Therefore, empirical studies, comparing the depth of different markets, should be based on the whole price impact function instead of a simple ratio. To allow the market depth to depend on trade volume could open promising avenues for further theoretical research. This could lead to quite different trading strategies as in traditional models.

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