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ORIGINAL PAPER

The economic incentive for risk taking in professional partnerships

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Abstract
Professional service firms are common in some areas, in particular auditing and law. They are organized as partnerships, private corporations, or public corporations. This paper discusses the first category of these three. When a partner leaves the partnership, her/his shares are redeemed. Two alternatives for redemption are at book value, the traditional alternative, or at fair market value. By means of a novel discounted dividends model that includes risk taking, it is shown that there may be an economic incentive for risk taking when the redemption value is equal to book value. There may also be an incentive for risk taking when the redemption value is equal to fair market value. However, the level of risk taking in the latter alternative is not higher than the level of risk taking in the former alternative. Switching from book value to fair market value as redemption value is hence suggested as one way to reduce client propensity for litigation. This paper's incentive for risk taking in a professional partnership has apparently not been noted in the literature.

Keywords Professional partnership · Horizon · Redemption value · Discounted dividends · Incentive for risk taking

JEL Classification M41 · G31 · G33 · C63

1 Introduction and overview

Professional service firm (PSF) denotes a class of firms that are characterized by knowledge intensity, low capital intensity, and professionalized workforce (von Nordenflycht 2010; see also Greenwood et al. 2005; Leblebici and Sherer 2015; Morris et al. 2010). PSFs are common in a number of areas, in particular auditing and law, but also in other areas such as management consulting, advertising, investment

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banking, and architecture. PSFs can be organized as partnerships, the topic of this paper, but also as private corporations or public corporations. Law and audit firms have traditionally been set up as partnerships. For discussions with emphasis on partnerships, see Clark et al. (2005), Fama and Jensen (1983, pp. 334–337), Greenwood and Empson (2003), Greenwood et al. (1990), Pickering (2015) (audit firms), Ribstein (2010) (law firms), and van Lent (1999) (audit firms).

In a professional partnership, there are no owner shares that are publicly traded. Instead, the shares are held by the partners, with restrictions on transferability. This means that shares are redeemed when a partner retires or leaves for some other reason. Clearly, a share in a partnership is quite different from a share of stock in a corporation. To emphasize that difference, other names are sometimes used, such as partnership interest (Milgrom and Roberts 1992, p. 523) and partnership stake (Morrison and Wilhelm 2004, p. 1682). However, (partnership) share is used consistently in this paper.

When a new partner is admitted, she/he acquires (by assumption on scaling) one share. It is assumed in this paper that part of the subsequent annual compensation to the partner is paid in accordance with the compensation system "you eat what you kill", a system that is increasing in importance (Brock et al. 2007, p. 237) and may be particularly important in smaller partnerships (Lennox and Wu 2018, p. 23); see also Saab Fortney (1995) on partnerships that are structured as confederations. More precisely, it is assumed here that part of the annual compensation to the partner is paid as dividend generated by the partner's own activities, and by the activities of assistants and other non-partner professionals that she/he supervises.

When a partner leaves, the redemption value of her/his share is typically (or at least traditionally) the *book value* of the assets that are associated with that share. Cf. Fama and Jensen (1983, p. 337): "... a departing partner is generally compensated for his share in assets, such as cash and accounts receivable."As an anecdotal example of share redemption at book value, Morrison and Wilhelm (2004, pp. 1682–1683) mention that for several decades it was stipulated in the consulting company McKinsey that new partners should buy out old partners at the book value rather than the higher market value of their shares.

The value of one partnership share at the present time is then discounted expected dividends over the years until retirement or otherwise leaving, plus the present value of the expected book value amount that is received when the share is redeemed. A partnership share with redemption value equal to book value hence has a *finite horizon*.

An alternative, apparently less frequent, redemption value of one partnership share is the present value (at the time of redemption) of subsequent expected dividends over infinitely many years. This present value is taken here to be based on assumptions that would be reasonable under prevailing market conditions and is therefore referred to as *fair market value*. With that redemption value, the value of one partnership share at the present time becomes discounted expected dividends over infinitely many years. Even so, a partnership share with redemption value equal to fair market value has the *same finite horizon* as with redemption value equal to book value, since in any case it is redeemed when the partner leaves the partnership. Expressed somewhat differently: A partnership share has the same finite horizon under both redemption values, but the *value calculation method* at the end of that same finite horizon is not the same (book value or fair market value).

The finite share horizon in a partnership is reminiscent of the classical horizon problem in an employee-owned firm that is not capital-light. This problem is mentioned in several places (e.g., Furubotn 1976; Hansmann 1988, p. 294; Jensen and Meckling 1979, pp. 481–484, 489) and can be summarized as follows: Workers lose their shares in capital that has been accumulated when they leave the firm, since (like in a professional partnership) they cannot freely sell their ownerships to other persons. Consequently, they have incentives not to consent to investment projects with pay-back periods longer than the expected remaining employment periods in the firm. This classical horizon problem is not immediately relevant for a capital-light professional partnership. However, one sees a resemblance to this paper's horizon problem: In a partnership, a partner may want to take risk, since her/his share has a finite horizon, and negative consequences of risk taking may show up only after that horizon.

The purpose of this paper is to show by means of a formal valuation model that there may indeed be an economic incentive for risk taking in a partnership with redemption value equal to book value. This incentive has apparently not been noted in the literature, at least not clearly, as will be suggested in the conclusion (Sect. 5). A similar incentive may also apply to a partnership with redemption value equal to fair market value. However, the optimal level of risk taking is not higher in the latter than in the former. Risk taking by partners can lead to litigation by clients. It can therefore be a good decision by the senior management team that is responsible for the long-term reputation of the partnership to change the redemption value from book value to fair market value, as one step to reduce client propensity for litigation. This is also suggested in the conclusion.

The valuation model is novel. It is outlined in the following Sect. 2, where the meaning of risk taking is also clarified. "Risk taking"is used throughout *only* as a polite synonym for a variety of expressions that are used in the literature on PSFs, such as "leeway" (Bronnenmayer et al. 2016, pp. 4–5, 10), "shoddy audit"or "shoddy work" (Eisenberg and Macey 2004, p. 267; Ribstein (2010), p. 807), and "low audit effort" (Gramling et al. 1998). The use of "risk taking"does not imply some reference to utility functions or risk attitudes of partners. Indeed, there is no assumption on utility functions at all. The one assumption that is made is that partnership shares can be valued by CAPM.

Section 3 displays present value formulas for partnership shares with redemption values equal to book value and to fair market value. For discounting, the certainty equivalent CAPM version is used in the main body of this paper. The reason for using the certainty equivalent CAPM version is that it is less complicated than the ordinary CAPM version (no complicated formulas for discount rates). Section 4 contains a series of numerical examples that illustrate the economic incentive for risk taking. Section 5 concludes with remarks on take-aways from this paper.

The notation is summarized in Appendix 1. Appendix 2 searches for counterexamples of optimal level of partnership risk taking that is higher with redemption value equal to fair market value than with redemption value equal to book value (no such example is found; a formal proof by taking derivatives and setting to zero cannot be obtained). Appendix 3 shows that the certainty equivalent CAPM version is totally equivalent to the ordinary CAPM version.

As is well known, the use of CAPM to obtain a cost of capital for discounting has been criticized in the literature (see, e.g., Fernandez 2015). However, according to Koller et al. (2020, pp. 58–59), CAPM has been the standard model for measuring differences in costs of capital, and so far no practical competing model has emerged. Actually, one particular present value is *assumed* in Sect. 4, so CAPM is used only in a limited sense, that is, only for discounting in a fashion that is consistent between different time horizons, levels of risk taking, and redemption values (book or fair market).

This paper is somewhat reminiscent of an earlier paper, Jennergren (2013). Like this paper, Jennergren (2013) allows for failure of the firm that is the object of the valuation. However, Jennergren (2013) does not incorporate an incentive for risk taking, does not use certainty equivalent discounting, and does not mention partnerships. Since a professional partnership is capital-light, this paper does not include property, plant and equipment that is a major concern in Jennergren (2013) (cf. also Jennergren 2008). Friedrich (2016) refers to Jennergren (2013) and uses binomial trees somewhat similar to this paper's Fig. 1 below. Cf. also Skogsvik et al. (2023).

