Providing a new three stage data envelopment analysis model (DEA) in fuzzy environment

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Abstract
The data envelopment analysis models (DEA) are provided to measure set of relative efficiency from decision making unit with different inputs to make different outputs. For a discussion of inaccurate data, the fuzzy concept is introduced. It is main objective of this paper to provide a new method for evaluating the efficiency in method of network data envelopment analysis that using inaccurate data fuzzy. The proposed methods to calculate the efficiency has been done in tow-stage yet, however in this study the three-stage are used to evaluate the efficiency.

Keywords: Data envelopment analysis, Efficiency, Fuzzy number, Network structure.

1 Introduction
The continuous improvement of organizational performance makes enormous forces of synergy, so that these forces can support the growth and development and create opportunities for organizational excellence. Government, organization and institutions are making progress in this regard. It is not possible to achieve the continuous improvement of performance without investigating and awareness of organizational progress levels and identifying challenges facing the organization and receiving feedback and understanding how well the policies are implemented and also cases that need improvement.[8] all of the above mentioned cases is not possible without measurement and evaluation.

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English physicist Lord Kelvin says about the necessary of measuring: “when you can measure what you are speaking about, and express it in numbers, you know something about it, when cannot express it in numbers, your knowledge is of meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts advanced to the stage of science.”

Measuring efficiency is one of the most important methods for evaluating the performance and productivity of a company and has always been at center of attention of researchers. Efficiency implies that an organization has been able to use its resources in production for a specified period of time. In other words, the efficiency of the use of resources to produce a certain amount of the product [10]

The first step on efficiency improvement is measuring. Efficiency and productivity provides the conditions for the organization's managers to find out where they are and thus, plan for improvement of the current situation [7].

In a general classification, the methods of efficiency measurement are divided into parametric and nonparametric group. In parametric methods, by using different statistical and econometric techniques, the efficiency is determined; and in nonparametric methods, using mathematical programming models (optimization of target functions), the efficiency of each unit is calculated. [10]

2 Literature review

2.1. Data Envelopment Analysis

In recent years, the Data Envelopment Analysis (DEA) method has been recognized as a useful tool in assessing the performance of manufacturing and service units. The DEA method involves a multi-factor productivity analysis model for measuring Relative performance values between sets of decision-making units (DMUs). Decision-making units include: An organization or system such as schools, hospitals, bank branches, and other similar items which has several input and output that are similar to each other. In literature, data envelopment analysis is used in order to avoid dispersion; instead of the input factors of the system, the concept of output and also instead of the system output products, the output concept is used [4]. The efficiency score in the case of multiple input and output factors is defined as:

\[
Efficiency = \frac{\text{Weighted sum of outputs}}{\text{Weighted sum of inputs}}
\]  

(2.1)

The desired weights in this regard are obtained from the models presented in data envelopment analysis methods. The first developed model in the Data Envelopment Analysis Method was proposed by Charles, Cooper, and Rhodes [1987], and is thus known as the CCR Model [1]. The CCR model, namely from the total weighted output ratio to the total weighted inputs, are used as scales for efficiency measures. If any DMU unit has M inputs to produce S output, then the fractional form of the classical model of data envelopment analysis that evaluates the related unit efficiency (or zero unit) will be as following [5]:
Max \( \theta_0 = \frac{u_1 y_{10} + u_2 y_{20} + \cdots + u_s y_{s0}}{v_1 x_{10} + v_2 x_{20} + \cdots + v_m x_{m0}} \)  

(2.2)

Subject to:
\[
\frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \leq 1
\]

\((j = 1, 2, \ldots, n)\)

\(v_1, v_2, \ldots, v_m \geq 0\)

\(u_1, u_2, \ldots, u_s \geq 0\)

The limitations of this problem mean that the ratio of output to input for each DMU should not exceed 1. The model is designed to gain weights in a way that maximizes the desired efficiency ratio for each DMU. In order to simplify, the problem of above fractional programming can be converted to a linear programming problem, as following [1]:

Max \( \theta_0 = \mu_1 y_{10} + \mu_2 y_{20} + \cdots + \mu_s y_{s0} \)  

