

Jean-Pierre Serre

# Trees

Translated from the French by John Stillwell



Springer

# Table of Contents

<b>Introduction</b> . . . . .	VII
<b>Chapter I. Trees and Amalgams</b> . . . . .	1
§1 <i>Amalgams</i> . . . . .	1
1.1 Direct limits . . . . .	1
1.2 Structure of amalgams . . . . .	2
1.3 Consequences of the structure theorem . . . . .	5
1.4 Constructions using amalgams . . . . .	8
1.5 Examples . . . . .	11
§2 <i>Trees</i> . . . . .	13
2.1 Graphs . . . . .	13
2.2 Trees . . . . .	17
2.3 Subtrees of a graph . . . . .	21
§3 <i>Trees and free groups</i> . . . . .	25
3.1 Trees of representatives . . . . .	25
3.2 Graph of a free group . . . . .	26
3.3 Free actions on a tree . . . . .	27
3.4 Application: Schreier's theorem . . . . .	29
Appendix: Presentation of a group of homeomorphisms	30
§4 <i>Trees and amalgams</i> . . . . .	32
4.1 The case of two factors . . . . .	32
4.2 Examples of trees associated with amalgams . . . . .	35
4.3 Applications . . . . .	36
4.4 Limit of a tree of groups . . . . .	37
4.5 Amalgams and fundamental domains (general case)	38
§5 <i>Structure of a group acting on a tree</i> . . . . .	41
5.1 Fundamental group of a graph of groups . . . . .	41
5.2 Reduced words . . . . .	45
5.3 Universal covering relative to a graph of groups . . . . .	50
5.4 Structure theorem . . . . .	54
5.5 Application: Kurosh's theorem . . . . .	56
§6 <i>Amalgams and fixed points</i> . . . . .	58
6.1 The fixed point property for groups acting on trees	58
6.2 Consequences of property (FA) . . . . .	59

VI Table of Contents

6.3	Examples . . . . .	60
6.4	Fixed points of an automorphism of a tree . . . . .	61
6.5	Groups with fixed points (auxiliary results) . . . . .	64
6.6	The case of $\mathbf{SL}_3(\mathbf{Z})$ . . . . .	67
<b>Chapter II. <math>\mathbf{SL}_2</math>.</b> . . . . .		69
§1	<i>The tree of <math>\mathbf{SL}_2</math> over a local field</i> . . . . .	69
1.1	The tree . . . . .	69
1.2	The groups $\mathbf{GL}(V)$ and $\mathbf{SL}(V)$ . . . . .	74
1.3	Action of $\mathbf{GL}(V)$ on the tree of $V$ ; stabilizers . . . . .	76
1.4	Amalgams. . . . .	78
1.5	Ihara's theorem. . . . .	82
1.6	Nagao's theorem . . . . .	85
1.7	Connection with Tits systems . . . . .	89
§2	<i>Arithmetic subgroups of the groups <math>\mathbf{GL}_2</math> and <math>\mathbf{SL}_2</math> over a function field of one variable</i> . . . . .	96
2.1	Interpretation of the vertices of $\Gamma \backslash X$ as classes of vector bundles of rank 2 over $C$ . . . . .	96
2.2	Bundles of rank 1 and decomposable bundles . . . . .	99
2.3	Structure of $\Gamma \backslash X$ . . . . .	103
2.4	Examples . . . . .	111
2.5	Structure of $\Gamma$ . . . . .	117
2.6	Auxiliary results . . . . .	120
2.7	Structure of $\Gamma$ : case of a finite field . . . . .	124
2.8	Homology. . . . .	125
2.9	Euler-Poincaré characteristic . . . . .	131
<b>Bibliography.</b> . . . . .		137
<b>Index</b> . . . . .		141