Unions, Growth and Unemployment

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We extend the standard quality-ladder model with heterogeneous workers by including efficiency wages and unions. We find that higher union bargaining power leads to a negative relationship between growth and unemployment. An increase in the supply of human capital, however, on the one hand induces firms to substitute high-skilled labour for jobs previously performed by low-skilled individuals and on the other hand, increases the demand for low-skilled labour as their productivity rises due to the higher skill-intensity. Depending on which effect dominates, either a positive or negative relationship between the growth and unemployment rates results.

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1 Introduction

Empirical data on unemployment shows several stylised facts for the OECD countries. First, since the mid-nineties the unemployment rate is slowly starting to fall. In fact, as can be seen by the “Greencard” debate in Germany, by which a large number of software programmers are to be enticed to come and work in Germany for several years, there are some branches of the economy which seem to be suffering from a severe shortage of workers. This is just one example of the fact that all analysis of unemployment must take different skill levels into account. A closer look at the empirical data confirms that low-skilled unemployment rates are now much higher (in absolute terms and also relative to high-skilled unemployment) than they were in the 1970s (see, e.g. Nickell, Bell 1997 and OECD 1997) and it is this rise in unskilled unemployment which contributes a large share to the rise in total unemployment. In Germany, for example, in 1998 the unemployment rate for workers with only a high-school diploma was more than six times as high as the corresponding rate for individuals with a university degree (see Reinberg 1999 for more detail). Second, although the unemployment rate for the “low-skilled” (i.e individuals with a high-school diploma) has always been higher than the corresponding rate for the “high-skilled” (meaning individuals who have graduated from a university), the difference between the two rates has grown substantially in the nineties. One of the main reasons attributed to this growing gap that is normally stated in the literature is the role of (skilled-biased) technical change in modern economies. Third, although the supply of skilled labour has increased dramatically in the last twenty years (see, for example, Acemoglu 2000 and Gregg, Manning 1997) wage inequality between low- and high-skilled wages has increased sharply in the United States and other countries with “liberalised” labour markets, whilst Continental Europe and the Scandinavian countries still have more or less the same degree of wage inequality as in the eighties, but have suffered from a substantial and persistent increase in unemployment. In the European context, one of the causes for this constant wage differential that is stated is the role of labour market institutions in general, and that of unions in particular. Fourth, on a balanced growth path, the unemployment rate is constant.

This paper integrates these empirical observations by extending the quality-ladder
growth model with endogenous technical change and heterogeneous workers as in Grossman, Helpman (1991, Ch.5) in two respects. First, we introduce unions which represent the interests of the low-skilled workers. Second, although not all low-skilled are in fact members of a union, due to “fair-wage” considerations, the negotiated wage between unions and employers affects the wage rate of all low-skilled workers in the economy. This means that union wage coverage is in fact far larger than the mere number of union members suggests.

There is without doubt a large literature on growth and unemployment in which most articles can be put into two major categories. First, there are models which combine Schumpeter’s idea of creative destruction with matching models of unemployment, see, for example, Aghion, Howitt (1994). Second, there are models which analyse the interaction between labour and capital, see, for example Blanchard (1998). Very few papers, however, have analysed how precisely unions affect the unemployment and growth rates. Notable exceptions are Bräuninger (2000) and Stadler (1999) who both, however, treat labour as homogeneous.¹ In the model by Bräuninger (2000) which is based on an overlapping-generations growth model, higher union bargaining power causes unemployment to increase and leads to a decrease in the savings of the young so that a negative relationship between growth and unemployment arises. Stadler (1999) on the other hand, finds that an increase in union bargaining power causes both a higher growth and unemployment rate. This positive relationship in his model results due to the assumption that the labour market in the research sector is perfectly competitive so that the net effect of higher union bargaining power is to shift labour into the research sector.

The economy considered here produces two final goods, a traditional one whose technology remains constant, and a composite high-tech manufacturing good which is produced using a multitude of intermediate goods. These intermediate goods are constantly being improved by researchers working in a separate sector. We find that an increase in the supply of skills (or human capital) always leads to a higher growth rate, but the effect on the unemployment rate is ambiguous. The reason for this ambiguity is that a higher level of human capital will also

¹ See also de Groot (2000) for a recent book devoted to the subject of growth and unemployment. This book also includes a chapter analysing the effects of unions, but again assuming that all workers are homogeneous.
raise the productivity amongst the low-skilled. Further, if the size of the population is constant, the supply of low-skilled workers decreases. Both effects raise the low-skilled wage rate. For a small elasticity of substitution between high- and low-skilled labour, for example when the production technology is specified as a COBB-DOUGLAS function, the resulting increase in demand due to the higher productivity more than outweighs the substitution effect induced by the higher relative low-skilled wage. These results are reversed for “high” values of the substitution elasticity, in which case both the growth and unemployment rates increase. The relationship between growth and unemployment is unambiguously negative if union bargaining power rises. In this case, the low-skilled wages increase although there is neither a corresponding increase in low-skilled productivity nor a decrease in the supply of low-skilled labour. The resulting higher low-skilled unemployment and real wage rate lower the effective research intensity leading to a lower product-improvement rate and thereby to a lower growth rate.

