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**CONTRACT: A Numerical Tool
for Contaminant Transport and
Chemical Transformations
– Theory and Program
Documentation –**

von

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