Boundary layer flow of MHD Casson-nanofluid with heat and mass transfer through porous medium over a semi infinite moving plate

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Abstract
The motion of a nano-Casson fluid with heat and mass transfer over a semi-infinite flat plate under the effect of a uniform magnetic field through porous medium is investigated. The Ohmic heating and viscous dissipation with chemical reaction are also considered. The problem is formulated mathematically by a system of non-linear partial differential equations (PDEs). Suitable similarity transformations are used to transform this system to non-linear ordinary differential equations (ODEs) in dimensionless form and solved numerically. The effect of the physical quantities of the problem, such as, permeability parameter, Casson parameter, chemical reaction parameter, Eckert parameter a well as Reynolds number and Hartmann number on the fluid behavior are discussed and illustrated graphically.

Keywords: Boundary layer; Casson fluids; Nanofluids; Moving surface; Magnetic field; Porous media; Chemical reaction; heat and mass transfer, scientific computations.

1 Introduction

Over the past two decades more interests are given to the study of convective transport of Nano fluids [1-4]. Fluids such as oil, water, ethylene and glycol mixture play an important role in manufacturing especially in petroleum industries. The need to improve the thermal conductivity of these fluids is a great challenge. Adding suspending Nano/micro or larger sized particle materials to the fluids improve fluid properties (Vleggaar [5]). Also it can improve the base fluid heat transfer property (Choi [6]). Recently nanotechnology is widely used in industry since materials with sizes of nanometers possess unique physical and chemical properties. Choi et al [7] proved that adding small amount of nanoparticles to convention heat transfer liquids increases the thermal conductivity of these liquids up to approximately two times. The highest thermal conductivity can be obtained in case of nanotubes; these results open the door to a wide range of nanotube applications in industry. Kuzentsov and Nield [8] have studied analytically the influence of nano particles on natural convection boundary layer flow past a vertical plate. They considered the simplest possible boundary conditions in which both temperature and Nano-particle fraction are constant along the wall. Further, Nield and Kuzentsov [9] discussed the Cheng–Minkowycz problem of natural convection past a
vertical plate, in porous medium saturated by nanofluid. Norfifah et al [10], studied the problem of heat transfer of steady two-dimensional boundary layer flow past a moving permeable flat plate in a nanofluid and proved that, the dual solutions exist only when the plate and the free stream flow move in opposite directions. Also, suction delays the boundary layer separation, while injection accelerates it. The study of the effects of external magnetic field on magnetohydrodynamic flow (MHD) is very important in fluid mechanics especially when the fluid under consideration is non-Newtonian (Casson fluid). The results of these studies have many applications in manufacturing, natural process, bio-rheology, chemical and petroleum industries, geophysical fluid dynamics, and blood flow in small blood vessels [11-25].

The aim of this paper is to extend the work of Norfifah et al [10] to investigate the behavior of non-Newtonian Casson fluid with heat and mass transfer through porous media under the influence of a uniform magnetic field. The Ohmic heating; viscous dissipation and chemical reaction are taken in consideration during this study.

2 Formulation of the problem

Consider two-dimensional steady boundary-layer flow of an incompressible Casson nanofluid over a moving semi-infinite flat plate. Use the Cartesian coordinates system (x, y) such that, the x-axis is taken along the direction of the continuous surface and the y-axis is measured normal to the surface of the plate. Assume the fluid occupies the region y≥ 0. A uniform magnetic field $\vec{B}$ is applied to the fluid normal to the moving surface, Fig. (1).

Let $U$ is the uniform free stream velocity and the flat plate velocity is $U_w = \lambda U$ ($\lambda$ is the plate velocity parameter). Moreover, assume the temperature $T$ and the nanoparticles fraction $C$ take the constant values $T_w$ and $C_w$ on the moving plate respectively, $T_\infty$ and $C_\infty$ are their values in the ambient fluid.