The paper is organised as follows: Section 2 presents the formal model in which all sectors of the economy are explained. The general equilibrium is derived in Section 3 and builds the basis for Section 4, in which some comparative statics are presented. Section 5 concludes.

2 Behaviour of Firms and Households

2.1 Production of Consumer Goods

There are two final goods produced in the economy: $T$ denotes the quantity of a “traditional” homogeneous good which is not improved over time. The production technology is assumed to be constant returns to scale with low- and high-skilled labour as inputs. However, it is assumed that this sector has the lowest high-skilled labour intensity. Further, firms operating in this sector face perfect competition on the goods market. Therefore, the price $p_T$ of the traditional good is given by

$$p_T = c_T(w_{LT}, w_{HT})$$

where $w_{LT}$ and $w_{HT}$ are the nominal wage rates in the traditional sector of low- and high-skilled labour respectively, and $c_T$ denotes unit costs. The resulting
demand for low-skilled labour can be derived by applying Shepard’s Lemma which yields

\[ L_T = a_{LT}(w_{LT}, w_{HT})T \]  

with \( a_{LT} \) as the partial derivative of the unit cost function with respect to low-skilled wages. The corresponding demand for high-skilled labour is

\[ H_T = a_{HT}(w_{LT}, w_{HT})T \]

The other final good is a high-technology manufacturing good which is produced using a whole range of intermediates which are constantly being improved with respect to quality. Assuming a constant returns to scale Cobb-Douglas technology means that output of this composite good \( M \) is given by the input index

\[ M = \exp \left\{ \sum_{j=1}^{n} \ln \left( \sum_{h_j} \lambda^{m_j} x_{j} \right)^{1/n} \right\} \]

where \( x_{j}, j = 1, \ldots, n \) denotes the various input goods which are each produced in a single intermediate sector. Each input can be of a different quality grade, arrayed along the rungs of a known quality ladder. Every time there is a successful innovation in sector \( j \), the quality of \( x_{j} \) is improved by a constant factor \( \lambda > 1 \). If the lowest quality of an intermediate good available at time \( t = 0 \) is normalised to one in each sector, then the highest quality available in sector \( j \) is \( \lambda^{m_j} \), where \( m_j = 0, 1, 2, \ldots \) denotes the number of quality improvements up to the present. The highest quality available at each point in time defines the state-of-the-art or top-of-the-line components.

As shown below, firms with the current highest quality good in sector \( j \) will charge a limit-price, so that their good has the lowest quality-adjusted price. This means that only the highest quality good in each intermediate sector will be demanded, so that the composite good index \( M \) simplifies to

\[ M = \exp \left\{ \sum_{j=1}^{n} \ln [\lambda^{m_j} x_{j}]^{1/n} \right\} \]

The minimum quality-adjusted price for each variant \( x_{j} \) is \( p_{x_{j}}/\lambda^{m_j} \). Therefore, the price index \( p_M \) which can be interpreted as the minimum cost of purchasing
one unit of the composite good $M$, can be determined by summing up over all $n$ variants in equation (5), to yield

$$p_M = \exp \left\{ \sum_{j=1}^{n} \ln \left[ np_j/\lambda_{mj} \right]^{1/n} \right\}$$

(6)

It can also be seen from equation (5) that the intermediate goods in each market $j$ are perfect substitutes for another if one adjusts for quality. Further, seeing as the elasticity of substitution between any pair of intermediate products is equal to one, all components will be used in equal quantities. Therefore, the intermediate output index becomes

$$M = A_M X$$

(7)

where $A_M$ defines the average quality and $X = nx$ the aggregate output (number of varieties multiplied with the output per variety) of intermediates. If each industry has its own POISSON-process with instantaneous arrival rate $\nu_j$, summing over all industries means that in the time interval $\tau$, the total number of expected quality improvements will be $I(\tau) = \int_0^\tau \nu(t) dt$. This means that the average quality of the varieties is

$$A_M(\tau) = \lambda I(\tau)$$

(8)

Production of the intermediate goods is assumed to be more human capital intensive than in the traditional sector, but less intensive than in the research sector. All firms producing the intermediate goods have identical cost functions given by

$$c x_j(w_{Lx_j}, w_{Hx_j})$$

(9)

As the qualities of the components in each intermediate market $j$ differ, the technological leader has the ability to capture the entire market demand by charging a quality-adjusted price which is marginally lower than that of his or her nearest competitor. With unit costs in the intermediate sector as given by equation (9), the optimal limit-pricing strategy is

$$p_{xj} = \lambda c x_j(w_{Lx_j}, w_{Hx_j})$$

(10)

See Stadler (1999) or Barro, Sala-i-Martin (1995, Ch. 7) for models in which one final good is produced using homogeneous labour and a quality-adjusted intermediate good. In this case, depending on the innovation size $\lambda$, firms can either engage in monopoly pricing if the quality improvements are “large” – Aghion, Howitt (1998, Ch. 2) label these innovations as “drastic” – or in limit-pricing if quality improvements are “small”.

