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Electromagnetic Theory for Microwaves and Optoelectronics

Second Edition

With 280 Figures and 13 Tables
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