(2.3)

Subject to:
\[
v_1 x_{10} + v_2 x_{20} + \cdots + v_m x_{m0} = 1
\]

\[
\mu_1 y_{1j} + \mu_2 y_{2j} + \cdots + \mu_s y_{sj} \leq v_1 x_{1f} + v_2 x_{2f} + \cdots + v_m x_{mf}
\]

\((j = 1, 2, \ldots, n)\)

\(v_1, v_2, \ldots, v_m \geq 0\)

\(\mu_1, \mu_2, \ldots, \mu_s \geq 0\)

2.2. Multi-stage Data Envelopment Analysis

In the real world, there are decision-making units, in which the production process can be considered as a two-stage or multi-stage process. Data envelopment analysis models deal with decision-making units like the "black box". Namely, inputs are used to generate outputs, without considering the communications within the internal parts of the units. Figure (1) shows the internal structure of a unit in a network model.

![Figure 1: Internal structure of a decision making unit in a network model](image)
2.2.1 Types of network structure models

The network models presented in this paper are divided into two categories: the first group is those whose total efficiency is defined based on the efficiency product of the sections. In the second group, the total efficiency is determined by the weighted average efficiency of the sections.

Kao [2] has developed models for evaluating network decision-making units by using both serial and parallel structures that are defined based on the product efficiency of the sections.

Serial structure: In a multi-section decision-making unit, when section activities are alongside each other, the system has a serial structure. In this case, the input of the entire system is entered into the first section and the final output of the system going out from the last section.

To evaluate the performance of each part of the binary systems, it is easy to apply the normal data envelopment analysis models to measure the performance of each section independently. But when the decision-making unit has several sections; the use of relational models are suggested. The major difference between independent models and relational models is to assign weight coefficients to the same data; because the importance of data usage varies from place to place. An interesting point in the relational model is that the overall system performance is achieved by the size of its parts. Meanwhile, for serial systems that consist of more sections, performance evaluation can be easily done by developing the model by recursive induction.

In order to introduce the first model, consider a decision-making unit with h section, in which the sections are arranged in series. Figure (2) depicts an image of a serial system.

![Serial system diagram]

Figure 2: Serial system

Show the direct inputs and outputs of the Jth unit respectively. \( z_{(k,t)}^j \) is the vector of the intermediate products from the K section to the t section. \( z_{(t,f)}^j \) is the vector of the intermediate products from t section to f section. The number of intermediary products for each section can be different from other sections. For convenience, we consider the number of intermediary products of all sections \( (d = 1,...,q) \).

Parallel structure:

In a multi-section decision-making unit, when the section activity is arranged in parallel to each other, the system has parallel structure. Figure (3) shows a parallel system.
In this structure, the total input is divided between all sections and the total output is obtained from the output of all segments namely $X_{ij}$. ($i = 1, ..., m$) is decision-making of Jth the total unit input ($j = 1, ..., n$). $X_{ij}^{(t)}$ is the value of the input assigned to the t section in the Jth decision-making unit. $Y_j$ is the total output of the jth unit, and $Y_{ij}^{(t)}$ is the output value generated by the t section of the jth unit.

2.3. Fuzzy Data Envelopment Analysis

To the lack of comprehensive knowledge and information, precise mathematics for modeling a complex system is not adequate and appropriate. Since in the real-world problems, the established decisions based on qualitative data are as well quantitative data, it seems that a fuzzy method is suitable for such problems [6]. The CCR model with fuzzy data can be written the form of the model:

$$\text{Max} \quad w_p = \sum_{r=1}^{s} u_r y_{rp}$$

St:

$$\sum_{i=1}^{m} v_i x_{ip} = 1 \quad \forall i$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \forall j$$

$$u_r, v_i \geq 0 \quad \forall i, r$$
In that, fuzzy inputs, $\tilde{x}_p$, and fuzzy outputs, $\tilde{y}_p$, are represented by weights; $u_r$ and $v_i$ respectively. Saati, Memariyani and Jahanshahlu, in their papers, under title of “Performance and Ranking of DMUs with fuzzy data”, suggested a fuzzy CCR model [3]. In this paper, in order to solve the model, instead of comparing in boundaries and distances, they choose variables that are both congruent in terms of constraints and maximize the objective function. They developed their proposed model as follows:

