

Contents

Part I Basic Stochastic Optimization Methods

1	Decision/Control Under Stochastic Uncertainty	3
1.1	Introduction	3
1.2	Deterministic Substitute Problems: Basic Formulation	5
1.2.1	Minimum or Bounded Expected Costs	6
1.2.2	Minimum or Bounded Maximum Costs (Worst Case)	8
2	Deterministic Substitute Problems in Optimal Decision Under Stochastic Uncertainty	9
2.1	Optimum Design Problems with Random Parameters	9
2.1.1	Deterministic Substitute Problems in Optimal Design	13
2.1.2	Deterministic Substitute Problems in Quality Engineering	16
2.2	Basic Properties of Substitute Problems	18
2.3	Approximations of Deterministic Substitute Problems in Optimal Design	19
2.3.1	Approximation of the Loss Function	20
2.3.2	Regression Techniques, Model Fitting, RSM	22
2.3.3	Taylor Expansion Methods	26
2.4	Applications to Problems in Quality Engineering	29
2.5	Approximation of Probabilities: Probability Inequalities	30
2.5.1	Bonferroni-Type Inequalities	31
2.5.2	Tschebyscheff-Type Inequalities	32
2.5.3	First Order Reliability Methods (FORM)	37

Part II Differentiation Methods

3	Differentiation Methods for Probability and Risk Functions	43
3.1	Introduction	43
3.2	Transformation Method: Differentiation by Using an Integral Transformation.....	46
3.2.1	Representation of the Derivatives by Surface Integrals..	51
3.3	The Differentiation of Structural Reliabilities	54
3.4	Extensions.....	57
3.4.1	More General Response (State) Functions	57
3.5	Computation of Probabilities and its Derivatives by Asymptotic Expansions of Integral of Laplace Type	62
3.5.1	Computation of Probabilities of Structural Failure and Their Derivatives	62
3.5.2	Numerical Computation of Derivatives of the Probability Functions Arising in Chance Constrained Programming.....	66
3.6	Integral Representations of the Probability Function $P(x)$ and its Derivatives.....	72
3.7	Orthogonal Function Series Expansions I: Expansions in Hermite Functions, Case $m = 1$	75
3.7.1	Integrals over the Basis Functions and the Coefficients of the Orthogonal Series.....	79
3.7.2	Estimation/Approximation of $P(x)$ and its Derivatives .	82
3.7.3	The Integrated Square Error (ISE) of Deterministic Approximations	88
3.8	Orthogonal Function Series Expansions II: Expansions in Hermite Functions, Case $m > 1$	89
3.9	Orthogonal Function Series Expansions III: Expansions in Trigonometric, Legendre and Laguerre Series	91
3.9.1	Expansions in Trigonometric and Legendre Series.....	92
3.9.2	Expansions in Laguerre Series.....	92

Part III Deterministic Descent Directions

4	Deterministic Descent Directions and Efficient Points.....	95
4.1	Convex Approximation.....	95
4.1.1	Approximative Convex Optimization Problem.....	99
4.2	Computation of Descent Directions in Case of Normal Distributions.....	101
4.2.1	Descent Directions of Convex Programs	105
4.2.2	Solution of the Auxiliary Programs	108
4.3	Efficient Solutions (Points)	113
4.3.1	Necessary Optimality Conditions Without Gradients...	116

4.3.2	Existence of Feasible Descent Directions in Non-Efficient Solutions of (4.9a), (4.9b)	117
4.4	Descent Directions in Case of Elliptically Contoured Distributions	118
4.5	Construction of Descent Directions by Using Quadratic Approximations of the Loss Function	121