This paper is intended as a contribution to financial modelling as defined by the EURO Working Group for Commodities and Financial Modelling, that is, as an aid

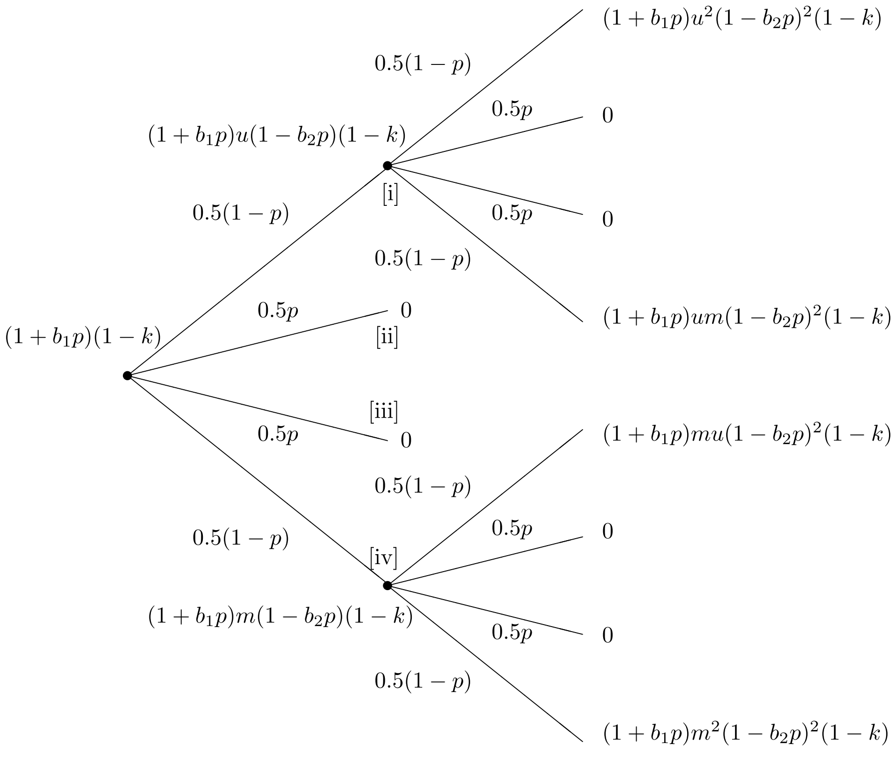

Fig. 1 Event tree showing different possible earnings in years 1 and 2

in solving problems that are faced by decision makers in firms, intermediaries and the investment community. Regarding this role of financial modelling, Jaap Spronk and Winfried Hallerbach have noted that there is a tension between finance theory and practical financial decision making, since finance theory is not so much concerned with individual decisions. Instead, finance theory is more concerned with effects of decisions and actions by "average"individuals (e.g., investors) (Spronk and Hallerbach 1997, pp. 113–117; Hallerbach and Spronk 2002, pp. 111–115; cf. also Spronk 2015, pp. 12–13, 31). However, financial modelling can be a bridge between finance theory and practical financial decision making (Spronk 1999, p. 217).

In the present paper, the decision making concerns the choice between book value and fair market value as redemption value when a partner leaves, as already indicated above. The theory that is connected to decision making by means of the financial modelling bridge is not only finance theory. Elements of corporate governance and organization theory are also included, as is clear from this introductory section (and can also be inferred from the reference list).

2 Model outline

The time of valuation is the end of year $t = 0$, the last historical year. All action takes place at year ends. The object of valuation is the one partner share (by scaling) belonging to a representative partner. $B_0 \geq 0$ is the book value of the assets at the end of year 0 that are associated with the partner in question. Working capital is understood as net, i.e., assets minus liabilities, and is assumed to be non-negative, a natural assumption for a professional partnership. There are only working capital (net) assets, more precisely only cash and net receivables since inventories are not produced by professional partnerships. There is no property, plant and equipment. That is also not unreasonable for a capital-light partnership. Hence, B_0 is the non-negative book value of working capital only at the end of year 0. For simplicity, the partnership has no interest-bearing debt. Earnings are sales minus costs, and costs are cash costs since there is no property, plant and equipment and hence no depreciation. In what follows, share value, sales, earnings, assets, and dividend are understood as pertaining to the activities of the representative partner.

It is assumed that there was no risk taking in year $t = 0$ and that earnings in that year are equal to $R_0(1 - k)$, where $R_0 > 0$ denotes sales and $0 \leq k \leq 1$ denotes cash costs as a fraction of sales. $w = B_0/R_0$ is book value of assets divided by sales, i.e., $B_0 = wR_0$.

By "base earnings"is meant earnings at the end of year 0 that are used as the starting point for forecasting earnings in the subsequent years. If there is no risk taking, the base earnings are equal to the actual earnings at the end of year 0, that is, $R_0(1 - k)$. It is assumed that risk taking starts in year $t = 1$, if the representative partner so decides. So, the choice whether or not to take risk is one that the partner makes. The decision variable is p, the annual failure probability that measures the amount of risk taken by the representative partner. If there is a partner failure, the partner's share is revoked, and the assets associated with that share are lost. This does not necessarily mean that the entire partnership also fails (Arthur Andersen's demise after the Enron scandal is one rare example of an entire audit partnership failure subsequent to a partner failure; but in any case, the reputation of the entire partnership is of course affected negatively by a partner failure). Apparently, there is a variety of possible failure consequences for a risk-taking partner, as seen by the following quote that refers to audit partnerships: "Individual partners acting alone in negligence or fraud typically lose future employment prospects, lose personal assets in damages settlements, and can be disbarred, fined, or jailed" (Ball 2009, p. 294; negligence and fraud are certainly included in this paper's general category of risk taking). However, for simplicity this paper only considers the two partner failure consequences just mentioned above, share revocation and loss of associated assets.

For a partner who does not take any risk, $p = 0$. For a risk-taking partner, $0 < p \leq 1$. The base earnings for a risk-taking partner (at the end of year 0) are $R_0(1 + b_1p)(1 - k)$, where $b_1 \geq 0$. In other words, risk taking means adding activities that increase sales and consequently also base earnings. b_1 indicates the maximal extent of that increase and can be thought of as anticipated sales from a *potential* maximal set of contracts with new clients and expansions of contracts with already existing clients. It is not assumed that the partner would be working alone on these potential contracts; assistants and other non-partner professionals would be involved under the partner's supervision. $R_0b_1p(1 - k)$ is the actual increase in base earnings that results from that particular amount of risk p that the partner decides on. There is also an effect at the end of year t, apart from the failure risk, through the factor $(1 - b_2p)^t$, where $0 \leq b_2 \leq 1$. In other words, $(1 - b_2p)^t$ enters into the earnings at the end of year t for a risk-taking partner. That factor expresses a gradual downward pressure on earnings, as the clients perceive that there is some lowering of work quality as a consequence of risk taking. The annual failure probability p is the only decision variable for the representative partner. In particular, b_1 and b_2 are not decision variables.

Without loss of generality, R_0 is assumed to be 1. Base earnings for a non-risk-taking partner are then $(1 - k)$, and for a risk-taking partner $(1 + b_1p)(1 - k)$. The book value of the assets at the end of year 0 becomes $w \times 1 = w$. By assumption above, $w \geq 0$. It also holds that $w \leq 1$, since w is a fraction of sales, and sales cannot generate more cash and receivables than the amount of sales.

The valuation model is embedded in a non-deterministic framework. See Fig. 1 for an event tree showing different possible earnings in years 1 and 2 for a risk-taking partner. At the end of year 0, the book value of the assets is w and base earnings $(1 + b_1p)(1 - k)$, as already indicated. Starting in year 1, for a risk-taking partner there is a probability p of failure in each year, also as already indicated above.