$$\text{Max} \quad w = \sum_{r=1}^{s} u_r y_{rp} \quad (2.5)$$

St:

$$\sum_{i=1}^{m} x_{ip} = 1$$

$$\sum_{r=1}^{s} y_{rj} - \sum_{i=1}^{m} x_{ij} \leq 0 \quad \forall j$$

$$v_i (ax_{ij}^m + (1 - a)x_{ij}^l) \leq x_{ij} \leq v_i (ax_{ij}^m + (1 - a)x_{ij}^u) \quad \forall i, j$$

$$u_r (ay_{rj}^m + (1 - a)y_{rj}^l) \leq x_{rj} \leq u_r (ay_{rj}^m + (1 - a)y_{rj}^u) \quad \forall r, j$$

$$u_r, v_i \geq 0 \quad \forall i$$

Where in $\alpha \in (0, 1)$. As previously stated, these three researchers presented a fuzzy CSW model, a MODM model, for evaluating and ranking decision-making in another paper entitled “Calculation of a common set of weights in fuzzy DEA.” The features of this model are common weights that are presented for all decision-making units to prevent the various weights of each factor in different units. In general, this model consists of three steps. Step 1: Determine the boundaries (limits), to determine the upper limits of input and output weights, consider models (2.9) and (2.10). The upper limit for output weights:

$$\text{Max} \quad u_p \quad (2.6)$$

St:

$$\sum_{i=1}^{m} x_{ip} \leq 1 \quad \forall j = 1, \ldots, n$$

$$\sum_{r=1}^{s} y_{rj} - \sum_{i=1}^{m} x_{ij} \leq 0 \quad \forall j = 1, \ldots, n$$

$$v_i (x_{ij}^m + (1 - \delta)x_{ij}^\beta) \leq x_{ij} \leq v_i (ax_{ij}^m + (1 - \delta)x_{ij}^\beta) \quad i = 1, \ldots, m \& j = 1, \ldots, n$$
\[ u_r(y_{rj}^m - (1 - \delta)y_{rj}^u) \leq y_{rj} \leq u_r(y_{rj}^m + (1 - \delta)y_{rj}^\beta) \quad r = 1, \ldots, S \& j = 1, \ldots, n \]

\[ x_{ij} \geq 0 \quad i = 1, \ldots, m \& j = 1, \ldots, n \]

\[ y_{rj} \geq 0 \quad r = 1, \ldots, S \& j = 1, \ldots, n \]

\[ u_r \geq 0 \quad r = 1, \ldots, S \]

\[ V_i \geq 0 \quad i = 1, \ldots, m \]

Upper limit for input weights:

\[ \text{Max} \quad V_i \]

St:

\[ \sum_{i=1}^{n} x_{ij} \leq 1 \quad j = 1, \ldots, n \]

\[ \sum_{r=1}^{s} y_{rj} - \sum_{i=1}^{m} x_{ij} \leq 0 \quad j = 1, \ldots, n \]

\[ v_i(x_{ij}^m - (1 - \delta)x_{ij}^u) \leq x_{ij} \leq v_i(x_{ij}^m + (1 - \delta)x_{ij}^\beta) \quad i = 1, \ldots, m \& j = 1, \ldots, n \]

\[ u_t(y_{rj}^m - (1 - \delta)y_{rj}^u) \leq y_{rj} \leq u_t(y_{rj}^m + (1 - \delta)y_{rj}^\beta) \quad t = 1, \ldots, S \& j = 1, \ldots, n \]

\[ x_{ij} \geq 0 \quad i = 1, \ldots, m \& j = 1, \ldots, n \quad y_{rj} \geq 0 \quad t = 1, \ldots, s \& j = 1, \ldots, n \]

\[ u_t \geq 0 \quad t = 1, \ldots, S \quad V_i \geq 0 \quad i = 1, \ldots, m \]

By solving the m + s linear programming problem, the upper bounds (limits) of inputs and outputs are determined.