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which implies a low-skilled labour demand in this sector of

\[ L_X = a_{LX}(w_{LX}, w_{HX})X \]  

(11)

and

\[ H_X = a_{HX}(w_{LX}, w_{HX})X \]  

(12)

for high-skilled labour, where \( a_{gX} \) denotes the partial derivative of the cost function in the intermediate goods sector with respect to labour of skill-group \( g \in \{L, H\} \).

From equation (10), profits \( \pi_{x_j} \) for a market leader can be written as

\[
\pi_{x_j} = p_{x_j}x_j - c_{x_j}x_j = p_{x_j}x_j(1 - 1/\lambda)
\]  

(13)

If market leaders invest in research efforts at the further improvement of their good, then by the same argument as above, the maximum price they could charge would be \( \lambda^2 c_{x_j} \). In this case they would earn a profit of \( p_{x_j}x_j(1 - 1/\lambda^2) \). This means that the marginal gain from being two steps ahead of the closest rival is \((1/\lambda)(1 - 1/\lambda)\) which is strictly less than the additional profit that would occur if it is possible to displace a current monopolist in another sector. Therefore, due to this “replacement effect” (Tirole 1988, 392), it is never optimal for the current market leader in sector \( j \) to undertake further research aimed at improving the quality of his own product. The knowledge required to improve the quality of the intermediate goods is obtained by researchers working in a separate sector discussed in the next section.

### 2.2 Research

The research technology exhibits constant returns to scale and requires both low- and high-skilled labour. As the economy modelled here is a developed one, we assume that the research sector is the most human capital intensive. The unit cost function in this sector is given by \( c_R(w_{LR}, w_{HR}) \) from which labour demand needed to achieve a research intensity \( \iota \) in this sector is derived as

\[ L_R = a_{LR}(w_{LR}, w_{HR})\iota \]  

(14)
\[ H_R = a_{HR}(w_{LR}, w_{HR}) \]  

(15)

with \( a_{LR} \) and \( a_{HR} \) as the respective partial derivatives of the cost function with respect to low- and high-skilled wages.

Innovations are financed through the emission of stocks. If a firm has a successful innovation, it will achieve a stock market value of \( v \) arising from the expected profit streams that a market leader enjoys. Therefore, any investor can purchase a share which, during the time interval \( dt \), pays a dividend of \( v dt \) with probability \( \nu dt \). At a research intensity of \( \nu \), total research costs during the time interval \( dt \) are given by \( (a_{LR}w_{LR} + a_{HR}w_{HR})\nu dt \). Given unit research costs of \( c_R \), each research firm maximises \( v\nu dt - c_R\nu dt \) which requires an infinite amount of research if \( v > c_R \) and zero research and no quality improvements if \( v < c_R \). Hence, finite research investments which lead to positive quality growth in equilibrium only occur if

\[ v = c_R(w_{LR}, w_{HR}) \]  

(16)

If \( f_j(m, \tau) \) denotes the probability that the product \( j \) will be improved \( m \) times during the time interval \( \tau \), then by the law of large numbers, \( f(m, \tau) \) also represents the fraction of industries which experience \( m \) improvements in this time interval. Assuming that the arrival rate of new innovations in each market follows a POISSON-process results in a growth rate for the intermediate index given by

\[ \frac{\dot{M}}{M} = \nu \ln \lambda \]  

(17)

In general equilibrium, the expected discounted value of a firm in the research sector is zero. Therefore, sectors with “high” profit flows also incur “high” expected research costs or have shorter time periods in which they realise the profit flows. This means that each industry has its own specific speed at which the quality of the good is improved and that the macroeconomic growth level is simply the average speed of quality improvements across all sectors.

This completes the production side of the economy. Demand for these goods is determined by household behaviour which is analysed in the next section.
2.3 Households

The economy consists of $N$ infinitely-lived dynasties. Each household consumes $C_T$ units of the traditional good and the amount $C_M$ of the high-tech good. Assuming that households have a common rate of time preference $\rho$ and that the intertemporal elasticity of substitution is equal to one, means that households maximise the time separable intertemporal utility function

$$U = \int_{0}^{\infty} e^{-\rho t} \left[ \phi \ln C_M(t) + (1 - \phi) \ln C_T(t) \right] dt, \quad 0 < \phi < 1$$  \hspace{1cm} (18)

subject to the intertemporal budget constraint

$$\int_{0}^{\infty} e^{-rt} p_C C dt \leq W(0)$$

where $p_C C = p_T C_T + p_M C_M$ are total consumption expenditures and $W(0)$ is the present value of the household’s assets at time $t = 0$ which are composed of future labour and interest income. Solving this intertemporal optimisation problem yields the Keynes-Ramsey rule

$$\frac{\dot{C}}{C} = r - \frac{\dot{p}_C}{p_C} - \rho$$  \hspace{1cm} (19)

Using the normalisation that aggregate spending $E(t) = p_C C$ is equal to one in all periods, it follows from (19) that

$$r = \rho$$  \hspace{1cm} (20)

With this normalisation and given the above utility function means that in the steady-state consumers will devote a fraction $\phi$ of their spending on the high-tech good $M$ and the remainder on the traditional good $T$. From this we can infer that demand for the manufacturing good equals $\phi/p_M$ and that for the traditional good $T$ is $(1 - \phi)/p_T$.

