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Media Markets with Habit Formation

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Abstract

The aim of this paper is to analyse the pricing behaviour of print media firms when consumption on reader markets is addictive or habituated. However, not only the reader but also the advertising market has to be considered by a publisher optimising profits. Because print media markets are highly concentrated a monopoly-monopoly model is built, where both markets are of monopolistic structure. Moreover, a monopoly-duopoly model is considered, where only the reader market is monopolistic but the advertising market is of duopolistic structure. To compare the results from the models regarding habit effects, simple static models are presented as a benchmark.

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"Do you read the papers today?"
Extreme, Cupid's Dead (1992)

"Nothing is stronger than habit."
Ovid, Ars Amatoria (1 BC)

1 Introduction

Media markets are interesting fields for economic research, from different points of view. Especially the dependency of the sub-markets served by a print media firm, like a newspaper or magazine publisher, has been the topic of a number of studies, both theoretical and empirical, since a considerable time (see Corden 1952, Bucklin et al. 1989, Dertrouzos and Trautman 1990, Blair and Romano 1993 or Chaudhri 1998). The main results of these studies focus on the pricing behaviour of the publisher with respect to the interrelationship of reader and advertising markets. And most of the models analysing the circulation industry assume positive feedback effects, therefore, the demand for copies is assumed to depend on the demand for advertising and vice versa. Consequently, a positive spiral of circulation and advertising is typically found, whereby copy prices are set possibly less than marginal costs in order to induce a high circulation and thus an increasing demand for advertising space. However, since it is not clear if advertising is informative or influencing, and because of focussing on habit effects, feedback effects from the advertising market are not considered in the following initially.

A further field of interest is the impact of high concentration on copy prices and advertising rates. While some of the studies find some positive impact of concentration on prices (see Stigler 1964, Landon 1971, Thompson 1984 or Dewenter and Kraft 2001), other authors reaching the opposite conclusion (see Reimer 1992, Ferguson 1983 or Bucklin et al. 1989). Nevertheless, most of these studies are motivated by highly concentrated media markets. In Germany, but also in other countries like the United States, the United Kingdom or Australia, the existence of monopolies, regarding local newspaper markets is typical and not the exception. Therefore, analysing a monopolistic or duopolistic print media publisher seems to be an adequate procedure.

A third characteristic of print media is the existence of habituated behaviour of readers. Although switching costs are negligible small, there is strong evidence for habit effects that are inherent with the consumption of magazines or newspaper (see Dewenter 2002). Typically, this effect is described as "newspaper habit". But also the consumers of alternative media like television, cinema, or Internet services can get used to those products.[1]

The phenomena of habit formation and addiction of human beings have been analysed in various disciplines like psychology, psychiatry or marketing, since a considerable time. But habit formation and addiction has also a long tradition in economic research, mostly with respect to consumer behaviour and less frequently with respect to the pricing decisions of firms which are faced with addicted consumers. The most popular study is certainly the rational addiction model by Becker and Murphy (1998) which analyse both, the consumption behaviour of a rational addict and the price setting of a monopolist which is faced with addictive consumers. But also others studies deal with habits and addiction, both rational or myopic (for a summary of the literature see Messinis 1999). Fethke and Jaggannathan (1996) examine consumption and pricing behaviour on markets with monopolistic and imperfect competitive firms. Showalter (1999) analyses the pricing of a monopolist which is faced with myopic addiction in a theoretical and empirical manner.

Admittedly, non of the articles, neither the media literature nor the studies investigating habit formation deal with habit effects on media markets.[2] Therefore, this paper analyses both effects, the typical network effects in media markets and habit formation. Thus, a media firm is considered serving two (inter-)related markets, a reader and an advertising market, where consumption on the reader

[1]Especially Internet addiction has become a popular topic in psychological and medical research during the last years (see Stein 1997, Griffith 1999, or Beard 2002).

[2]Exceptions are some empirical studies by Cameron (1999) who tests the model of rational addiction using data of cinema attendance or Dewenter 2002 testing the Becker Murphy model using data of German popular magazines.

market is characterised by habit formation.