Step 2: determine a common weights set

\[ \text{Max} \quad \emptyset \]

St:

\[ \sum_{r=1}^{s} y_{rj} - \sum_{i=1}^{m} x_{ij} \leq 0 \quad j = 1, \ldots, n \]

\[ v_i(x_{ij}^m - (1 - \delta)x_{ij}^u) \leq x_{ij} \leq v_i(x_{ij}^m + (1 - \delta)x_{ij}^\beta) \quad i = 1, \ldots, m \& j = 1, \ldots, n \]

\[ u_t(y_{rj}^m - (1 - \delta)y_{rj}^u) \leq y_{rj} \leq u_t(y_{rj}^m + (1 - \delta)y_{rj}^\beta) \quad r = 1, \ldots, S \& j = 1, \ldots, n \]
\[ \varnothing u_t \leq u_t \leq (1 - \varnothing)u_t \quad r = 1, \ldots, S \]
\[ \varnothing v_i \leq v_t \leq (1 - \varnothing)v_i \quad i = 1, \ldots, m \]

By solving model 11, a common set of weights is obtained, and the performance of each DMU is calculated in the following way.

Step 3: Obtaining DMUs
\[ e_j = (e_j^m, e_j^u, e_j^\beta) \quad \forall j \quad (2.9) \]
\[ e_j = \frac{\sum_{r=1}^s u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \quad \forall j \]

As a result, we have:
\[ e_j^m = \frac{\sum_{r=1}^s u_t y_{rj}^m}{\sum_{i=1}^m v_i x_{ij}^m} \quad (2.10) \]
\[ e_j^\beta = \frac{\sum_{r=1}^s u_r y_{rj}^m + \sum_{i=1}^m v_i x_{ij}^m + \sum_{r=1}^s u_t y_{rj}^\beta \sum_{i=1}^m v_i x_{ij}^m}{(\sum_{i=1}^m v_i x_{ij}^m)^2} \]

3 Methodology of research

We first introduce the objective functions and restrictions applicable to them in non-fuzzy mode. The target functions are presented with the aim of achieving maximum efficiency.
\[ \max \quad \theta^l = \frac{\sum_{r=1}^n u_r y_{rp}}{\sum_{i=1}^m v_i x_{ip}^u} \quad (3.11) \]
\[ \frac{\sum_{r=1}^n u_r y_{rp}^l}{\sum_{i=1}^m v_i x_{ip}^l} \leq 1 \]
\[ v_i, u_r \geq 0 \]
\[ \max \quad \theta^m = \frac{\sum_{r=1}^n u_r y_{rp}^m}{\sum_{i=1}^m v_i x_{ip}^m} \]
\[ \frac{\sum_{r=1}^n u_r y_{rp}^m}{\sum_{i=1}^m v_i x_{ip}^m} \leq 1 \]
\[ v_i, u_r \geq 0 \]
\[ \max \quad \theta^u = \frac{\sum_{r=1}^n u_r y_{rp}^u}{\sum_{i=1}^m v_i x_{ip}^l} \]
\[ \frac{\sum_{r=1}^n u_r y_{rp}^u}{\sum_{i=1}^m v_i x_{ip}^l} \leq 1 \]
\[ v_i, u_r \geq 0 \]
If we want to evaluate the model in fuzzy mode; with triangular fuzzy numbers, the objective function is capable of dividing in three objective functions and constraints will be evaluable in each state. In lower limit, we have the lowest output from the largest input. In upper limits, we will evaluate the highest output with low inputs and in the middle limits, namely, in the middle norm of inputs and the output, the objective function and constrains have defined. Initially, in non-phase mode, and assuming the equal weights, the objective function and constraints, we will investigate the three-step DEA network mode; using the Cooper conversions, we linearize the objective function.