With the end of year 0 as a starting point, there are four branches to four subsequent possible states of the world at the end of year 1. The four possible states are denoted by [i], [ii], [iii], and [iv] in Fig. 1 and defined as follows:

State of the world [i]*:* Probability $0.5(1 - p)$. The market rate of return is M_{h} that is fairly high. The end of year 1 sales are $(1 + b_1p)u(1 - b_2p)$, and earnings are $(1 + b_1p)u(1 - b_2p)(1 - k)$. u is a fairly high change factor for earnings, or equivalently for sales since earnings are proportional to sales. $(1 - b_2p)$ is the factor that depresses earnings due to risk taking (as already mentioned). To obtain *dividend* from earnings, one first deducts the multiple w of sales

$(1+b_1p)u(1-b_2p)$ that expresses working capital tied up in the operations. Then one adds back the book value of working capital w at the end of year 0. The end result, i.e., the dividend, is $(1+b_1p)u(1-b_2p)(1-k)-(1+b_1p)u(1-b_2p)w+w$ $=(1+b_1p)u(1-b_2p)(1-k-w)+w$.

State of the world [ii]*:* Probability $0.5p$. The market rate of return is M_{h} that is fairly high. The representative partner has failed. Consequently, her/his share is revoked and the associated assets are lost (so there is no dividend).

State of the world [iii]*:* Probability $0.5p$. The market rate of return is M_{l} that is fairly low. The consequences for the partner are the same as in state of the world [ii] (her/his share is revoked and the associated assets are lost).

State of the world [iv]*:* Probability $0.5(1-p)$. The market rate of return is M_{l} that is fairly low. The end of year 1 earnings are $(1+b_1p)m(1-b_2p)(1-k)$. m is a fairly low change factor for earnings. The consequences for the partner are the same as in state of the world [i], except that m replaces u in the formulas for earnings and dividend.

If the representative partner finds her/himself in state [ii] or [iii] at the end of year 1, there are no further branches in the event tree in Fig. 1. If the partner finds her/himself in state [i] at the end of year 1, there are the same possible four branches in year 2 as in year 1, with the same probabilities and the same change factors u and m. If it happens that the change factor is fairly high (u), then earnings at the end of year 2 are $(1+b_1p)u^2(1-b_2p)^2(1-k)$. Again deducting working capital that is tied up in the operations at the end of year 2 and adding back working capital at the end of year 1, one obtains the dividend as $(1+b_1p)u^2(1-b_2p)^2(1-k-w)+(1+b_1p)u(1-b_2p)w$. Similarly, if the partner finds her/himself in state [iv] at the end of year 1, there are the same possible four branches in year 2 as in year 1, with the same probabilities and the same change factors u and m. The formulas for all possible earnings at the end of year 2 are seen in Fig. 1. It is clear that dividends are obtained by deducting working capital that is tied up in the operations, that is, this year's sales multiplied by the factor w, and adding back working capital at the end of the previous year, that is, the previous year's sales multiplied by w. Setting the working capital as proportional to the activity level of the firm measured by sales is standard in firm valuation by the discounted cash flow model (see, e.g., Jennergren (2008), p. 1553).

At any node in the event tree, the probability of a market fairly high rate of return M_{h} in the next year is apparently 0.5, and the same for a market fairly low rate of return M_{l}. The expected market rate of return over one year is hence $0.5(M_{\mathrm{h}}+M_{\mathrm{l}})$, and the variance of the market rate of return $(0.5(M_{\mathrm{h}}-M_{\mathrm{l}}))^2$. If $p=0$, the probability of a change factor for earnings u is also 0.5 (in state [i]), and the same for a change factor m (in state [iv]). The expected change factor for earnings is then $0.5(u+m)$. The covariance between the market rate of return and the change factor for earnings when there is no risk taking is $(0.5(M_{\mathrm{h}}-M_{\mathrm{l}}))(0.5(u-m))$. These simple formulas for variance and covariance follow from the very simple assumed distributions. However, it is seen in the next section that the distribution assumptions are not restrictive, and that the valuation model is actually more general than it would appear from Fig. 1.

To recapitulate, risk taking means a choice of $p > 0$, leading to three different consequences: In the first place, there is an increase in base earnings immediately, i.e., starting at the end of year $t = 0$, from $(1 - k)$ to $(1 + b_1 p)(1 - k)$. In the second place, there is a probability p of partner failure in each year t. In the third place, there is a gradual downward pressure on earnings year after year, expressed by the factor $(1 - b_2 p)^t$, as the reputation of the partner declines when the clients notice that there is risk taking going on.

The expected earnings at the end of year t are equal to

$$(1 + b_1 p)((0.5(1 - p)(u + m)(1 - b_2 p))^t)(1 - k)$$

for $t \geq 1$, where the annual earnings growth rate is apparently

$$c = (0.5(1 - p)(u + m)(1 - b_2 p)) - 1.$$

The expected book value of the assets at the end of year $t \geq 1$ is

$$(1 + b_1 p)((0.5(1 - p)(u + m)(1 - b_2 p))^t)w.$$

The expected dividend at the end of year $t = 1$ (possibly negative) is

$$(1 + b_1 p)(0.5(1 - p)(u + m)(1 - b_2 p))(1 - k - w) + w(1 - p),$$

and at the end of year $t \geq 2$

$$\begin{aligned}&(1 + b_1 p)((0.5(1 - p)(u + m)(1 - b_2 p))^t)(1 - k - w)\\&+ (1 + b_1 p)((0.5(1 - p)(u + m)(1 - b_2 p))^{(t-1)})w(1 - p).\end{aligned}$$

The formulas for expected dividend are seen to include expected change in working capital.

It is clear from Sect. 1 that irrespective of the redemption value, book value or fair market value, a partner share has a limited time horizon (the same horizon for both redemption values). In the sequel, the horizon is the remaining time span of the one representative partner share that will eventually be redeemed. The horizon year is denoted by $T \geq 1$.

3 Certainty equivalent discounting and a comment on assumptions

The value of the one representative partner share at the end of year $t = 0$ with redemption value equal to book value is denoted by $V^{\mathrm{BV}}_{\mathrm{CEQ}}(p, T)$, and by $V^{\mathrm{FMV}}_{\mathrm{CEQ}}(p)$ for the value at the same point in time of the one share with redemption value equal to fair market value (the arguments are sometimes suppressed below). $V^{\mathrm{FMV}}_{\mathrm{CEQ}}$ does not depend on T, even though the share is redeemed at the end of year T, when the partner leaves the partnership. Values will be determined by the following formulas, (2) plus (3) for $V^{\mathrm{FMV}}_{\mathrm{CEQ}}(p)$, and (4) for $V^{\mathrm{BV}}_{\mathrm{CEQ}}(p, T)$, for discounting certainty equivalents by the risk-free interest rate, denoted by i. As explained in Appendix 3, these formulas provide exactly the same present values as using the ordinary CAPM version to obtain risk-adjusted discount rates and then discounting expected free cash flow. The subscript CEQ obviously means certainty equivalent, as opposed to the subscript ORD that means ordinary CAPM and will be encountered in Appendix 3.

A definition:

$$Q = \frac{1}{0.5(u+m)} \times \left(0.5(M_{\mathrm{h}} + M_{\mathrm{l}}) - i\right) \times \frac{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))(0.5(u-m))}{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))^2} \tag{1}$$

is a convenient formula shortener in what follows. The second factor in (1) is the (non-negative; cf. (8) in Appendix 2 below) market risk premium. The third factor in (1) is called "cash-flow beta"in Sick (1986), p. 24.