We write the rheological equations for an isotropic incompressible flow of a non-Newtonian Casson fluid as ([26]-[28]):
\[
\tau_{ij} = \begin{cases} 
2(\mu_b + \frac{p_y}{2\pi})e_{ij}, & \pi > \pi_c \\
2(\mu_b + \frac{p_y}{2\pi})e_{ij}, & \pi < \pi_c
\end{cases}
\] (2.1)

Here, \( \pi = e_{ij}e_{ij} \) where \( e_{ij} \) is the \( (i, j) \)th component of the deformation rate. \( \pi_c \) is the critical value of \( \pi \). \( \mu_b \) is the plastic dynamic viscosity and \( P_y \) is the yield stress of the fluid where,

\[
p_y = \frac{\mu_b\sqrt{2\pi}}{\beta}.
\] (2.2)

In case of non-Newtonian Casson fluid, we can write

\[
\mu = \mu_b + \frac{1}{\sqrt{2\pi}} p_y = \mu_b \left( 1 + \frac{p_y}{\mu_b\sqrt{2\pi}} \right) = \mu_b \left( 1 + \frac{1}{\beta} \right), \quad \beta = \frac{\sqrt{2\pi}\mu_b}{p_y}, \quad \pi > \pi_c
\] (2.3)

\[
\tau_{ij} = \mu_b \left( 1 + \frac{1}{\beta} \right) 2e_{ij} ; \quad e_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)
\] (2.4)

\[
\tau_{ij} = \mu_b \left( 1 + \frac{1}{\beta} \right) \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)
\] (2.5)

The governing equations, in usual notations, can be written as following:

The continuity equation:

\[
\nabla \cdot \mathbf{V} = 0,
\] (2.6)

The momentum equation:

\[
\rho_f \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\nabla p + \nabla \cdot \mathbf{\tau} - \frac{\mu_b}{\kappa} \mathbf{V} + \mathbf{J} \times \mathbf{B}.
\] (2.7)

The heat equation with heat generation:

\[
(\rho c)_f \left( \frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = k_b \nabla^2 T + (\rho c)_p [D_b \nabla C \cdot \nabla T + \frac{D_T}{T_\infty} \nabla T \cdot \nabla T] + \mathbf{\tau} \cdot \nabla \mathbf{V} + \frac{J^2}{\sigma}.
\] (2.8)

The concentration equation with chemical reaction,

\[
\frac{\partial C}{\partial t} + \nabla \cdot C = D_b \nabla^2 C + \frac{D_T}{T_\infty} \nabla^2 T + k_c (C - C_\infty).
\] (2.9)

Here, \( \mathbf{V} \) is the velocity of the fluid, \( t \) is the time, \( \rho_f \) is the density of the base fluid.

\( \rho, \mu_b, k_b, k_c, \kappa, \sigma, J, B \) and \( c \) are density, dynamic viscosity, thermal conductivity, Permeability coefficient, coefficient of the chemical reaction, electrical conductivity of the fluid, current density, total magnetic field and volumetric volume expansion coefficient of the nanofluids, \( (\rho c)_f \) is the heat capacity of the fluid and \( (\rho c)_p \) is the effective heat capacity of the nanoparticles material. \( D_b \) and \( D_T \) are the Brownian diffusion coefficient and the thermophoretic diffusion coefficient respectively.

For steady-state flow, the boundary layer governing equations (2.6)-(2.9) can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\] (2.10)
\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho_f} \left( \sigma B^2 + \frac{\mu_b}{k} \right) u , \tag{2.11}
\]

\[
\frac{u}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_b \frac{\partial C}{\partial y} + \frac{D_T}{T_c} \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu_b}{(\rho c)_f} \left( 1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial y} + \frac{\sigma}{(\rho c)_f} B^2 u^2 , \tag{2.12}
\]

\[
\frac{u}{\partial x} + v \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_c} \left( \frac{\partial^2 T}{\partial y^2} \right) + k_c (C - C_\infty) . \tag{2.13}
\]

Subject to the boundary conditions:
\[
v = 0, \quad u = U_w = \lambda U, \quad T = T_w, \quad C = C_w \text{ at } y = 0 , \tag{2.14}
\]
\[
u \rightarrow U, \quad T \rightarrow T_\infty, \quad C \rightarrow C_\infty \text{ at } y \rightarrow \infty
\]
where, \(u\) and \(v\) are the velocity components in \(x\) and \(y\) directions, \(\alpha = k_c / (\rho c)_f \) is the thermal diffusivity of the fluid, \(\nu\) is the kinematic viscosity coefficient and \(\tau = (\rho c)_P / (\rho c)_f \) is the heat capacity ratio.