Further, due to, for example, different learning abilities amongst individuals, a fraction $s$ of the population $N$ is high-skilled and consequently, a fraction $(1 - s)$ is low-skilled.\(^3\)

\(^3\) See, for example, Stokey (1991) and Lucas (1993), for models where the number of high-skilled workers is endogenously determined.
2.4 Wages

One of the main justifications for unions and one of the reasons why workers decide to become members is the expected redistribution effect. In the present model, this means that on the one hand it is the low-skilled workers who can profit most from union membership, with the aim of reducing the wage differential between the two skill groups. On the other hand, this implies that high-skilled workers have no incentive to join the union and the labour market for these workers is fully competitive.\(^4\)

As there are only rents in the intermediate sector, unions only have an incentive to operate here. Therefore, low-skilled workers will earn different wages, depending on which sector they work in. This corresponds to the empirical evidence of inter-industry wage differentials.\(^5\)

According to the right-to-manage approach which is adopted here, in each intermediate market \(j\), unions negotiate the wage level with employers who subsequently unilaterally set their profit-maximising level of labour demand. This means that firms and unions together maximise the NASH-product \(\Omega\) given by

\[
\Omega = \max_{w_{Lx_j}} \left[ (w_{Lx_j} - \bar{w}_{LX}) L_{x_j} \right]^\beta \left[ \pi_{x_j} - \bar{\pi}_{x_j} \right]^{1-\beta}
\]  

with \(\bar{w}_{LX}\) as the low-skilled reservation wage in the intermediate sector and \(\beta\) as union bargaining power. Assuming that the two types of labour are gross complements as inputs in production, means that if no wage agreement is reached with the union and the low-skilled workers go on strike, the firm is forced to completely stop production and therefore has a negative fallback position \(\bar{\pi}_{x_j}\) as it would still have to pay the high-skilled workers their wages. Thus, the employer’s net bargaining position which enters the NASH-product is given by

\[
\pi_{x_j} - \bar{\pi}_{x_j} = p_{x_j} x_j - w_{Lx_j} L_{x_j}
\]  

\(^4\) See Agell, Lommerud (1992) for a model where workers do not know \textit{ex-ante} whether they will get a low or high-paying job so that both groups are interested in lowering the wage differential.

\(^5\) See, for example, Gibbons, Katz (1992) and Haisken-DeNew, Schmidt (1999) for empirical evidence and Dickens, Lang (1988) and Wapler (1999) for surveys of so-called “dual” or “segmented” labour market theories which analyse these types of wage differentials in more detail.
Inserting equation (22) into (21) and applying standard optimisation techniques leads to a low-skilled nominal wage given by

\[
w_{Lx,j} = \left[ 1 - \frac{\beta(1 - \epsilon_{x,L})}{\beta(1 - \epsilon_{x,L})\epsilon_{L,wL} - \epsilon_{x,L} + \beta + \mu(1 - \beta)} \right] \bar{w}_{LX} \tag{23}
\]

where \(\epsilon_{L,wL} < 0\) denotes the elasticity of low-skilled labour demand with respect to low-skilled wages in the intermediate sector and \(\epsilon_{x,L}\) is the output elasticity with respect to low-skilled labour, both of which will be constant given the constant returns to scale production technology. Finally, with \(R_j \equiv p_{x,j}x_j\) as the revenue function of firm \(j\) in the intermediate sector and again making use of the assumption of a constant returns to scale production function, means that \(\mu \equiv w_{Lx,j}w_{Hx,j}(\partial H_{x,j}/\partial w_{Lx,j})/R_j < 0\) is also a constant.

The low-skilled reservation wage is

\[
\bar{w}_{LX} = (1 - u_L)w_{LX} + u_Lb \tag{24}
\]

where \(u_L\) is the low-skilled unemployment rate, \(w_{LX}\) is the low-skilled wage paid by other firms in the intermediate goods sector, and \(b\) are unemployment benefits which are assumed to be strictly less than average low-skilled wages.\(^7\) Assuming symmetric firms-union pairs, so that \(w_{Lx,j} = w_{LX}\) and inserting equation (24) into (23), leads to nominal low-skilled wages in the intermediate sector given by

\[
w_{LX} = \left[ 1 - \frac{\beta(1 - \epsilon_{x,L})}{\beta(1 - \epsilon_{x,L}) + [\beta(1 - \epsilon_{x,L})\epsilon_{L,wL} - (\epsilon_{x,L} - \mu)(1 - \beta)]u_L} \right] b \tag{25}
\]

It can be seen from equation (25) that an interior solution (i.e. a positive low-skilled wage) only exists if there is unemployment, i.e. \(u_L > 0\). How high this minimum unemployment rate is depends on the parameters of the production function and on the degree of union bargaining power. If unemployment falls below this critical level, the (nominal) wage would need to fall below the income level guaranteed by unemployment benefits, so that no worker would accept a

\(^6\) A detailed appendix containing derivations of all important equations is available form the author upon request.