The procedure of the paper is as follows. At first within a monopoly-monopoly framework the effects of habituation are examined. Therefore, a simple static model of a print media firm optimising profits on both markets but neglecting habit effects and interdependency is presented. The results from this model will be used as a benchmark for later outcomes. Next, the analysis is expanded by habit formation with respect to the reader market. Simultaneously, a comparison of both models will be carried out. The next section will repeat the analysis assuming a monopoly-duopoly situation, where the reader market is still monopolistic, but the advertising market is assumed to be of duopolistic structure.

To come closer to a more realistic model, the next step is to relax the assumption of related but not interrelated markets. Therefore, at first the monopoly-monopoly approach is repeated within an interrelated markets framework. More exact, not only the reader market is assumed to have an impact on the advertising market but also vice versa. The most extended model will follow in terms of a monopoly-duopoly approach assuming interrelated demands. Some concluding remarks are offered in the last section.

2 Monopolistic media markets

2.1 A static approach neglecting habit effects

At first a media firm, say a newspaper publisher, is considered serving a reader and an advertising market. Moreover, a monopolistic situation is assumed to be existent for both products, copies and advertising space. A situation which is typical for regional newspaper markets in many German cities, but also in the United States, Australia and other Countries.[3] Habit effects are, at first, neglected thus a simple static model can be used to analyse optimal prices and

[3]See Stigler (1964), Thompson (1984) or Chaudhri (1998) for concentration of regional newspaper markets in the United States, Australia or the Republic of Ireland. Regarding Germany (see Dewenter and Kraft 2001), most of the cities have monopolistic or duopolistic newspapers markets. In only few cities are more than two independent regional newspapers available.

quantities. The outcomes of this model are well suited for comparison with the dynamic approach. The markets are assumed to have linear demand structures of the form

$$Q_c = a - bP_c \tag{1}$$

and

$$Q_a = e - fP_a + gQ_c, \tag{2}$$

where P_c and Q_c is the price and the demand for copies, Q_a and P_a are the demand for advertising and the advertising rate, respectively. Thus, inverse demand functions of both markets can be represented by:

$$P_c = \alpha - \gamma Q_c, \tag{3}$$

for the reader market and

$$P_a = \delta + \varepsilon Q_c - \mu Q_a, \tag{4}$$

for the advertising market, where the parameters $\alpha = a/b$, $\gamma = 1/b$, $\delta = e/f$, $\varepsilon = g/f$ and $\mu = 1/f$ determining the respective demand structures are all assumed to be positive. Note that, at first, only the advertising market is assumed to be linked with the demand for copies but not vice versa. Correspondingly, no feedback effects from advertising space to the demand for copies exists. The readers of the print media are, therefore, assumed to be indifferent with respect to the advertising volumes. Thus, the profit function of the publisher is:

$$\Pi = (\alpha - \gamma Q_c - C_c)Q_c + (\delta + \varepsilon Q_c - \mu Q_a - C_a)Q_a, \tag{5}$$

where C_c and C_a are the constant marginal costs from the reader and advertising markets. Profit maximising behaviour leads to the following first order conditions with respect to Q_c and Q_a as

$$\frac{\partial \pi}{\partial Q_c} = \alpha - 2\gamma Q_c - C_c + \varepsilon Q_a = 0 \tag{6}$$

and

$$\frac{\partial \pi}{\partial Q_a} = \delta + \varepsilon Q_c - 2\mu Q_a - C_a = 0. \tag{7}$$

Solving equation (7) for the advertising volumes yields the optimal advertising space in dependence of demand for copies:

$$Q_a = \frac{\delta + \varepsilon Q_c - C_a}{2\mu}. \tag{8}$$

Replacing Q_a in equation (6) with the right hand side of (8) leads to the optimal quantity of copies

$$Q_c^* = \frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\gamma\mu - \varepsilon^2}, \tag{9}$$

as a function of the markets parameters only. In comparison to the standard monopoly output $(\alpha - C_c)/2\gamma$, Q_c^* is extended by the effects from the advertising market. Intuitively, one would expect that the relationship of reader and advertising markets would lead to an increase in circulation, compared with the usual monopoly case.