Part IV Semi-Stochastic Approximation Methods

5	RSM-Based Stochastic Gradient Procedures	129
5.1	Introduction	129
5.2	Gradient Estimation Using the Response Surface Methodology (RSM)	131
5.2.1	The Two Phases of RSM	134
5.2.2	The Mean Square Error of the Gradient Estimator	138
5.3	Estimation of the Mean Square (Mean Functional) Error	142
5.3.1	The Argument Case	143
5.3.2	The Criterial Case	147
5.4	Convergence Behavior of Hybrid Stochastic Approximation Methods	147
5.4.1	Asymptotically Correct Response Surface Model	148
5.4.2	Biased Response Surface Model	150
5.5	Convergence Rates of Hybrid Stochastic Approximation Procedures	153
5.5.1	Fixed Rate of Stochastic and Deterministic Steps	158
5.5.2	Lower Bounds for the Mean Square Error	169
5.5.3	Decreasing Rate of Stochastic Steps	173
6	Stochastic Approximation Methods with Changing Error Variances	177
6.1	Introduction	177
6.2	Solution of Optimality Conditions	178
6.3	General Assumptions and Notations	179
6.3.1	Interpretation of the Assumptions	181
6.3.2	Notations and Abbreviations in this Chapter	182
6.4	Preliminary Results	183
6.4.1	Estimation of the Quadratic Error	183
6.4.2	Consideration of the Weighted Error Sequence	185
6.4.3	Further Preliminary Results	188
6.5	General Convergence Results	190
6.5.1	Convergence with Probability One	190
6.5.2	Convergence in the Mean	192
6.5.3	Convergence in Distribution	195
6.6	Realization of Search Directions Y_n	204
6.6.1	Estimation of G^*	209

6.6.2	Update of the Jacobian	210
6.6.3	Estimation of Error Variances	216
6.7	Realization of Adaptive Step Sizes	220
6.7.1	Optimal Matrix Step Size	221
6.7.2	Adaptive Scalar Step Size	227
6.8	A Special Class of Adaptive Scalar Step Sizes	236
6.8.1	Convergence Properties	237
6.8.2	Examples for the Function $Q_n(r)$	241
6.8.3	Optimal Sequence (w_n)	247
6.8.4	Sequence (K_n)	247

Part V Reliability Analysis of Structures/Systems

7	Computation of Probabilities of Survival/Failure by Means of Piecewise Linearization of the State Function	253
7.1	Introduction	253
7.2	The State Function s^*	256
7.2.1	Characterization of Safe States	258
7.3	Probability of Safety/Survival	259
7.4	Approximation I of p_s, p_f : FORM	262
7.4.1	The Origin of \mathbf{R}' Lies in the Transformed Safe Domain	262
7.4.2	The Origin of \mathbf{R}' Lies in the Transformed Failure Domain	266
7.4.3	The Origin of \mathbf{R}' Lies on the Limit State Surface	268
7.4.4	Approximation of Reliability Constraints	269
7.5	Approximation II of p_s, p_f : Polyhedral Approximation of the Safe/Unsafe Domain	270
7.5.1	Polyhedral Approximation	273
7.6	Computation of the Boundary Points	279
7.6.1	State Function s^* Represented by Problem A	280
7.6.2	State Function s^* Represented by Problem B	280
7.7	Computation of the Approximate Probability Functions	282
7.7.1	Probability Inequalities	282
7.7.2	Discretization Methods	289
7.7.3	Convergent Sequences of Discrete Distributions	293

Part VI Appendix

A	Sequences, Series and Products	301
A.1	Mean Value Theorems for Deterministic Sequences	301
A.2	Iterative Solution of a Lyapunov Matrix Equation	309

B	Convergence Theorems for Stochastic Sequences	313
B.1	A Convergence Result of Robbins–Siegmund	313
B.1.1	Consequences	313
B.2	Convergence in the Mean	316
B.3	The Strong Law of Large Numbers for Dependent Matrix Sequences	318
B.4	A Central Limit Theorem for Dependent Vector Sequences....	319
C	Tools from Matrix Calculus	321
C.1	Miscellaneous	321
C.2	The v. Mises-Procedure in Case of Errors	322
	References	327
	Index	335