\(^7\) For simplicity we do not include a government budget constraint as doing so and assuming that unemployment benefits are financed by a lump-sum tax would not affect the qualitative steady-state results of the model.
job in this sector. For unemployment levels above this critical value, the low-skilled wage is a negative function of unemployment, in other words, the mark-up over unemployment benefits (which effectively determines the reservation wage) decreases as the unemployment rate rises.

For simplicity, it is assumed that low-skilled wages in the research and traditional sectors are identical. However, in line with the fair-wage hypothesis as first formulated by Akerlof, Yellen (1990) and backed up by a large body of empirical evidence, see, for example, Agell, Lundborg (1995) and Bewley (2000), it is assumed that workers in these sectors compare the wages they receive with those of similarly qualified workers in the intermediate sector. Only if they perceive their wages as “fair”, are they prepared to provide effort and thereby contribute to output.\(^8\)

\[ w_{LT} = w_{LR} \geq \eta w_{LX}, \quad \eta > 0 \]  

(26)

If there are only two levels that effort by low-skilled workers in the traditional and research sectors can take on, namely either zero or a positive minimum amount, then firms in these sectors have no incentive to pay higher wages than required and equation (26) will hold with equality. Further, although from equation (25) it is obvious that low-skilled wages in the intermediate sector are higher than their market-clearing level, it would be theoretically possible for all low-skilled workers who do not find a job in the unionised sector, to take up jobs in the other sectors. It is therefore assumed that \( \eta > \eta^* \) where \( \eta^* \) is defined as the largest value of \( \eta \) at which all unskilled workers not employed in the intermediate sector would find employment in either of the other two sectors. The assumption that the wage level agreed upon between unions and employers in one sector is treated as a benchmark by firms in other branches of the economy, so that the actual union wage coverage is larger than the number of unionised employees suggests, is also in line with the empirical evidence for many Western European countries.\(^9\)

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8 See Grossman (2000) for a similar assumption in a fair-wage model. However, in his model, low-skilled workers in the final goods sector make comparisons across skill groups, whereas in the intermediate sector low-skilled workers compare their wages within their skill group. It does not become clear, why otherwise identical workers should have different notations of fair-wages.

9 For example, in Germany Franz (1999, Ch.7) calculates that, although only approximately
The determination of wages fully closes the model so that it is now possible to derive the equilibrium in the next section.

3 Equilibrium

Equilibrium for the traditional good is given by

\[ T = \frac{1 - \phi}{p_T} \]  

(27)

The value of demand for the high-tech good \( M \) is \( \phi \), which in equilibrium must be equal to the value of output in this sector. This equals total production costs \( p_M M \) which are given by the aggregate component costs \( p_x X \), i.e.

\[ p_M M = p_x X = \phi \]  

(28)

For the capital market to be in equilibrium, there cannot be any arbitrage possibilities. This means that the expected return on any shares invested in an innovative firm must be equal to the rate of return on a riskless asset. In the time interval \( dt \), these shares pay a dividend of \( \pi dt \). With the probability \( (1 - \iota dt) \), no other firm will develop a successful innovation during this time interval, in which case the current value of future profits (and thus the price of the shares) increases by \( (\dot{v} / v) dt \). However, if another firm does innovate successfully (which happens with the probability \( \iota dt \)), this new firm captures all the demand in this market so that the previous monopolist will earn zero profits in the future and thus the capital owners incur a financial loss of \( v \). Equating this dividend with that which a riskless asset valued at \( v \) pays during the same time interval leads to:

\[ \pi dt + \dot{v} dt (1 - \iota dt) - \iota v dt = rv dt \]

Ignoring terms to the power of two and noting that in the steady-state equilibrium \( \dot{v} = 0 \) leads to

\[ \frac{\pi}{v} = r + \iota \]

30% of the workforce are members of a union, union wage agreements cover roughly 90% of all work contracts. See also FRANZ et al. (2000) for a recent survey of union coverage amongst manufacturing and service sector firms which comes to a similar result.
which from equations (13), (16), (20) and (28) gives

\[
\frac{(1 - 1/\lambda)\phi}{nc_R} = \rho + \iota
\]  

(29)

Amongst low-skilled workers, unemployment will occur so that the labour-market condition for these workers is

\[
a_{LR}(w_{LR}, w_H)\iota + a_{LX}(w_{LX}, w_H)X + a_{LT}(w_{LT}, w_H)T = (1 - u_L)(1 - s)N
\]  

(30)

The market for high-skilled labour always clears and is characterised by:

\[
a_{HR}(w_{LR}, w_H)\iota + a_{HX}(w_{LX}, w_H)X + a_{HT}(w_{LT}, w_H)T = sN
\]  

(31)

noting that by assumption \(a_{HR}/a_{LR} > a_{HX}/a_{LX} > a_{HT}/a_{LT}\).