To determine the optimal copy price (P_c^*), the quantity Q_c^* can be inserted into the inverse demand equation for copies. Hence, the copy price under profit maximising behaviour is

$$P_c^* = \frac{2\mu\gamma(\alpha + C_c) - \varepsilon\left[\gamma(\delta - C_a) + \alpha\varepsilon\right]}{4\mu\gamma - \varepsilon^2}. \tag{10}$$

Again, the effects from advertising affecting the price for copies in an ambiguous manner. But intuition expects a lower price than for simple monopoly goods.

Proposition 1 *(i) The media firm monopolist offers an optimal quantity of copies (Q_c^*) that is greater than the usual monopoly output (Q_M^*). (ii) The price, in contrast, is less than the usual monopoly price.*

Proof: (i) $Q_c^* > Q_M^*$, if

$$\frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{2\gamma\mu - \varepsilon^2} > \frac{\alpha - C_c}{2\gamma}.$$

Simple algebra yields the inequality

$$2\gamma\varepsilon(\delta - C_a) > -\varepsilon^2(\alpha - C_a),$$

which holds if all parameters are positive and $\alpha > C_c$ and $\delta > C_a$.
Proposition 1 (ii) holds if

$$\frac{2\mu\gamma(\alpha + C_c) - \varepsilon\left[\gamma(\delta - C_a) + \alpha\varepsilon\right]}{4\mu\gamma - \varepsilon^2} < \frac{\alpha + C_c}{2}.$$

Using simple algebra yields the inequality

$$-2\gamma(\delta - C_a) < \varepsilon(\alpha - C_c),$$

which is true, if all parameters are positive and $\delta > C_a$ and also $\alpha > C_c$. ■

Thus, the media firm produces a greater output than a standard monopolist, because of the reinforcing relation to the advertising market; taking into account that generating a large demand for copies will stimulate the demand for advertising space. An effect that can be described as some kind of network effect. The larger the network of readership, the larger the demand for advertising space. Not surprisingly, this outcome is in line with other studies (e.g. Blair and Romano 1993 or Chaudhri 1998).

Since the monopolisation of *reader markets* leads to both, higher output and lower prices than usually known from monopolistic markets the outcome is less inefficient than for a *normal* monopoly. But note, that this is only true for the reader market. To consider the advertising rate and quantity, P_c^* and Q_c^* have to be included into the respective inverse demand function and first order condition, respectively. Thus, the optimal advertising volume and optimal advertising rate can be determined by:

$$Q_a^* = \frac{2\gamma(\delta - C_a) + \varepsilon(\alpha - C_c)}{4\gamma\mu - \varepsilon^2} \tag{11}$$

and

$$P_a^* = \frac{2\gamma\mu(\delta + C_a) + \varepsilon\left[(\alpha - C_c) - \varepsilon C_a\right]}{4\gamma\mu - \varepsilon^2}. \tag{12}$$

Seemingly, the quantity of advertising is symmetrically parameterised to the optimal quantity of copies. Hence, one would expect a higher demand for advertising space as the standard monopoly output. The optimal advertising rate, however, seems to be much more complicated.

Proposition 2 *(i) Also the quantity supplied on the advertising market exceeds the quantity offered by an usual monopolist. (ii) The advertising rate, in contrast to the copy price, could probably exceed the monopoly price.*

Proof: Because of the analogy to the copy market, the proof of Proposition 3 (i) is analogous to the proof of Proposition 2. Proposition 3 (ii) can be proofed using the inverse demand equation for P_a. A comparison of both prices (P_a^* and P_M^*) yields

$$\delta + \varepsilon Q_c^* - \mu Q_a^* > \delta - \mu Q_M^*,$$

where Q_M^* is the optimal quantity from a usual monopoly. Inserting quantities and simple algebra leads to

$$\mu(\alpha - C_c) + \varepsilon(\delta - C_a)(1 - \mu/2) > 0.$$

Consequently, the inequality is valid as long as $\mu \leq 2$, regardless of the magnitude of the other parameters. And more exact, the advertising rate exceeds the normal monopoly price if

$$\mu > 2\varepsilon \frac{\delta - C_a}{\varepsilon(\delta - C_a) - 2(\alpha - C_c)}. \ \blacksquare$$

Hence, the dependency of the advertising market from the copy market leads to greater advertising volumes than in simple monopolies in all respects. But also the advertising rates exceed monopoly prices, if demand for advertising space is elastic. The stronger the impact of copies on the demand for advertising the less decisive is the advertising rate for the demand for advertising space. Thus, the consequences of monopolistic market power in the media sector rather develops on advertising than on reader markets, when markets are related but not interdependent.