Non-fuzzy model:
Because the weights are equal

\[ w_1 = w_2 = w_3 \]

\[ \max \theta_t = w_1, \theta_1 + w_2, \theta_2 + w_3, \theta_3 \]  \hspace{1cm} (3.12)

\[ \max \theta_t = 1/3(\theta_1 + \theta_2 + \theta_3) \]

\[ \max \theta = 1/3\left( \frac{\sum_{i=1}^{n} h_i f_i p + \sum_{g=1}^{o} c_g b_{gp}}{\sum_{i=1}^{m} v_i x_{ip}} + \frac{\sum_{d=1}^{H} q_d t_{dp}}{\sum_{t=1}^{n} h_t f_{tp} + \sum_{e=1}^{F} w_e z_{ep}} + \frac{\sum_{r=1}^{S} u_r y_{rp}}{\sum_{d=1}^{H} q_d t_{dp} + \sum_{g=1}^{o} c_g b_{gp}} \right) \]

S.t

\[ \frac{\sum_{t=1}^{n} h_t f_{tp} + \sum_{g=1}^{o} c_g b_{gp}}{\sum_{i=1}^{m} v_i x_{ip}} \leq 1 \]

\[ \frac{\sum_{d=1}^{H} q_d t_{dp}}{\sum_{t=1}^{n} h_t f_{tp} + \sum_{e=1}^{F} w_e z_{ep}} \leq 1 \]

\[ \frac{\sum_{r=1}^{S} u_r y_{rp}}{\sum_{d=1}^{H} q_d t_{dp} + \sum_{g=1}^{o} c_g b_{gp}} \leq 1 \]

\[ v_i, u_r, q_d, c_g, h_t, w_e \geq 0 \]

Then, in a fuzzy state with a fuzzy DEA network model, and considering three models to examine the model in fuzzy mode, we will evaluate the network. The lower objective function and the constraints are set somehow that will have the highest inputs and the lowest output in each step, the limit of objective function and constraints are in such a way that we have the highest outputs from the lowest inputs, and finally, using the Cooper conversions, the related models of upper and lower limit, in the form of following relationships can be evaluated:
Fuzzy Model:

\[
\max \quad \tilde{\theta} = 1/3(\tilde{\theta}_1 + \tilde{\theta}_2 + \tilde{\theta}_3)
\]

(3.13)

Lower limit

\[
\begin{align*}
\max \theta^l &= 1/3\left(\frac{\sum_{t=1}^n h_t f_{tp}^l + \sum_{g=1}^o c_g b_{gp}^l}{\sum_{i=1}^m v_i x_{lp}^i} + \frac{\sum_H^d q_d t_{dp}^l}{\sum_{t=1}^n h_t f_{tp}^l + \sum_{e=1}^F w_e z_{ep}^u} + \frac{\sum_{r=1}^s u_r y_{rp}^l}{\sum_{d=1}^H q_d t_{dp}^l + \sum_{g=1}^o c_g b_{gp}^l}\right) \\
\text{s.t.} \\
\frac{\sum_{t=1}^n h_t f_{tp}^l + \sum_{g=1}^o c_g b_{gp}^l}{\sum_{i=1}^m v_i x_{lp}^i} &\leq 1 \\
\frac{\sum_H^d q_d t_{dp}^l}{\sum_{t=1}^n h_t f_{tp}^l + \sum_{e=1}^F w_e z_{ep}^u} &\leq 1 \\
\frac{\sum_{r=1}^s u_r y_{rp}^l}{\sum_{d=1}^H q_d t_{dp}^l + \sum_{g=1}^o c_g b_{gp}^l} &\leq 1 \\
v_i, u_r, q_d, c_g, h_t, w_e &\geq 0
\end{align*}
\]