The value of the one share with redemption value equal to fair market value is divided into two parts (2) plus (3) (Appendix 3 refers to this division into two parts):

$$V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p) = \frac{(1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)(1-Q) + w(1-p)}{1+i} \tag{2}$$

$$\begin{aligned} &+ \Bigg[\Bigg\{ (1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^2)(1-k-w)(1-Q)^2 \\ &+ (1+b_1p)(0.5(1-p)(u+m)(1-b_2p))w(1-p)(1-Q) \Bigg\} \\ &\div \Bigg\{ 1+i-(0.5(1-p)(u+m)(1-b_2p))(1-Q) \Bigg\}\Bigg] \Bigg/ \Bigg[1+i\Bigg]. \end{aligned} \tag{3}$$

Row 2 in (2) is the present value at the end of year 0 of the certainty equivalent expected dividend at the end of year 1. The first factor in curly brackets (rows 1 and 2 of (3)) divided by the second factor in curly brackets (row 3) is the certainty equivalent Gordon formula value, at the end of year 1, of subsequent expected dividends over an infinite horizon. The second factor in curly brackets in (3) is seen to be equal to $\{1 + i - (1 + c)(1 - Q)\}$. Discounting from the end of year 1 to the end of year 0 is done at the risk-free interest rate i (end of row 3 of (3)). (2) plus (3) can be simplified, if $p = 0$. See formula (5) in Sect. 4 below.

For $T \geq 1$, the value of the one share with redemption value equal to book value is

$$
\begin{aligned}
V^{\mathrm{BV}}_{\mathrm{CEQ}}(p,T) &= V^{\mathrm{FMV}}_{\mathrm{CEQ}}(p) \\
&- \Bigg[\bigg\{(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{(T+1)})(1-k-w)(1-Q)^{(T+1)} \\
&+ (1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{T})w(1-p)(1-Q)^{T}\bigg\} \\
&\div \bigg\{1+i-(0.5(1-p)(u+m)(1-b_2p))(1-Q)\bigg\}\Bigg] \Big/ \Big[(1+i)^T\Big] \\
&+ \Big[(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{T})w(1-Q)^{T}\Big] \Big/ \Big[(1+i)^T\Big].
\end{aligned}
\tag{4}
$$

Equation (4) for $V^{\mathrm{BV}}_{\mathrm{CEQ}}(p,T)$ is seen to consist of three components of value at the end of year 0: The value of one share with redemption value equal to fair market value with the same p (row 1); minus the value of expected dividends from year $T+1$ of one share with redemption value equal to fair market value (rows 2–4); plus the value of the expected book value of the assets at the end of year T (row 5).

A comment on assumptions: As illustrated in Fig. 1 in Sect. 2, the distribution of market rates of return is very simple, two states of the world M_{h} with probability 0.5 and M_{l} with probability 0.5. This is not restrictive, since in the end it is only the mean $0.5(M_{\mathrm{h}}+M_{\mathrm{l}})$ and the variance $(0.5(M_{\mathrm{h}}-M_{\mathrm{l}}))^2$ that matter, as can be seen in the formulas (1), (2) plus (3), and (4) above. M_{h} and M_{l} can be set to obtain any desired mean and variance, while keeping the two states with probabilities 0.5. In the absence of risk taking, the distribution of change factors for earnings is also very simple, the same two states of the world u with probability 0.5 and m with probability 0.5. Again, this is not restrictive, since it is only the change factor expectation $0.5(u+m)$, and the covariance $(0.5(M_{\mathrm{h}}-M_{\mathrm{l}}))(0.5(u-m))$ between market rate of return and change factor for earnings (in the absence of risk taking), that matter. This can also be seen in the formulas (1), (2) plus (3), and (4). One can set u and m, while keeping the two states with probabilities 0.5, to obtain any desired expectation and covariance (given $0.5(M_{\mathrm{h}}-M_{\mathrm{l}})$; it is not necessary that $u>m$). These simple distributions of market rates of return and change factors were used in order to construct the simple illustrative event tree in Fig. 1, to facilitate the derivation of formulas (1), (2) plus (3), and (4) above, and also to facilitate the calculation of the ordinary CAPM version value in Appendix 3. It is seen in (2) plus (3) and (4) that partner failure due to risk taking is a non-systematic risk. That is the most restrictive assumption.

4 Examples of the economic incentive to take risks

Consider the following assumptions: Sales in year $t=0$, R_0, have already been assumed to be 1. Cash costs k as a fraction of sales are 0.9. The initial book value of assets, B_0, is equal to $wR_0 = 0.2\times 1 = 0.2$, so $w=0.2$. The fairly high market rate of return M_{h} is equal to 10%, and the fairly low market rate of return M_{l} is equal to 3%. The risk-free interest rate i is equal to 2%.

Suppose for the moment that $p = 0$, and the redemption value is fair market value. The simplified variant of (2) plus (3) is then

$$V_{\mathrm{CEQ}}^{\mathrm{FMV}}(0) = \left\{ (0.5(u+m))(1-k-w)(1-Q)+w \right\} \Big/ \left\{ 1+i-0.5(u+m)(1-Q) \right\}. \tag{5}$$

The change factors u and m are now set so that $V_{\mathrm{CEQ}}^{\mathrm{FMV}}(0) = 1.65$ and $0.5(u-m) = 0.04140$ (rounded). 1.65 is hence the assumed present value that was mentioned in Sect. 1 above. One obtains $u = 1.05140$ and $m = 0.96860$ (rounded). Also, $0.5(u+m) = 1.01$.

b_2 in the factor for downward pressure $(1 - b_2 p)$ is set to 1. The representative partner horizon T is equal to 15 years.

Three different examples of incentives, corresponding to different assumptions about b_1, will now be considered. The annual failure probability with redemption value equal to fair market value is denoted by p^{FMV}, and that with redemption value equal to book value by p^{BV}.

In the first example, $b_1 = 60$. With all assumptions now specified, the optimal p^{FMV} is 0.0396, resulting in the value $V_{\mathrm{CEQ}}^{\mathrm{FMV}} = 2.10184$. The optimal p^{BV} is 0.0615, resulting in $V_{\mathrm{CEQ}}^{\mathrm{BV}} = 1.95450$. In both cases, the representative partner has an incentive to take risk, but the optimal p^{BV} is higher than the optimal p^{FMV}. $V_{\mathrm{CEQ}}^{\mathrm{FMV}}$ is seen to be higher than $V_{\mathrm{CEQ}}^{\mathrm{BV}}$. Figure 2 illustrates by showing $V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}})$ and $V_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}}, T)$ for different values of p^{FMV} and p^{BV} between 0 and 1. The upper curve in the figure is valid for redemption value equal to fair market value, and the lower curve for redemption value equal to book value. The maximizing values of p^{FMV} and p^{BV} and the resulting values of $V_{\mathrm{CEQ}}^{\mathrm{FMV}}$ and $V_{\mathrm{CEQ}}^{\mathrm{BV}}$ in the first example are indicated by the peaks of the two curves. The curve that applies to book value is initially lower than the one that applies to fair market value, but the two separate curves cannot be distinguished in the figure for values of p higher than 0.15. (The book value curve is actually very slightly higher than the fair market value curve for p between 0.25 and 0.71.) Evidently, $V_{\mathrm{CEQ}}^{\mathrm{BV}}$ converges to $V_{\mathrm{CEQ}}^{\mathrm{FMV}}$ as p goes to 1.

In the second example, $b_1 = 24$. The optimal p^{FMV} is 0, that is, no risk taking. The resulting value $V_{\mathrm{CEQ}}^{\mathrm{FMV}}$ is 1.65000. The optimal p^{BV} is 0.0276, resulting in a value $V_{\mathrm{CEQ}}^{\mathrm{BV}}$ of 1.18019. In this case, there is no incentive for risk taking when the redemption value is equal to fair market value, only when the redemption value is equal to book value. It still holds that $V_{\mathrm{CEQ}}^{\mathrm{FMV}}$ is higher than $V_{\mathrm{CEQ}}^{\mathrm{BV}}$.