2.2 Monopolistic media markets and habit formation

In contrast to the previous section, we now turn to the optimisation process of a media firm, which is confronted with habit formation. Under the assumption that readers of newspaper get used to the print media, the optimisation problem becomes intertemporal because of the relationship of current and past demand. For comparability, the present linear demand function for copies from section 2 is extended by past consumption:

$$Q_t^c = a - bP_t^c + dQ_{t-1}^c, \tag{13}$$

where Q_{t-1}^c is the demand for copies in period $t-1$. The demand function from the advertising market remains unchanged with respect to the parameters but is transformed to a dynamic function. Thus, the inverse demand functions are

$$P_t^c = \alpha - \gamma Q_t^c + \eta Q_{t-1}^c, \tag{14}$$

and

$$P_t^a = \delta + \varepsilon Q_t^c - \mu Q_t^a \tag{15}$$

where $\eta = d/b$. Assuming an infinite lifetime the newspaper publisher maximises the current value of all, current and future, profits:

$$\max_{Q_t^c, Q_t^a} \sum_{t=1}^{\infty} \beta^{t-1} \left[(\alpha + \eta Q_{t-1}^c - \gamma Q_t^c - C_c) Q_t^c + (\delta + \varepsilon Q_t^c - \mu Q_t^a - C_a) Q_t^a \right], \quad (16)$$

where $\beta \in [0, 1]$ is a constant discount factor. Again, only the advertising market depends on the quantity of copies, but not vice versa. The demand for copies is habituated and the readers are assumed to be myopic. Thus, past but not anticipated future demand will affect current demand for newspaper. Advertising customers, in contrast, are not assumed to be subject of any habit effect.

The Euler equations are[4]

$$\beta^t \eta Q_{t+1}^c + \beta^{t-1} \left[(\alpha + \eta Q_{t-1}^c - 2\gamma Q_t^c - C_c) + \varepsilon Q_t^a \right] = 0, \quad (17)$$

for the reader market and

$$\beta^{t-1} \left[\delta + \varepsilon Q_t^c - 2\mu Q_t^a - C_c \right] = 0, \quad (18)$$

for the advertising market. Rearranging equation (18) yields

$$Q_t^a = \frac{\delta + \varepsilon Q_t^c - C_a}{2\mu}. \quad (19)$$

which seems to be identical to Q_a from the static approach. Again, the demand for advertising space is positively affected by the quantities of copies. Combining the Euler equations yields the second order difference equation

$$A Q_{t+1}^c + B Q_t^c + C Q_{t-1}^c = -D \quad (20)$$

where $A = 2\beta\eta\mu$, $B = -(4\gamma\mu - \varepsilon^2)$, $C = 2\eta\mu$ and $D = 2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)$. Equation (20) is useful to determine the optimal consumption path and

[4]The Euler equations are derived using the first derivatives of equation (16) with respect to Q_t^c and Q_t^a. Note that, of course, the derivatives have to be carried out for all leads and lags of Q_t^c and Q_t^a.

all quantities and prices in the steady state. Interestingly, even if consumers are myopic, not only current and past consumption determines the optimal quantity of copies, but also anticipated future consumption Q^c_{t+1}. Note, that because of $D > 0$ equality (20) only holds if $A + C < |B|$ and as long as all parameters are positive and $\alpha - C_c > 0$ and $\delta - C_a > 0$, respectively.