Upper limit

\[
\begin{align*}
\max \theta^u &= 1/3\left(\frac{\sum_{t=1}^n h_t f_{tp}^u + \sum_{g=1}^o c_g b_{gp}^u}{\sum_{i=1}^m v_i x_{lp}^i} + \frac{\sum_H^d q_d t_{dp}^u}{\sum_{t=1}^n h_t f_{tp}^u + \sum_{e=1}^F w_e z_{ep}^u} + \frac{\sum_{r=1}^s u_r y_{rp}^u}{\sum_{d=1}^H q_d t_{dp}^u + \sum_{g=1}^o c_g b_{gp}^u}\right) \\
\text{s.t.} \\
\frac{\sum_{t=1}^n h_t f_{tp}^u + \sum_{g=1}^o c_g b_{gp}^u}{\sum_{i=1}^m v_i x_{lp}^i} &\leq 1 \\
\frac{\sum_H^d q_d t_{dp}^u}{\sum_{t=1}^n h_t f_{tp}^u + \sum_{e=1}^F w_e z_{ep}^u} &\leq 1 \\
\frac{\sum_{r=1}^s u_r y_{rp}^u}{\sum_{d=1}^H q_d t_{dp}^u + \sum_{g=1}^o c_g b_{gp}^u} &\leq 1 \\
v_i, u_r, q_d, c_g, h_t, w_e &\geq 0
\end{align*}
\]
Middle limit

\[
\max \theta^m = 1/3 \left( \frac{\sum_{t=1}^{n} h_{tf}^m + \sum_{g=1}^{o} c_g b_{gp}^m}{\sum_{i=1}^{m} v_i x_{ip}^m} + \frac{\sum_{d=1}^{H} q_d t_{dp}^m}{\sum_{t=1}^{m} h_{tf}^m + \sum_{e=1}^{F} w_e z_{ep}^m} + \frac{\sum_{r=1}^{s} u_r y_{rp}^m}{\sum_{d=1}^{H} q_d t_{dp}^m + \sum_{g=1}^{o} c_g b_{gp}^m} \right)
\]

(3.15)

S.t

\[
\sum_{t=1}^{n} h_{tf}^m + \sum_{g=1}^{o} c_g b_{gp}^m \leq 1
\]

\[
\sum_{d=1}^{H} q_d t_{dp}^m + \sum_{e=1}^{F} w_e z_{ep}^m \leq 1
\]

\[
\sum_{r=1}^{s} u_r y_{rp}^m \leq 1
\]

\[
\sum_{d=1}^{H} q_d t_{dp}^m + \sum_{g=1}^{o} c_g b_{gp}^m
\]

\[
\sum_{i=1}^{n} v_i x_{ip}^m = 1
\]

\[
\sum_{t=1}^{n} h_{tf}^m + \sum_{g=1}^{o} c_g b_{gp}^m - \sum_{i=1}^{m} v_i x_{ip}^m \leq 0
\]

\[
\sum_{d=1}^{H} q_d t_{dp}^m - \sum_{t=1}^{n} h_{tf}^m - \sum_{e=1}^{F} w_e z_{ep}^m \leq 0
\]

\[
\sum_{r=1}^{s} u_r y_{rp}^m - \sum_{d=1}^{H} q_d t_{dp}^m - \sum_{g=1}^{o} c_g b_{gp}^m \leq 0
\]

\[
\sum_{i=1}^{n} v_i, u_r, q_d, c_g, h_r, w_e \geq 0
\]

Using the Cooper conversions, we linearize the objective function

Middle limit

\[
\max \theta^m = 1/3 \left( \sum_{t=1}^{n} h_{tf}^m + \sum_{g=1}^{o} c_g b_{gp}^m + \sum_{d=1}^{H} q_d t_{dp}^m + \sum_{r=1}^{s} u_r y_{rp}^m \right)
\]

(3.16)

S.t:

\[
\sum_{d=1}^{H} q_d t_{dp}^m + \sum_{g=1}^{o} c_g b_{gp}^m = 1
\]

\[
\sum_{t=1}^{n} h_{tf}^m + \sum_{e=1}^{F} w_e z_{ep}^m = 1
\]

\[
\sum_{i=1}^{m} v_i x_{ip}^m = 1
\]

\[
\sum_{t=1}^{n} h_{tf}^m + \sum_{g=1}^{o} c_g b_{gp}^m - \sum_{i=1}^{m} v_i x_{ip}^m \leq 0
\]