In the third example, $b_1 = 14$. Now it is optimal to set $p^{\mathrm{FMV}} = 0$ and $p^{\mathrm{BV}} = 0$. Optimal values are $V_{\mathrm{CEQ}}^{\mathrm{FMV}} = 1.65000$ and $V_{\mathrm{CEQ}}^{\mathrm{BV}} = 1.09473$.

It is noted in the preceding three examples that there may be an incentive for risk taking by the representative partner, irrespective of whether the redemption value is fair market value or book value. However, the optimal p^{BV} is higher than or equal to the optimal p^{FMV}. This means that optimal risk taking with fair market value as redemption value is not higher than optimal risk taking with book

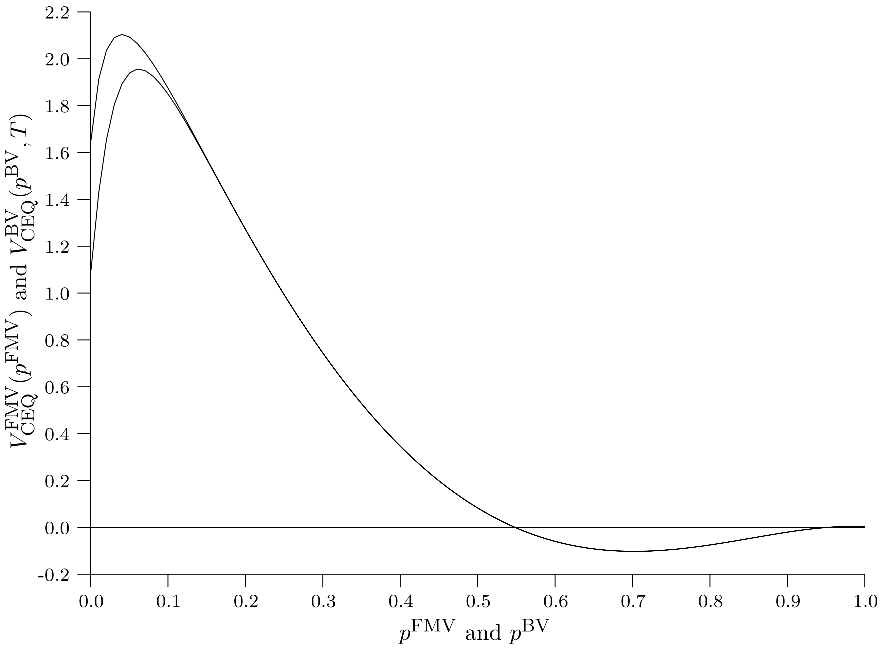

Fig. 2 Partnership values $V_{\text{CEQ}}^{\text{FMV}}(p^{\text{FMV}})$ and $V_{\text{CEQ}}^{\text{BV}}(p^{\text{BV}}, T)$ for different values of p^{FMV} and p^{BV} ($b_1 = 60, T = 15$)

value as redemption value, and may well be lower. This is illustrated in Fig. 3 that is an extension of the first example above. It shows optimal values of p^{FMV} and p^{BV} for all horizons T from 1 to 100. The optimal p^{FMV}, 0.0396, does not depend on T and is therefore displayed as a straight line across the figure. The optimal p^{BV} starts at 0.3055 for $T = 1$ and converges to 0.0396 as T goes to 100. The optimal value 0.0615 of p^{BV} for $T = 15$ can be read from the upper curve in the figure. In Fig. 3, indeed, there is no T for which the optimal p^{FMV} is higher than the optimal p^{BV}.

Hence, the conclusion that optimal risk taking by the representative partner with redemption value equal to fair market value is not higher than with redemption value equal to book value follows from a comparison of the the optimal p^{FMV} and p^{BV}. In other words, lower (or at most equal) risk taking follows from $p^{\text{FMV}} \leq p^{\text{BV}}$.

It is intuitively plausible that optimal risk taking with fair market value as redemption value is not higher than optimal risk taking with book value as redemption value. A formal proof appears elusive, however, as already mentioned in Sect. 1 above. Taking derivatives of the functions $V_{\text{CEQ}}^{\text{FMV}}(p)$ in (2) plus (3) and $V_{\text{CEQ}}^{\text{BV}}(p, T)$ in (4), setting to zero, and solving for p is not feasible, since those derivatives include polynomial factors of higher degree than four. Also, the boundaries at $p = 0$ and $p = 1$ must be considered. Actually, both value functions in Fig. 2 have three extreme points, even though only two are visible in the figure. More precisely, $V_{\text{CEQ}}^{\text{FMV}}(p) = 0$ for $p = 1$ and $V_{\text{CEQ}}^{\text{BV}}(p, 15) = 0$ for $p = 1$, but

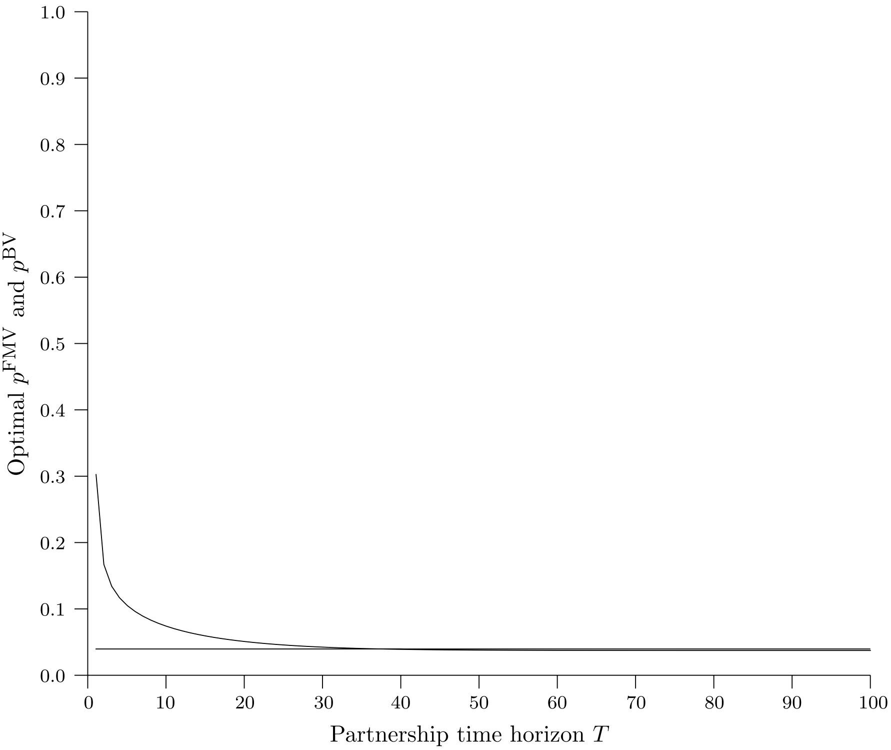

Fig. 3 Optimal p^{FMV} and p^{BV} for different partnership time horizons T ($b_1 = 60$)

$V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p)$ and $V_{\mathrm{CEQ}}^{\mathrm{BV}}(p, 15)$ are both very slightly positive for p slightly below 1. This discussion is continued in Appendix 2 below that searches for counterexamples where $p^{\mathrm{FMV}} > p^{\mathrm{BV}}$.

5 Concluding remarks

Intuitively, it is not surprising that there should be an economic incentive for risk taking in a partnership when the leaving representative partner's one share is redeemed at the book value of associated assets. But it is not obvious how that intuition should be modelled. The modelling in this paper is parsimonious, yet realistic. As mentioned in Sect. 3, only a limited number of assumptions is needed: The expectation and variance of the market rate of return, the expectation of the change factor for earnings when there is no risk taking, and the covariance between the market rate of return and the change factor for earnings (again when there is no risk taking). Three distinctive aspects of risk taking are included, an initial increase in base earnings, a subsequent possibility of failure, and (in the absence of failure) a gradual pressure on earnings due to decline of reputation as the risk taking is noted by the clients. Failure due to risk taking is assumed to be a non-systematic risk. These aspects of risk taking fit well with a discounted dividends valuation model. This combination of risk taking, on the one hand, and valuation model, on the other, is suggested as a first take-away from this paper. The same combination of risk taking and valuation model is applicable also when the leaving partner's share is redeemed at fair market value. As seen in Sect. 4 above, in that case there is also an incentive for risk taking, but not as strong as when the partner's share is redeemed at book value.

