

Contents

1	Probabilities and Games of Chance	1
1.1	Illustrious Minds Play Dice	1
1.2	The Classic Notion of Probability	2
1.3	A Few Basic Rules	4
1.4	Testing the Rules	8
1.5	The Frequency Notion of Probability	11
1.6	Counting Techniques	14
1.7	Games of Chance	18
1.7.1	Toto	19
1.7.2	Lotto	20
1.7.3	Poker	21
1.7.4	Slot Machines	22
1.7.5	Roulette	24
2	Amazing Conditions	25
2.1	Conditional Events	25
2.2	Experimental Conditioning	27
2.3	Independent Events	28
2.4	A Very Special Reverend	29
2.5	Probabilities in Daily Life	33
2.6	Challenging the Intuition	36
2.6.1	The Problem of the Sons	36
2.6.2	The Problem of the Aces	37

2.6.3	The Problem of the Birthdays	39
2.6.4	The Problem of the Right Door	40
2.6.5	The Problem of Encounters	41
3	Expecting to Win	45
3.1	Mathematical Expectation	45
3.2	The Law of Large Numbers	48
3.3	Bad Luck Games	51
3.4	The Two-Envelope Paradox	52
3.5	The Saint Petersburg Paradox	54
3.6	When the Improbable Happens	57
3.7	Paranormal Coincidences	59
3.8	The Chevalier de Méré Problem	60
3.9	Martingales	62
3.10	How Not to Lose Much in Games of Chance	63
4	The Wonderful Curve	67
4.1	Approximating the Binomial Law	67
4.2	Errors and the Bell Curve	69
4.3	A Continuum of Chances	72
4.4	Buffon's Needle	75
4.5	Monte Carlo Methods	76
4.6	Normal and Binomial Distributions	78
4.7	Distribution of the Arithmetic Mean	79
4.8	The Law of Large Numbers Revisited	82
4.9	Surveys and Polls	84
4.10	The Ubiquity of Normality	86
4.11	A False Recipe for Winning the Lotto	89
5	Probable Inferences	93
5.1	Observations and Experiments	93
5.2	Statistics	95
5.3	Chance Relations	96
5.4	Correlation	98
5.5	Significant Correlation	101
5.6	Correlation in Everyday Life	102

6	Fortune and Ruin	107
6.1	The Random Walk	107
6.2	Wild Rides	109
6.3	The Strange Arcsine Law	111
6.4	The Chancier the Better	113
6.5	Averages Without Expectation	114
6.6	Borel's Normal Numbers	117
6.7	The Strong Law of Large Numbers	120
6.8	The Law of Iterated Logarithms	121
7	The Nature of Chance	125
7.1	Chance and Determinism	125
7.2	Quantum Probabilities	128
7.3	A Planetary Game	134
7.4	Almost Sure Interest Rates	136
7.5	Populations in Crisis	139
7.6	An Innocent Formula	142
7.7	Chance in Natural Systems	146
7.8	Chance in Life	149
8	Noisy Irregularities	153
8.1	Generators and Sequences	153
8.2	Judging Randomness	155
8.3	Random-Looking Numbers	157
8.4	Quantum Generators	159
8.5	When Nothing Else Looks the Same	160
8.6	White Noise	163
8.7	Randomness with Memory	166
8.8	Noises and More Than That	169
8.9	The Fractality of Randomness	171
9	Chance and Order	175
9.1	The Guessing Game	175
9.2	Information and Entropy	178
9.3	The Ehrenfest Dogs	181

XII Contents

9.4	Random Texts	184
9.5	Sequence Entropies	188
9.6	Algorithmic Complexity	191
9.7	The Undecidability of Randomness	194
9.8	A Ghost Number	195
9.9	Occam's Razor	196
10	Living with Chance	199
10.1	Learning, in Spite of Chance	199
10.2	Learning Rules	204
10.3	Impossible Learning	207
10.4	To Learn Is to Generalize	209
10.5	The Learning of Science	211
10.6	Chance and Determinism: A Never-Ending Story	213
	Some Mathematical Notes	215
A.1	Powers	215
A.2	Exponential Function	216
A.3	Logarithm Function	217
A.4	Factorial Function	218
A.5	Sinusoids	219
A.6	Binary Number System	220
	References	221