Using the equilibrium demand equations (27) together with (28) and the price-setting equations (1) and (10), makes it possible to express the labour-market equations (30) and (31) as

\[
\eta \theta_{LR}(w_{LR}, w_H)c_R(w_{LR}, w_H)\iota + \frac{\phi \theta_{LX}(w_{LX}, w_H)}{\lambda} + \eta(1 - \phi)\theta_{LT}(w_{LT}, w_H) = w_{LX}(1 - u_L)(1 - s)N
\]  

(32)

and

\[
\theta_{HR}(w_{LR}, w_H)c_R(w_{LR}, w_H)\iota + \frac{\phi \theta_{HX}(w_{LX}, w_H)}{\lambda} + (1 - \phi)\theta_{HT}(w_{LT}, w_H) = w_{H}sN
\]  

(33)

where \(\theta_{gk} \equiv a_{gk}w_{gk}/c_k\) represents the share of labour of skill-group \(g \in \{L, H\}\) in costs incurred in sector \(k \in \{R, T, X\}\).

Equations (32) and (33) together with the zero-profit condition (29) endogenously determine the steady-state innovation rate and factor prices, and by equation (25), the equilibrium unemployment rate. This equilibrium is characterised by constant wages in all sectors for both skill groups and positive unemployment and growth rates.
4 Growth and Unemployment

Using the fact that the proportional changes in the cost shares can be expressed as \( \hat{\theta}_{lk} = \theta_{hk}(1 - \sigma_k)(\hat{w}_{lk} - \hat{w}_{hk}) \) and \( \hat{\theta}_h = -\theta_{lk}(1 - \sigma_k)(\hat{w}_{lk} - \hat{w}_{hk}) \), where a “hat” over a variable denotes the proportional rate of change, e.g. \( \hat{w}_{lk} \equiv \frac{dw_{lk}}{w_{lk}} \) and \( \sigma_k \) is the (absolute) elasticity of substitution between low- and high-skilled labour in sector \( k \), as well as the condition \( \theta_{lk}\hat{w}_{lk} + \theta_{hk}\hat{w}_{hk} = \hat{c}_k \) (see, for example, Jones (1965) for derivations), makes it possible to simultaneously analyse the effects on the innovation rate \( \iota \) as well as on the factor prices \( w_{lk} \) and \( w_{hk} \) caused by, for example, a change in the fraction of the population that becomes skilled, and leads to\(^{10}\)

\[
\begin{pmatrix}
\Phi_{11} & \Phi_{12} & \Phi_{13} \\
\Phi_{21} & \Phi_{22} & \Phi_{23} \\
\theta_{LR} & \theta_{HR} & \frac{1}{\rho + \iota}
\end{pmatrix}
\begin{pmatrix}
\hat{w}_{lx} \\
\hat{w}_h \\
\iota
\end{pmatrix}
= 
\begin{pmatrix}
-\hat{s} \\
\hat{s} \\
0
\end{pmatrix}
\] (34)

where the \( \Phi \)-coefficients are defined as

\[
\Phi_{11} = - \left\{ \theta_{HR}\sigma_R L_R + \theta_{HX}\sigma_X L_X + \theta_{HT}\sigma_T L_T + \theta_{LX} L_X + \theta_{LT} L_T - (1 - s)w_{lx}\frac{\partial u_L}{\partial w_{lx}} N \right\} / (1 - u_L)sN
\]

\[
\Phi_{12} = \left[ \theta_{HR}\sigma_R L_R + \theta_{HX}\sigma_X L_X + \theta_{HT}\sigma_T L_T - \theta_{HXLX} - \theta_{HTLX} \right] / (1 - u_L)sN
\]

\[
\Phi_{13} = L_R / (1 - u_L)sN
\]

\[
\Phi_{21} = \left[ \theta_{LR}\sigma_R H_R + \theta_{LX}\sigma_X H_X + \theta_{LT}\sigma_T H_T - \theta_{LXX} + \theta_{LTHX} + \theta_{LTHX} \right] / sN
\]

\[
\Phi_{22} = - \left[ \theta_{LR}\sigma_R H_R + \theta_{LX}\sigma_X H_X + \theta_{LT}\sigma_T H_T + \theta_{HXR} + \theta_{HTH} \right] / sN
\]

\[
\Phi_{23} = H_R / sN
\]

Noting from equation (25) that \( \partial u_L / \partial w_{lx} < 0 \), means that the sign of determinant of the above 3 × 3 matrix \( \Theta \), can unambiguously be determined as

\[
|\Theta| = (\Phi_{11}\Phi_{22} - \Phi_{12}\Phi_{21})\iota / (\rho + \iota) + \theta_{HR}(\Phi_{21}\Phi_{13} - \Phi_{11}\Phi_{23}) + \theta_{LR}(\Phi_{12}\Phi_{23} - \Phi_{22}\Phi_{13}) > 0 \quad (35)
\]