Apparently, the horizon problem in this paper has not attracted much attention in the literature. More precisely, one finds only weak hints suggesting that redemption value at the horizon equal to book value could be a (partial) explanation for risk taking in a partnership. One such hint is that partners may avoid participating in firm-building efforts such as coaching younger staff members who aspire to become partners (Clark et al. 2005, p. 294; Greenwood and Empson 2003, p. 915). Even if non-participation in such efforts is not the same as this paper's meaning of risk taking, one sees a parallel to the classical horizon problem in Sect. 1 above, that workers in an employee-owned firm may not consent to investment projects with payback consequences that extend beyond expected remaining employment periods.

It has been noted repeatedly in the literature that for some time there has been a tendency for partnerships to reorganize as private corporations (or as public corporations; for PSFs, the boundary between private and public corporations is diffuse). See, e.g., Empson and Chapman (2006), Greenwood and Empson (2003, pp. 911–912), Leblebici and Sherer (2015, p. 208), Malhotra et al. (2006, p. 196), Morris et al. (2010, p. 285), von Nordenflycht (2014) (cf. also Smets et al. 2017). A number of reasons have been suggested for this tendency. These reasons can generally not be linked to specific economic incentives. However, there is one exception, increasing propensity for litigation by clients (Greenwood and Empson 2003, p. 925). van Lent (1999, p. 225) writes that a growing number of audit firms have altered organizational form from partnership to corporation in order to cap liability exposure.

For a professional partnership, it is plausible that client propensity for litigation is related to this paper's meaning of risk taking by the partners, since clients may perceive a lowering of work quality that is due to risk taking. Also, circumstances relating to failures by partners due to risk taking may lead to litigation. For a partnership that applies the compensation system "you eat what you kill", one could therefore recommend to the senior management team to switch from redemption value equal to book value to redemption value equal to fair market value as one way of reducing client propensity for litigation. This recommendation follows, since it is a conclusion of this paper that risk taking with redemption value equal to fair market value is not higher than, and may well be lower than, risk taking with redemption value equal to book value.

A switch from book value to fair market value as redemption value would seem to be a fairly easy step, compared to a switch from partnership to corporation. It is proposed here as a second take-away from this paper.

Appendix 1: Notation

t	Index for years (0, 1, 2...),
R_0	Sales without risk taking at the end of year $t = 0$,
k	Cash costs as a fraction of sales ($0 \leq k \leq 1$),
B_0	Book value of assets (net working capital only) at the end of year $t = 0$ ($B_0 \geq 0$),
w	Book value of assets divided by sales ($= B_0/R_0$, $0 \leq w \leq 1$),
p	Annual failure probability (selected by the representative partner, indicator of level of risk taking),
b_1	Maximal increase in potential sales contracts, enters into sales with risk taking $R_0(1 + b_1 p)$ at the end of year $t = 0$ ($b_1 \geq 0$),
b_2	Effect of risk taking on the annual earnings decrease factor $(1 - b_2 p)$ ($0 \leq b_2 \leq 1$),
M_{h}	Fairly high market rate of return,
M_{l}	Fairly low market rate of return,
u	Fairly high change factor for earnings,
m	Fairly low change factor for earnings,
c	Annual earnings growth rate, equal to $(0.5(1 - p)(u + m)(1 - b_2 p)) - 1$,
T	Index for last year of partner horizon with share redemption value equal to either fair market value or book value,
$V_{\mathrm{CEQ}}^{\mathrm{BV}}(p, T)$	Value of the representative partner's one share at the end of year 0 when the redemption value is equal to book value and the certainty equivalent CAPM version is used,
$V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p)$	Value of the representative partner's one share at the end of year 0 when the redemption value is equal to fair market value and the certainty equivalent CAPM version is used,
$V_{\mathrm{ORD}}^{\mathrm{BV}}(p, T)$	Value of the representative partner's one share at the end of year 0 when the redemption value is equal to book value and the ordinary CAPM version is used,
$V_{\mathrm{ORD}}^{\mathrm{FMV}}(p)$	Value of the representative partner's one share at the end of year 0 when the redemption value is equal to fair market value and the ordinary CAPM version is used,
i	Risk-free interest rate,
Q	Formula shortener, equal to $[1/(0.5(u + m))] \times (0.5(M_{\mathrm{h}} + M_{\mathrm{l}}) - i) \times [\{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))(0.5(u - m))\}/\{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))^2\}]$,
p^{BV}	Annual failure probability (selected by the representative partner) when redemption value is equal to book value,
p^{FMV}	Annual failure probability (selected by the representative partner) when redemption value is equal to fair market value,
r	Ordinary CAPM version discount rate (only in Appendix 3),
V	Discounted value with ordinary CAPM version discounting (only in Appendix 3).

Appendix 2: Unsuccessful search for counterexamples

Very long formulas for $dV_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}})/dp^{\mathrm{FMV}}$ and $dV_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}},T)/dp^{\mathrm{BV}}$ can be obtained, but the equations $dV_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}})/dp^{\mathrm{FMV}}=0$ and $dV_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}},T)/dp^{\mathrm{BV}}=0$ cannot be solved for p^{FMV} and p^{BV}. For that reason, different heuristic searches have been undertaken to find counterexamples where the optimal p^{FMV} is higher than the optimal p^{BV}. No such counterexample has been found. Only the most basic search will be outlined here. For different values of T, the following optimization problem has been formulated (and solved with Excel's Solver):

$$\begin{aligned}
&\text{Maximize } p^{\mathrm{FMV}}-p^{\mathrm{BV}}\\
&\quad\text{with respect to } p^{\mathrm{FMV}},p^{\mathrm{BV}},k,b_1,b_2,w,u,m,M_{\mathrm{h}},M_{\mathrm{l}}, \text{ and } i\\
&\quad\text{subject to}\\
&dV_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}})/dp^{\mathrm{FMV}}=0, \qquad (6)\\
&dV_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}},T)/dp^{\mathrm{BV}}=0,\\
&0.01\le V_{\mathrm{CEQ}}^{\mathrm{FMV}}(0)\le V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}}),
\end{aligned}$$

$$0\le V_{\mathrm{CEQ}}^{\mathrm{BV}}(0,T)\le V_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}},T), \tag{7}$$

$$\begin{aligned}
&0\le p^{\mathrm{FMV}}\le 1,\\
&0\le p^{\mathrm{BV}}\le 1,\\
&0\le k\le 1,\\
&0\le b_1,\\
&0\le b_2\le 1,\\
&0\le w\le 1,\\
&0\le i\le 0.5(M_{\mathrm{h}}+M_{\mathrm{l}}).
\end{aligned} \tag{8}$$

The restrictions (6) and (7) (attempt to) force interior maximal values (like in Fig. 2). The condition $0.01\le V_{\mathrm{CEQ}}^{\mathrm{FMV}}(0)$ eliminates trivial solutions with $Q=1$ and $w=0$, in which case $V_{\mathrm{CEQ}}^{\mathrm{FMV}}(p^{\mathrm{FMV}})$ and $V_{\mathrm{CEQ}}^{\mathrm{BV}}(p^{\mathrm{BV}},T)$ are both equal to 0 for any values of p^{FMV} and p^{BV} between 0 and 1. The restriction (8) imposes a non-negative market risk premium. Also, it must hold that

$$\left\{1+i-(0.5(1-p)(u+m)(1-b_2p))(1-Q)\right\}>0.$$

Numerous different combinations of T and initial values of p^{FMV}, p^{BV}, k, b_1, b_2, w, u, m, M_{h}, M_{l}, and i have been tried. No solution has been found for which $p^{\mathrm{FMV}}>p^{\mathrm{BV}}$. But many solutions exist for which $p^{\mathrm{FMV}}=p^{\mathrm{BV}}$ (with the precision that is possible with Excel).