Introducing the stream function \(\psi\) where \(u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}\), we find that the continuity equation (2.10) is identically satisfied and equations (2.11)-(2.13) take the following form:

\[
\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = \nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^3 \psi}{\partial y^3} - \frac{1}{\rho_f} \left( \sigma B^2 + \frac{\mu_b}{k} \right) \frac{\partial \psi}{\partial y} , \tag{2.15}
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left[ D_b \frac{\partial C}{\partial y} + \frac{D_T}{T_c} \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\mu_b}{(\rho c)_f} \left( 1 + \frac{1}{\beta} \right) \frac{\partial \psi}{\partial y} + \frac{\sigma}{(\rho c)_f} B^2 \left( \frac{\partial \psi}{\partial y} \right)^2 , \tag{2.16}
\]

\[
\frac{\partial \psi}{\partial y} \frac{\partial C}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial C}{\partial y} = D_b \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_c} \left( \frac{\partial^2 T}{\partial y^2} \right) + k_c (C - C_\infty) . \tag{2.17}
\]

With the help of the similarity transformations:

\[
\psi = \sqrt{2 \nu U x f(\eta)}, \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} ; \eta = y \sqrt{\frac{U}{2 \nu x}} . \tag{2.18}
\]

The left hand side of equation (2.15) can be written as:

\[
\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = - \left( \frac{U^2}{2x} \right) f'(\eta) f''(\eta) ,
\]

and the right hand side can be written as:

\[
\nu \left( 1 + \frac{1}{\beta} \right) \frac{\partial^3 \psi}{\partial y^3} = \frac{1}{\rho_f} \left( \sigma B^2 + \frac{\mu_b}{k} \right) \frac{\partial \psi}{\partial y} =
\]

\[
\nu \left( 2 U \nu x \right)^{1/2} \left( \frac{U}{2 \nu x} \right)^{3/2} f' \left( \eta \right) - \left( 2 U \nu x \right)^{1/2} \left( \frac{U}{2 \nu x} \right)^{1/2} \left( \frac{\sigma}{\rho_f} B^2 + \frac{\mu}{\rho_f} \right) \frac{1}{k} f'(\eta) .
\]

Therefore the transformed non-dimensional ordinary differential equation of (2.15) is:
\( (1 + \frac{1}{\beta}) f''(\eta) + f(\eta) f''(\eta) - (M + \frac{1}{K}) f'(\eta) = 0, \) \hspace{1cm} (2.19)

Similarly the transformed non-dimensional ordinary differential equations of (2.16) and (2.17) can be written as:

\[
\frac{1}{P_r} \theta'' + f \theta' + N_t \theta' \varphi' + N_b \theta'' + E_v (1 + \frac{1}{\beta}) f'' + ME_v f'^2 = 0,
\] \hspace{1cm} (2.20)

\[ \varphi'' + \frac{N_t}{N_b} \varphi' + L_v f \varphi' + 2 \gamma R_{ex} L_v \varphi = 0, \] \hspace{1cm} (2.21)

Also, the boundary conditions (2.14) are transformed to:

\[ f(0) = 0, \quad f'(0) = \lambda, \quad \theta(0) = 1, \quad \varphi(0) = 1, \text{ and } \] \hspace{1cm} (2.22)

\[ f'(\eta) \to 1, \quad \theta(\eta) \to 0, \quad \varphi(\eta) \to 0, \text{ as } \eta \to \infty \]

The prime means differentiation with respect to the independent variable \( \eta \).

Here, \( M, \frac{1}{K}, E_v, L_v, R_{ex}, \gamma, P_r, N_b \) and \( N_t \) are respectively magnetic parameter, permeability parameter, Eckert number, Lewis number, Reynolds number, chemical reaction parameter, Prandtl number, Brownian motion parameter and thermophoresis parameter.

The mathematical forms of these parameters are defined as:

\[
M = \frac{2 \alpha \sigma B^2}{U \rho_f}, \quad K = \frac{U \rho_f k}{2 \pi \mu_b}, \quad E_v = \frac{U^2}{(C_{\rho_f})_f (T_w - T_\infty)}, \quad L_v = \frac{v}{D_B}, \quad R_{ex} = \frac{U x}{v},
\]

\[
\gamma = \frac{vk}{U^2}, \quad P_r = \frac{v}{\alpha}, \quad N_b = \frac{(\rho C_p) D_b (C_w - C_\infty)}{v(\rho C_f)}, \quad N_t = \frac{(\rho C_p) D_b (T_w - T_\infty)}{v(\rho C_f) T_\infty}.
\] \hspace{1cm} (2.23)

The physical quantities \( c_f \) (skin-friction coefficient), \( Nu_x \) (local Nusselt number) and \( Sh_x \) (Local Sherwood number) are defined as:

\[
c_f = \frac{\tau_w}{\rho U^2}, \quad Nu_x = \frac{x q_w}{k(T_w - T_\infty)}, \quad Sh_x = \frac{x q_m}{D_b (C_w - C_\infty)},
\] \hspace{1cm} (2.24)

\[ \tau_w = \mu_b \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad q_w = -\kappa \left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0}, \quad q_m = -D_B \left(\frac{\partial T}{\partial \bar{y}}\right)_{\bar{y}=0},
\] \hspace{1cm} (2.25)

where, \( \tau_w, q_w \) and \( q_m \) are the values of the shear stress, heat flux and mass flux on the surface respectively.