\(^{10}\) As will become clear below, expressing all comparative-static effects as proportional changes, makes it possible to explain the underlying economic effects more precisely and intuitively.
Using Cramer’s Rule we derive

\[
\frac{\hat{w}_{LX}}{\hat{s}} = \frac{1}{sN|\Theta|} \left[ \frac{\ell}{\rho + \ell} \left( \theta_{LR} \sigma_R H_R + \theta_{LX} \sigma_X H_X + \theta_{LT} \sigma_T H_T + \theta_{HX} H_X + \theta_{HT} H_T \right) + \frac{1}{\left(1 - u_L\right)sN|\Theta|} \left[ \theta_{HR} L_R - \frac{\ell}{\rho + \ell} \left( \theta_{HR} \sigma_R L_R + \theta_{HX} \sigma_X L_X + \theta_{HT} \sigma_T L_T - \theta_{HX} L_X - \theta_{HT} L_T \right) \right] \right]
\]

for the effect of a higher supply of skills on the low-skilled wage rate. This is unambiguously positive only if \(\sigma_T, \sigma_X \leq 1\) and \(\sigma_R \leq \left(\rho + \ell\right)/\ell\). There are two counteracting effects which determine whether low-skilled wages increase with a higher supply of human capital or not. First, seeing as the size of the population is constant, a higher supply of human capital automatically means that the supply of low-skilled workers declines. Second, as shown below, the higher number of high-skilled workers will always lead to a reduction in the relative high-skilled wage. This means that the skill intensity in each sector rises whereby the productivity amongst the remaining low-skilled individuals increases. For relatively low values of the elasticities of substitution, for example if the production technology is given by a Cobb-Douglas function with a unitary substitution elasticity, the increased demand for low-skilled workers due to the increase in their productivity will dominate the substitution effect, whereby the fall in the relative high-skilled wage will cause firms to substitute high-skilled workers for low-skilled ones. In this case, total demand for low-skilled workers increases, leading to a higher low-skilled wage rate. For larger values of the elasticities of substitution, the substitution effect outweighs the demand effect, so that now total demand and thereby wages for the low-skilled decrease.\(^{11}\)

\(^{11}\) Empirical estimations of the substitution elasticity between low- and high-skilled labour vary greatly. For the U.S., Bound, Johnson (1992) and Katz, Murphy (1992) estimate (absolute) values between 1.4 and 1.7. Estimates for Germany vary a great deal depending on the economic sector and time period which is analysed. For example, whilst Entorf (1996) estimates a value of 1 between low- and high-skilled blue-collar manufacturing workers and between 0.5 and 1.5 for white-collar workers, both Fitzenberger, Franz (1998) and Steiner, Wagner (1998) find values in the range of 0.3 and 0.4. For an overview of these results and more detailed estimates which differentiate between various sectors and between males and females, see Steiner, Mohr (1998).
The effect of a higher supply of skills on the high-skilled wage is

\[
\frac{\bar{w}_H}{\bar{s}} = \frac{1}{sN|\Theta|} \left[ \frac{\ell}{\rho + \ell} \left( \theta_{LR} \sigma_R H_R + \theta_{LX} \sigma_X H_X + \theta_{LT} \sigma_T H_T - \theta_{LX} H_X - \theta_{LT} H_T \right) - \theta_{LR} H_R \right] - \frac{1}{(1 - u_L) sN|\Theta|} \left[ \frac{\ell}{\rho + \ell} \left( \theta_{HR} \sigma_R L_R + \theta_{HX} \sigma_X L_X + \theta_{HT} \sigma_T L_T + \theta_{LX} L_X + \theta_{LT} \sigma_T L_T - \theta_{LX} L_X \right) - \theta_{LR} L_R \right]
\]

which is unambiguously negative only if \( H_R > L_R \). However, seeing as the research sector is the most skill intensive and the economy we are considering is a highly developed one, it is realistic to assume that wages for high-skilled workers decrease when the supply of skills rises. Even if \( H_R < L_R \) so that the high-skilled wage increases, a comparison of equations (36) and (37) shows that the increase in the high-skilled wage is always less than the corresponding rise in the low-skilled wage level. This means that, as stated above, a higher level of human capital always lowers the relative high-skilled wage.

Similarly, the connection between the supply of human capital and the growth rate is found to be

\[
\hat{i} = \frac{1}{(1 - u_L) sN|\Theta|} \left\{ \theta_{HR} \sigma_R L_R + \theta_{HX} \sigma_X L_X + \theta_{HT} \sigma_T L_T + L_X(\theta_{HR} - \theta_{HX}) + L_T(\theta_{HR} - \theta_{HT}) - \theta_{HR}(1 - s) w_{LX} \frac{\partial u_L}{\partial w_{LX}} N \right\} - \frac{1}{sN|\Theta|} \left\{ \theta_{LR} \sigma_R H_R + \theta_{LX} \sigma_X H_X + \theta_{LT} \sigma_T H_T - H_X(\theta_{HR} - \theta_{HX}) - H_T(\theta_{HR} - \theta_{HT}) \right\} > 0
\]

As the research sector is the most skill-intensive in the economy, this sector benefits most from increase in the supply of human capital. With more individuals becoming highly skilled, the relative high-skilled wage falls, so that research costs fall. The new equilibrium is therefore characterised by a higher research intensity which leads to a faster innovation (and therefore higher growth) rate.