Appendix 3: Discounting by the ordinary CAPM version

Formulas for $V_{\text{ORD}}^{\text{FMV}}$ and $V_{\text{ORD}}^{\text{BV}}$, i.e., the ordinary CAPM version values corresponding to $V_{\text{CEQ}}^{\text{FMV}}$ and $V_{\text{CEQ}}^{\text{BV}}$, will now be derived. Ordinary CAPM version discounting will first be applied to the two parts of the formula for $V_{\text{ORD}}^{\text{FMV}}$ that correspond to (2) plus (3) in the formula for $V_{\text{CEQ}}^{\text{FMV}}$ in Sect. 3 above. Consider the first part corresponding to (2). Using

$$\frac{(1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p)}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}=(1+r),$$

it holds for the ordinary CAPM version discount rate r for that first part:

$$\begin{aligned}
r = i &+ \frac{0.5(M_{\text{h}}+M_{\text{l}})-i}{(0.5(M_{\text{h}}-M_{\text{l}}))^2}\times\Bigg[(M_{\text{h}}-0.5(M_{\text{h}}+M_{\text{l}}))\times 0.5(1-p)\\
&\times\left(\frac{(1+b_1p)u(1-b_2p)(1-k-w)+w}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}-(1+r)\right)\\
&+(M_{\text{h}}-0.5(M_{\text{h}}+M_{\text{l}}))\times 0.5p\\
&\times\left(\frac{0}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}-(1+r)\right)\\
&+(M_{\text{l}}-0.5(M_{\text{h}}+M_{\text{l}}))\times 0.5p\\
&\times\left(\frac{0}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}-(1+r)\right)\\
&+(M_{\text{l}}-0.5(M_{\text{h}}+M_{\text{l}}))\times 0.5(1-p)\\
&\times\left(\frac{(1+b_1p)m(1-b_2p)(1-k-w)+w}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}-(1+r)\right)\Bigg]\\
&= i+\frac{0.5(M_{\text{h}}+M_{\text{l}})-i}{(0.5(M_{\text{h}}-M_{\text{l}}))^2}\times(M_{\text{h}}-0.5(M_{\text{h}}+M_{\text{l}}))\times 0.5(1-p)\\
&\times\frac{(1+b_1p)(u-m)(1-b_2p)(1-k-w)}{((1+b_1p)(0.5(1-p)(u+m)(1-b_2p))(1-k-w)+w(1-p))/(1+r)}.
\end{aligned} \tag{9}$$

The equality $(M_{\text{l}}-0.5(M_{\text{h}}+M_{\text{l}}))=-(M_{\text{h}}-0.5(M_{\text{h}}+M_{\text{l}}))$ is used in (9) to simplify. (This equality is also used below.) After further (tedious) simplification and manipulation, the Eq. (9) can be solved for r as

$$r=\frac{(i+Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+iw}{(1-Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+w}. \tag{10}$$

With r as in (10), the first value part, corresponding to (2) in Sect. 3, becomes

$$\begin{aligned}
&\Big[(1+b_1p)(0.5(u+m)(1-p)(1-b_2p))(1-k-w)+w(1-p)\Big]\\
&\div\left[1+\frac{(i+Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+iw}{(1-Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+w}\right].
\end{aligned} \tag{11}$$

The ordinary CAPM version formula corresponding to the second part (3) of $V^{\mathrm{FMV}}_{\mathrm{CEQ}}$ in Sect. 3 will now be derived in two steps. The first step computes the value at the end of year 1. For the ordinary CAPM version discount rate r it holds:

$$\begin{aligned}
r = i + &\frac{0.5(M_{\mathrm{h}} + M_{\mathrm{l}}) - i}{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))^2} \times (M_{\mathrm{h}} - 0.5(M_{\mathrm{h}} + M_{\mathrm{l}})) \times 0.5(1-p) \\
&\times \Big\{ \Big((1 + b_1 p)(0.5(1-p)(u+m)(1-b_2 p)) \times (u-m)(1-b_2 p)(1-k-w) \\
&+ \Big[(1 + b_1 p)(0.5(1-p)(u+m)(1-b_2 p)) \times (u-m)(1-b_2 p) \\
&\times ((0.5(1-p)(u+m)(1-b_2 p))(1-k-w) + w(1-p)) \Big] / \Big[r - c \Big] \Big) \\
&\div \Big(\Big[(1 + b_1 p)((0.5(1-p)(u+m)(1-b_2 p))^2)(1-k-w) \\
&+ (1 + b_1 p)(0.5(1-p)(u+m)(1-b_2 p)) \times w(1-p) \Big] / \Big[r - c \Big] \Big) \Big\}.
\end{aligned}$$

Again after (tedious) simplification and manipulation, the discount rate r can be written as

$$r = \frac{(i+Q)0.5(u+m)(1-b_2 p)(1-k-w) + iw + Q0.5(u+m)(1-b_2 p)w(1-p)}{(1-Q)0.5(u+m)(1-b_2 p)(1-k-w) + w}. \tag{12}$$

With the discount rate r as in (12), the value V at the end of year 1 is

$$\begin{aligned}
V = &\Big\{ (1 + b_1 p)((0.5(1-p)(u+m)(1-b_2 p))^2)(1-k-w) \\
&+ (1 + b_1 p)(0.5(1-p)(u+m)(1-b_2 p))w(1-p) \Big\} \\
&\div \Big\{ \frac{(i+Q)0.5(u+m)(1-b_2 p)(1-k-w) + iw + Q0.5(u+m)(1-b_2 p)w(1-p)}{(1-Q)0.5(u+m)(1-b_2 p)(1-k-w) + w} \\
&- c \Big\}.
\end{aligned} \tag{13}$$

In the second step, the ordinary CAPM version value corresponding to (3) in Sect. 3 is obtained by discounting to a present value at the end of year 0. Then it holds for the ordinary CAPM version discount rate r:

$$\begin{aligned}
r = i + &\frac{0.5(M_{\mathrm{h}} + M_{\mathrm{l}}) - i}{(0.5(M_{\mathrm{h}} - M_{\mathrm{l}}))^2} \times (M_{\mathrm{h}} - 0.5(M_{\mathrm{h}} + M_{\mathrm{l}})) \times 0.5(1-p) \\
&\times \frac{[1/(1+c)]V(u-m)(1-b_2 p)}{V/(1+r)} \\
&= (i+Q)/(1-Q).
\end{aligned} \tag{14}$$

Using (11), (13), and (14), the ordinary CAPM version value of one share with redemption value equal to fair market value is

$$\begin{aligned}
&V_{\text{ORD}}^{\text{FMV}}(p) = \\
&\Big[(1+b_1p)(0.5(u+m)(1-p)(1-b_2p))(1-k-w)+w(1-p)\Big] \\
&\div\left[1+\frac{(i+Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+iw}{(1-Q)(1+b_1p)0.5(u+m)(1-b_2p)(1-k-w)+w}\right] \\
&+\Bigg[\Bigg\{(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^2)(1-k-w) \\
&+(1+b_1p)(0.5(1-p)(u+m)(1-b_2p))w(1-p)\Bigg\} \\
&\div\Bigg\{\frac{(i+Q)0.5(u+m)(1-b_2p)(1-k-w)+iw+Q0.5(u+m)(1-b_2p)w(1-p)}{(1-Q)0.5(u+m)(1-b_2p)(1-k-w)+w} \\
&-c\Bigg\}\Bigg]\Bigg/\Big[1+(i+Q)/(1-Q)\Big].
\end{aligned} \tag{15}$$

For the same assumptions, the value $V_{\text{ORD}}^{\text{FMV}}(p)$ in (15) is *exactly* the same as the value $V_{\text{CEQ}}^{\text{FMV}}(p)$ in (2) plus (3) in Sect. 3. For instance, the optimal value $V_{\text{CEQ}}^{\text{FMV}}(p)$ was stated as 2.10185 in the first example of incentive for risk taking in Sect. 4. That optimal value can now be reported more exactly as 2.10184496116898 for *both* $V_{\text{ORD}}^{\text{FMV}}(p)$ and $V_{\text{CEQ}}^{\text{FMV}}(p)$. Equation (2) plus (3) is actually nothing more than a step-by-step rewritten variant of (15).