Also, with the help of the similarity transformations (2.18) and expressions (2.24) – (2, 25) we can write:

\[
\sqrt{2 R_{ex}} C_f = \left(1 + \frac{1}{\beta}\right) f''(0), \quad -\theta'(0) = \frac{2}{R_{ex}} Nu_x, \quad -\varphi'(0) = \frac{2}{R_{ex}} Sh_x,
\] \hspace{1cm} (2.26)

where, \( R_{ex} \) is the local Reynolds number.

Here, \( \sqrt{2 R_{ex}} Nu_x \) and \( \sqrt{2 R_{ex}} Sh_x \) referred to as the reduced Nusselt number \( \bar{Nu}_x \) and Sherwood number \( \bar{Sh}_x \) respectively.

It is interesting to note that if the motion of a Newtonian fluid with no external magnetic field and no chemical reaction is considered, we obtain the work of Norfifah et al [10]. Moreover, if no nanoparticles are considered, we obtain the work of Weidman et al [16].
3 Results and Discussion

In this problem, we study the effects of various emerging parameters on velocity, temperature and concentration profiles over a semi-infinite moving flat plate with heat and mass transfer through porous medium. The similarity transformations are used to transform the governing partial differential Eqs. (2.15)-(2.17) to nonlinear coupled ordinary differential Eqs. (2.19)–(2.21) subject to the boundary conditions (2.22). The system of equations (2.19)–(2.22) is solved numerically using the Mathematica program package. The results for different values of the physical parameters $P_r$, $R_{ex}$, $N_t$, $N_b$, $L_c$, $E_c$, $\lambda$, $M$, $k$ and $\gamma$ are:

Case 1: ($P_r = R_{ex} = 1$, $N_t = N_b = 0.1$, $L_c = E_c = 2$, $\lambda = 0.2$, $M = k = \gamma = 0.5$)

In this case, we study the variation of velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\varphi(\eta)$ for different values of the Casson parameter $\beta$. The results of this case are shown in Fig. 1(a). It is observed that the increase in $\beta$ increases the fluid viscosity but decreasing fluid velocity and the increase in $\beta$ decreases temperature but increase concentration.

Case 2: ($P_r = R_{ex} = 1$, $N_t = N_b = 0.1$, $L_c = E_c = 2$, $\lambda = 0.2$, $\beta = 1$, $k = \gamma = 0.5$)

In this case, we study the effect of the magnetic field $M$ on velocity $f'(\eta)$, temperature $\theta(\eta)$ and concentration $\varphi(\eta)$, the results are shown in Fig. 1(b). It is found that when $M$ increases the velocity...
decreases because of the induced Lorentz forces created by the transverse magnetic field. Also, increasing the magnetic field $M$ increases temperature but decreases fluid concentration.

Figure 1 (b)

**Case 3:** ($P = R = 1$, $N = N_b = 0.1$, $L_c = E_c = 2$, $\lambda = 0.2$, $\beta = 1$, $M = \gamma = 0.5$)

In this case, we study the effect of the permeability parameter $k$ on velocity $f''(\eta)$, temperature $\theta(\eta)$ and concentration $\phi(\eta)$ (Fig. 1(c)). We observed that increasing $k$ increases velocity and concentration but decreases fluid temperature.
Figure 1(c)

Case 4: \( P_i = R_{xx} = 1, N_i = N_b = 0.1, L_c = 2, \lambda = 0.2, \beta = 1, M = \gamma = k = 0.5 \) 

In this case, we study the effect of Eckert number \( Ec \) on the temperature \( \theta(\eta) \) and the concentration \( \varphi(\eta) \) (Fig. 2(a)). It is observed that increasing Eckert number \( Ec \) increases fluid temperature but decreases the fluid concentration.

