# Contents

Chapter I. Probability Distributions and Characteristic Functions . 1

§ 1. Random variables and probability distributions . . . . . . . 1
§ 2. Characteristic functions . . . . . . . . . . . . . . . . . . 7
§ 3. Inversion formulae . . . . . . . . . . . . . . . . . . . 12
§ 4. The convergence of sequences of distributions and characteristic functions . . . . . . . . . . . . . . . . . . . . . . 14
§ 5. Supplement. . . . . . . . . . . . . . . . . . . . . . . 18

Chapter II. Infinitely Divisible Distributions . . . . . . . . . . 25

§ 1. Definition and elementary properties of infinitely divisible distributions . . . . . . . . . . . . . . . . . . . . . . 25
§ 2. Canonical representation of infinitely divisible characteristic functions . . . . . . . . . . . . . . . . . . . . . . 26
§ 3. An auxiliary theorem . . . . . . . . . . . . . . . . . . 32
§ 4. Supplement. . . . . . . . . . . . . . . . . . . . . . . 36

Chapter III. Some Inequalities for the Distribution of Sums of Independent Random Variables . . . . . . . . . . . . . . 38

§ 1. Concentration functions . . . . . . . . . . . . . . . . . 38
§ 2. Inequalities for the concentration functions of sums of independent random variables. . . . . . . . . . . . . . . . . 43
§ 3. Inequalities for the distribution of the maximum of sums of independent random variables . . . . . . . . . . . . . . . 49
§ 4. Exponential estimates for the distributions of sums of independent random variables . . . . . . . . . . . . . . . . . . 52
§ 5. Supplement. . . . . . . . . . . . . . . . . . . . . . . 56

Chapter IV. Theorems on Convergence to Infinitely Divisible Distributions . . . . . . . . . . . . . . . . . . . . . . . 63

§ 1. Infinitely divisible distributions as limits of the distributions of sums of independent random variables . . . . . . . . . . 63

§ 2. Conditions for convergence to a given infinitely divisible distribution . . . . . . . . . . . . . . . . . . . . . . 75
§ 3. Limit distributions of class $L$ and stable distributions . . . . 82
§ 4. The central limit theorem . . . . . . . . . . . . . . . . 91
§ 5. Supplement . . . . . . . . . . . . . . . . . . . . . . 102

Chapter V. Estimates of the Distance Between the Distribution of a Sum of Independent Random Variables and the Normal Distribution . . . . . . . . . . . . . . . . . . . . . . 104

§ 1. Estimating the nearness of functions of bounded variation by the nearness of their Fourier-Stieltjes transforms . . . . . . 104
§ 2. The Esseen and Berry-Esseen inequalities . . . . . . . . . 109
§ 3. Generalizations of Esseen's inequality . . . . . . . . . . . 112
§ 4. Non-uniform estimates . . . . . . . . . . . . . . . . . . 120
§ 5. Supplement . . . . . . . . . . . . . . . . . . . . . . 126

Chapter VI. Asymptotic Expansions in the Central Limit Theorem 134

§ 1. Formal construction of the expansions . . . . . . . . . . . 134
§ 2. Auxiliary propositions . . . . . . . . . . . . . . . . . 139
§ 3. Asymptotic expansions of the distribution function of a sum of independent identically distributed random variables . . . . 158
§ 4. Asymptotic expansions of the distribution function of a sum of independent non-identically distributed random variables, and of the derivatives of this function . . . . . . . . . . . . . 172
§ 5. Supplement . . . . . . . . . . . . . . . . . . . . . . 181

Chapter VII. Local Limit Theorems . . . . . . . . . . . . . . 187

§ 1. Local limit theorems for lattice distributions . . . . . . . . 187
§ 2. Local limit theorems for densities . . . . . . . . . . . . . 198
§ 3. Asymptotic expansions in local limit theorems . . . . . . . 204
§ 4. Supplement . . . . . . . . . . . . . . . . . . . . . . 213

Chapter VIII. Probabilities of Large Deviations . . . . . . . . . 217

§ 1. Introduction . . . . . . . . . . . . . . . . . . . . . . 217
§ 2. Asymptotic relations connected with Cramér's series . . . . . 218
§ 3. Necessary and sufficient conditions for normal convergence in power zones . . . . . . . . . . . . . . . . . . . . . . 231
§ 4. Supplement . . . . . . . . . . . . . . . . . . . . . . 248

Chapter IX. Laws of Large Numbers . . . . . . . . . . . . . . 256

§ 1. The weak law of large numbers . . . . . . . . . . . . . . 256
§ 2. Convergence of series of independent random variables . . . 263

§ 3. The strong law of large numbers . . . . . . . . . . . . . . 268
§ 4. Convergence rates in the laws of large numbers . . . . . . . . 283
§ 5. Supplement. . . . . . . . . . . . . . . . . . . . . . . 286

Chapter X. The Law of the Iterated Logarithm . . . . . . . . . . 292

§ 1. Kolmogorov's theorem . . . . . . . . . . . . . . . . . . 292
§ 2. Generalization of Kolmogorov's theorem . . . . . . . . . . . 301
§ 3. The central limit theorem and the law of the iterated logarithm 304
§ 4. Supplement. . . . . . . . . . . . . . . . . . . . . . . 313

Notes on Sources in the Literature . . . . . . . . . . . . . . . 319

References . . . . . . . . . . . . . . . . . . . . . . . . 326

Subject Index . . . . . . . . . . . . . . . . . . . . . . . 344

Table of Symbols and Abbreviations. . . . . . . . . . . . . . . 346