Now it is not difficult to see that the ordinary CAPM version value of one share with redemption value equal to book value is

$$\begin{aligned}
&V_{\text{ORD}}^{\text{BV}}(p,T) = V_{\text{ORD}}^{\text{FMV}}(p) \\
&-\Bigg[\Bigg\{(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{(T+1)})(1-k-w) \\
&+(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{T})w(1-p)\Bigg\} \\
&\div\Bigg\{\frac{(i+Q)0.5(u+m)(1-b_2p)(1-k-w)+iw+Q0.5(u+m)(1-b_2p)w(1-p)}{(1-Q)0.5(u+m)(1-b_2p)(1-k-w)+w} \\
&-c\Bigg\}\Bigg]\Bigg/\Big[(1+(i+Q)/(1-Q))^{T}\Big] \\
&+\Big[(1+b_1p)((0.5(1-p)(u+m)(1-b_2p))^{T})w\Big]\Big/\Big[(1+(i+Q)/(1-Q))^{T}\Big].
\end{aligned} \tag{16}$$

Equation (16) is seen to consist of the same three components as Eq. (4) in Sect. 3: The value of one share with redemption value equal to fair market value; minus the value of expected dividends of one share with redemption value equal to fair market value from year $T+1$; plus the value of the expected book value of the assets at the end of year T. Again, for the same assumptions, the value of $V_{\text{ORD}}^{\text{BV}}(p,T)$ in (16) is exactly the same as that of $V^{BV}_{CEQ}(p,T)$ in (4). The optimal value of $V^{BV}_{CEQ}(p,T)$ was stated as 1.95450 in the first example of incentive for risk taking in Sect. 4. That optimal value can now be reported more precisely as 1.95449755928187 for *both* of $V^{BV}_{ORD}(p,T)$ and $V^{BV}_{CEQ}(p,T)$. Again, Eq. (4) is actually nothing more than a step-by-step rewritten variant of (16).

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References

Ball R (2009) Market and political/regulatory perspectives on the recent accounting scandals. J Account Res 47(2):277–323

Brock DM, Powell MJ, Hinings CR (2007) Archetypal change and the professional service firm. Res Organ Chang Dev 16:221–251

Bronnenmayer M, Wirtz BW, Göttel V (2016) Success factors of management consulting. RMS 10:1–34

Clark KD, Doh JP, Stumpf SA (2005) Management and governance of professional services firms. In: Doh JP, Stumpf SA (eds) Handbook on responsible leadership and governance in global business. Edward Elgar, Cheltenham, pp 275–303

Eisenberg T, Macey JR (2004) Was Arthur Andersen different? An empirical examination of major accounting firm audits of large clients. J Empir Leg Stud 1(2):263–300

Empson L, Chapman C (2006). Partnership versus corporation: implications of alternative forms of governance in professional service firms. In: Greenwood R, Suddaby R (eds) Professional service firms—research in the sociology of organizations, vol 24, pp 139–170

Fama EF, Jensen MC (1983) Agency problems and residual claims. J Law Econ 26(2):327–349

Fernandez P (2015) CAPM: An absurd model. Bus Valuat Rev 34(1):4–23

Friedrich T (2016) Corporate valuation and risk of insolvency. Die Betriebswirtschaft 76(6):521–541

Furubotn EG (1976) The long-run analysis of the labor-managed firm: An alternative interpretation. Am Econ Rev 66(1):104–123

Gramling AA, Schatzberg JW, Bailey AD Jr, Zhang H (1998) The impact of legal liability regimes and differential client risk on client acceptance, audit pricing, and audit effort decisions. J Account Audit Finance 13(4):437–460

Greenwood R, Empson L (2003) The professional partnership: Relic or exemplary form of governance? Organ Stud 24(6):909–933
Greenwood R, Hinings CR, Brown J (1990) P2-form strategic management: Corporate practices in professional partnerships. Acad Manag J 33(4):725–755
Greenwood R, Li SX, Prakash R, Deephouse DL (2005) Reputation, diversification, and organizational explanations of performance in professional service firms. Organ Sci 16(6):661–673
Hallerbach W, Spronk J (2002) A Multidimensional framework for financial-economic decisions. J Multi-Criteria Decis Anal 11:111–124
Hansmann H (1988) Ownership of the firm. J Law Econ Organ 4(2):267–304
Jennergren LP (2008) Continuing value in firm valuation by the discounted cash flow model. Eur J Oper Res 185:1548–1563
Jennergren LP (2013) Firm valuation with bankruptcy risk. J Bus Valuat Econ Loss Anal 8(1):91–131
Jensen MC, Meckling WH (1979) Rights and production functions: An application to labor-managed firms and codetermination. J Bus 52(4):469–506
Koller T, Goedhart M, Wessels D (2020) Valuation - Measuring and managing the value of companies, 7th edn. Wiley, Hoboken
Leblebici H, Sherer PD (2015) Governance in professional service firms. In: Empson L, Muzio D, Broschak J, Hinings B (eds) The Oxford handbook of professional service firms. Oxford University Press, Oxford, pp 189–212
Lennox CS, Wu X (2018) A review of the archival literature on audit partners. Account Horiz 32(2):1–35
Malhotra N, Morris T, Hinings CR (2006) Variation in organizational form among professional service organizations. In: Greenwood R, Suddaby R (eds) Professional service firms—research in the sociology of organizations, vol 24, pp 171–202
Milgrom P, Roberts J (1992) Economics, organization and management. Prentice-Hall International, London
Morris T, Greenwood R, Fairclough S (2010) Decision making in professional service firms. In: Nutt PC, Wilson DC (eds) Handbook of decision making. Wiley, Chichester, pp 275–306
Morrison AD, Wilhelm WD Jr (2004) Partnership firms, reputation, and human capital. Am Econ Rev 94(5):1682–1692
Pickering ME (2015) An exploratory study of organizational governance in publicly-quoted professional service firms. Organ Stud 36(6):779–807
Ribstein LE (2010) The death of big law. Wis Law Rev 2010(3):749–815
Saab Fortney S (1995) Am I my partner's keeper? Peer review in law firms. Univ Colo Law Rev 66:329–373
Sick GA (1986) A certainty-equivalent approach to capital budgeting. Financ Manag 15:23–32
Skogsvik K, Skogsvik S, Andersson H (2023) Bankruptcy risk in discounted cash flow equity valuation. J Risk Financ Manag 16:476
Smets M, Morris T, von Nordenflycht A, Brock DM (2017) 25 years since P2: Taking stock and charting the future of professional firms. J Prof Organ 4:91–111
Spronk J (1999) Financial modelling as a bridging feature (editorial). Eur J Oper Res 114:217–218
Spronk J (2015) Let's change finance. Farewell address series, Research in management, EIA-2015-063-F &A. Rotterdam University, Rotterdam
Spronk J, Hallerbach W (1997) Financial modelling: Where to go? With an illustration for portfolio management. Eur J Oper Res 99:113–125
van Lent L (1999) The economics of an audit firm: The benefits of partnership governance. Br Account Rev 31:225–254
von Nordenflycht A (2010) What is a professional service firm? Toward a theory and taxonomy of knowledge-intensive firms. Acad Manag Rev 35(1):155–174
von Nordenflycht A (2014) Does the emergence of publicly traded professional service firms undermine the theory of the professional partnership? A cross-industry historical analysis. J Prof Organ 1:137–160