Figure 2(a)

Case 5: \( P_i = R_{xx} = 1, N_i = N_b = 0.1, L_c = 2, \lambda = 0.2, \beta = 1, M = k = 0.5 \) 

In this case, we study the effect of the chemical reaction \( \gamma \) on the temperature \( \theta(\eta) \) and the concentration \( \varphi(\eta) \) (Fig. 2(b)). We observes that the temperature and concentration increases slightly as the chemical reaction \( \gamma \) increases.
Case 6: \((R_{\text{ex}} = 1, N_i = N_b = 0.5, L_c = E_c = 2, \lambda = 0.2, \beta = 0.1, M = k = \gamma = 0.5)\)

In this case, we study the effect of Prandtl number \(P_r\) on the temperature \(\theta(\eta)\) and the concentration \(\varphi(\eta)\) (Fig. 3(a)). With increasing Prandtl number increases fluid temperature but decreases the concentration.

Case 7: \((P_r = 1, R_{\text{ex}} = 1, N_i = 0.5, L_c = E_c = 2, \lambda = 0.2, \beta = 0.1, M = k = \gamma = 0.5)\)

In this case, we study the effect of the Brownian motion parameter \(N_b\) on temperature \(\theta(\eta)\) and the concentration \(\varphi(\eta)\) (Fig. 3(b)). With increasing Brownian motion parameter increases the temperature and concentration.
Case 8: \( P = 1, R_{ex} = 1, N_b = 0.5, L = E_c = 2, \lambda = 0.5, \beta = 0.1, M = k = \gamma = 0.5 \)

In this case, we study the effect of the thermophoretic parameter \( N_t \) on temperature \( \theta(\eta) \) and concentration \( \phi(\eta) \) (Fig. 4(a)). It is clear that increasing the values of the thermophoretic parameter decreases nanoparticle concentration and fluid temperature.

![Figure 4(a)](image)

Case 9: \( P = 1, R_{ex} = 1, N_b = 0.5, N_t = 1, E_c = 2, \lambda = 0.5, \beta = 0.1, M = k = \gamma = 0.5 \)

In this case, we study the effect of the Lewis number \( Le \) on temperature \( \theta(\eta) \) and the concentration \( \phi(\eta) \) (Fig.4(b)). The increase in Lewis number values reduces fluid concentration and temperature.

![Figure 4(b)](image)

Case 10: \( P = 1, N_b = 0.5, L = 0.5, N_t = 1, E_c = 2, \lambda = 0.5, \beta = 0.1, M = k = \gamma = 0.5 \)

In this case, we study the effect of the local Reynolds number \( R_{ex} \) on the temperature \( \theta(\eta) \) and the concentration \( \phi(\eta) \) (Fig.4(c)). It is observed that the temperature slightly decreases with the increasing of the Reynolds number \( Rex \) values.
4 Conclusions

In this paper, the motion of nano non-Newtonian Casson fluid with heat and mass transfer over a semi-infinite moving plate is studied. The effects of a uniform magnetic field, Ohmic heating and viscous dissipation with chemical reaction are also included in this study. The variation of the physical parameters values such as Hartmann number and permeability parameter are discussed. The effects of Casson, chemical, Eckert parameters and Reynolds number on the obtained solutions are examined. The main results of this study are summarized as following:

- The effects of some physical quantities (Eckert number $E_c$, Prandtl number $Pr$, Lewis number $Le$, Reynolds number $Re_x$, Chemical reaction $\gamma$, Brownian motion Parameter $N_b$ and thermophoresis parameter $N_t$) on velocity distribution are are very small.
- Increasing the Casson parameter $\beta$ decreases fluid velocity and temperature.
- Increasing Hertman number $M$ reduces fluid velocity and concentration but raises fluid temperature.
- Increasing the value of the permeability parameter $k$ increases both velocity field and fluid concentration but decreases fluid temperature.
- Increasing Eckert number $Ec$ value increases fluid temperature but decreasing fluid concentration.

5 Applications

The most interesting applications of the considered problem are in petroleum industries, human blood flow and rheology, as following:

- The demand of a better drilling fluid increases when oil and gas companies turn their attention to new search areas such as the unstable shale areas and high pressure. Nanotechnology can improve viscosity, density, specific gravity and any other physical properties of the drilling fluid to overcome any difficulties.
- Human blood can be treated as a kind of Casson fluid due to the presence of several substances like, protein, fibrinogen and globulin in aqueous base plasma.
- The importance of rheology science comes from its industrial applications where synthetic polymers and their solution in different solvents is necessary for polymers and industrial applications.
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