From equations (38) and (25) we find

\[
\frac{du}{ds} \left[ \frac{dw_{LX}}{ds} / \frac{dw_{LX}}{du_L} \right]
\]

There are two counteracting effects which determine whether the unemployment rate rises or falls with a higher supply of skills and thus whether there is a positive
or negative relationship between the growth and unemployment rate. On the one hand, a higher supply of skills automatically reduces the supply of low-skilled workers so that there is a negative effect on the unemployment rate. On the other hand, demand for these workers can either increase or decrease depending on the production technology. For low elasticities of substitution in all sectors, i.e. $\sigma_T, \sigma_X \leq 1$ and $\sigma_R \leq (\rho + \iota) / \iota$, the higher supply of skills leads to an increase in the low-skilled wage rate. However, as shown above, in this case total demand for low-skilled workers increases so that their unemployment rate falls while at the same time the growth rate increases. This is the case if all sectors produce according to a Cobb-Douglas output technology. For larger substitution elasticities, the fall in low-skilled labour demand means that the net result of a higher supply of skills is now a higher growth and unemployment rate.

Looking at the effects of union bargaining power on growth, we can use the same techniques as used to derive equation (34) to obtain

$$
\begin{pmatrix}
\Psi_{11} & \Psi_{12} & \Psi_{13} \\
\Psi_{21} & \Psi_{22} & \Psi_{23} \\
\theta_{LR} & \theta_{HR} & \frac{\iota}{\rho + \iota}
\end{pmatrix}
\begin{pmatrix}
\hat{w}_{LX} \\
\hat{w}_H \\
\hat{i}
\end{pmatrix}
= \begin{pmatrix}
-\hat{\beta} \\
0 \\
0
\end{pmatrix}
$$

(40)

where the $\Psi$-coefficients are defined as

$$
\Psi_{11} = -\left\{ \theta_{HR}\sigma_R L_R + \theta_{HX}\sigma_X L_X + \theta_{HT}\sigma_T L_T + \theta_{LX} L_X + \theta_{LT} L_T - \frac{w_{LX}}{\beta} \frac{\partial u_L}{\partial \beta} (1 - s)\beta N \right\} / \left[ \partial u_L / \partial \beta \right](1 - s)\beta N
$$

$$
\Psi_{12} = [\theta_{HR}\sigma_R L_R + \theta_{HX}\sigma_X L_X + \theta_{HT}\sigma_T L_T - \theta_{HX} L_X - \theta_{HT} L_T] / \left[ \partial u_L / \partial \beta \right](1 - s)\beta N
$$

$$
\Psi_{13} = L_R / (1 - s)\beta N
$$

and the remaining coefficients the same as the corresponding $\Phi$-coefficients in equation (34).

The determinant of the above $3 \times 3$-matrix is always positive. Therefore, again applying Cramer’s Rule, we obtain

$$
\frac{\iota}{\beta} = -\frac{\theta_{LR}}{sN|\Lambda|} \left[ \theta_{LR}\sigma_R H_R + \theta_{LX}\sigma_X H_X + \theta_{LT}\sigma_T H_T + \theta_{HX} H_X + \theta_{HT} H_T \right] -
$$
In this case, an increase in union bargaining power leads to higher low-skilled wages (in all sectors) but there is no corresponding increase in productivity. This lowers unskilled labour demand, thereby increasing their unemployment rate. At the same time, with fewer low-skilled workers in the research sector, the productivity of high-skilled labour in this sector declines, thereby lowering the growth rate.

5 Conclusion

Although there is without doubt a large literature (but still little consensus) on the connection between growth and unemployment, the role that unions play has been largely neglected. Integrating unions into a quality-ladder model with heterogeneous workers, allows us to analyse the effects of technical change on both the growth and unemployment rate. Firms in the intermediate sector supplying the highest quality good in their market, are able to set limit-prices and thus make profits. For this reason, low-skilled workers in this sector are organised in a union in order to capture some of the accruing rents. The wage level that results through bilateral bargaining between unions and employers also affects the wages workers in other sectors demand, so that union wage coverage is larger than union membership numbers suggest. Within this framework, we find that an increase in the supply of skills leads to a higher growth rate and may or may not lower the unemployment rate amongst the low-skilled. If there is a “high” elasticity of substitution between low- and high-skilled labour in all sectors, then the unemployment rate will rise at the same time as the growth rate increases. For “low” substitution elasticities, the opposite is true so that as the growth rate increases, the unemployment rate falls. A negative relationship between the growth and unemployment rate also results if union bargaining power rises in which case a higher low-skilled unemployment rate lowers the effective research intensity leading to a lower growth rate. However, because the low-skilled wage is higher than it would be without unions, we find exactly what can be observed on the two sides of the atlantic: the United States with its larger wage differential
but also higher growth rates than its Continental European counterparts which have a lower wage inequality but also lower growth rates as a result.
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