\[
\sum_{d=1}^{H} q_d t_{dp}^m - \sum_{t=1}^{n} h_{tf}^m - \sum_{e=1}^{F} w_e z_{ep}^m \leq 0
\]

\[
\sum_{r=1}^{s} u_r y_{rp}^m - \sum_{d=1}^{H} q_d t_{dp}^m - \sum_{g=1}^{o} c_g b_{gp}^m \leq 0
\]

\[
v_i, u_r, q_d, c_g, h_r, w_e \geq 0
\]
Lower limit

\[ \max \theta^l = 1/3 \left( \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{g=1}^{o} c_{g} b_{gp}^l + \sum_{d=1}^{H} q_{d} t_{dp}^l + \sum_{r=1}^{s} u_{r} y_{rp}^l \right) \]  

\[ \sum_{r=1}^{s} u_{r} y_{rp}^l = 1 \]

\[ \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{e=1}^{F} w_{e} z_{ep}^u = 1 \]

\[ \sum_{d=1}^{H} q_{d} t_{dp}^u + \sum_{g=1}^{o} c_{g} b_{gp}^u = 1 \]

\[ \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{g=1}^{o} c_{g} b_{gp}^l - \sum_{i=1}^{m} v_{i} x_{ip}^l = 0 \]

\[ \sum_{d=1}^{H} q_{d} t_{dp}^l - \sum_{t=1}^{n} h_{tf_{tp}} - \sum_{e=1}^{F} w_{e} z_{ep}^u \leq 0 \]

\[ \sum_{r=1}^{s} u_{r} y_{rp}^l - \sum_{d=1}^{H} q_{d} t_{dp}^u - \sum_{g=1}^{o} c_{g} b_{gp}^u \leq 0 \]

\[ v_{i}, u_{r}, q_{d}, c_{g}, h_{t}, w_{e} \geq 0 \]

Upper limit

\[ \max \theta^l = 1/3 \left( \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{g=1}^{o} c_{g} b_{gp}^u + \sum_{d=1}^{H} q_{d} t_{dp}^u + \sum_{r=1}^{s} u_{r} y_{rp}^u \right) \]  

S.t:

\[ \sum_{r=1}^{s} u_{r} y_{rp}^u = 1 \]

\[ \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{e=1}^{F} w_{e} z_{ep}^l = 1 \]

\[ \sum_{d=1}^{H} q_{d} t_{dp}^l + \sum_{g=1}^{o} c_{g} b_{gp}^l = 1 \]

\[ \sum_{t=1}^{n} h_{tf_{tp}} + \sum_{g=1}^{o} c_{g} b_{gp}^u - \sum_{i=1}^{m} v_{i} x_{ip}^l = 0 \]
\[
\sum_{d=1}^{H} q_{d} t_{d}^{u} - \sum_{t=1}^{n} h_{t} f_{t p}^{l} - \sum_{e=1}^{F} w_{e} z_{e p}^{l} \leq 0
\]

\[
\sum_{r=1}^{s} v_{r} y_{r p}^{u} - \sum_{d=1}^{H} q_{d} t_{d p}^{l} - \sum_{g=1}^{o} c_{g} b_{g p}^{l} \leq 0
\]

\[v_{i}, u_{r}, q_{d}, c_{g}, h_{t}, w_{e} \geq 0\]

4 Conclusion

The model presented in this paper is an extension of two-stage network models with considering the fuzzy parameters and variables. Considering fuzzy parameters and variables are more applicable than non-phase and absolute parameters, because real-world concepts also embody the fuzzy concepts and new one. Hence, the review of efficiency with regard to fuzzy concepts in network structures has been investigated in this paper. In future researches, one can Combine mentioned generalized model with other concepts from data envelopment analysis topics.

The model presented in this paper is an extension of two-stage network models with consideration to fuzzy parameters and variables. Considering fuzzy parameters and variables are more useful than non-phase and non-phase parameters because real-world concepts are made. It also covers the fuzzy and new concepts. Hence, the efficiency review with respect to fuzzy concepts in network structures has been investigated in this paper. In future researches, one can generalize this model with other concepts Combined with data envelopment analysis topics.

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