Steady State Equilibrium

In order to compare the results from the static model, neglecting habit formation, we now analyse the steady state prices and quantities from both markets. Simultaneously, the differences to the static model will be considered. Setting all intertemporal quantities equal to Q^c_*, the steady state quantity of copies is

$$Q^c_* = \frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)}, \tag{21}$$

which is very similar to the static approach. Only the denominator distinguishes from the static case. Hence, the larger the discount factor, the habit effect or the influence of the advertising rate on the demand for advertising space, the larger is the optimal quantity of copies. Moreover, setting $\eta = 0$ and, therefore, neglecting habit effects yields the optimal quantity Q^*_c.

The steady state copy price can easily be calculated using the inverse demand function as

$$P^c_* = \frac{2\mu\left[\gamma(\alpha + C_c) - \eta(\alpha\beta + C_c)\right] - \varepsilon\left[(\gamma - \eta)(\delta - C_a) + \varepsilon\alpha\right]}{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)}, \tag{22}$$

which is quite different to the static approach. But again, setting $\eta = 0$ reduces P^c_* to P^*_c. Because of the occurrence of η in the denominator and nominator of equation (22), it is not clear, whether the price P^c_* is larger or less than P^*_c. However, economic intuition would expect a higher price in the steady state equilibrium, when consumption is determined by habit formation.

Proposition 3 *(i) The steady state output on the copy market, when consumption of newspaper is habituated, is greater than the output neglecting habit effects.*

(ii) Copy prices in media markets with habit effects are probably larger than prices without regarding any habituation.

Proof: (i) Since the nominators of both equations (21) and (9) are identical, only the denominators have to be considered. Assuming positive parameters yields $2\mu\eta(\beta+1) > 0$ and therefore $Q_*^c > Q_c$. (ii) $P_*^c > P_c^*$ can be proofed by the comparison of the inverse demand functions which yields:

$$\alpha + \eta Q_{t-1}^c - \gamma Q_t^c < \alpha - \gamma Q_c^*.$$

Taking into account that $Q_{t-1}^c = Q_t^c = Q_*^c$ in the steady state, and simple algebra leads to the inequality

$$2\gamma\mu(1-\beta) - \varepsilon^2 > 0,$$

which is an ambiguous result. It strongly depends on the magnitude of the parameters, whether $P_*^c \gtreqless P_c^*$. ■

Hence, regarding copy markets, not only the connection of the sub-markets but also the habit formation leads to an increasing optimal quantity of copies. Moreover, the impact of habit effects is stronger than the impact of the connection of the sub-markets with respect to the optimal number of copies.

Habituated demand for newspapers with a positive link to advertising demand leads to increased quantities and only possibly to higher prices. One the one hand, habit formation leads to a higher demand for copies. Hence, one would expect that, if media firms are able to apply intertemporal price differentiation, prices are low in an early period and high in a later period. But regarding the steady state, the media firm is possibly able to set higher prices than usually, because demand is not expected to change. The critical variables for the magnitude of the copy prices are γ, μ, β and ε. If the relative influence of the demand for copies on the advertising market (ε) is large enough, the habit effects lead to a decreasing copy price. This is intuitive, because if the network effect form the reader market leads to a strong enough increase in advertising volumes, it is more profitable to lower copy prices. Moreover, the copy price could probably lie below marginal costs, for specific situations.[5]

Turning to the advertising market, the steady state advertising volume is determined by:

$$Q^a_* = \frac{\delta + \varepsilon Q^c_* - C_a}{2\mu}, \tag{23}$$

which is the well known result from the static approach. Simple algebraic manipulation and inserting Q^c_* yields the optimal advertising volume in the steady state

$$Q^a_* = \frac{[2\gamma - \eta(\beta+1)](\delta - C_a) + \varepsilon(\alpha - C_c)}{4\gamma\mu - \varepsilon^2 - 2\eta\mu(\beta+1)}, \tag{24}$$

which reduces to the static optimal advertising volume if habit effect are neglected. The effect from habituation is not obvious regarding the optimal quantity of advertising. On the one hand η occurs negatively in the nominator but on the other hand it occurs also negatively in the denominator.

The steady state advertising rate is:

$$P^a_* = \delta + \varepsilon Q^c_* - \mu Q^a_*, \tag{25}$$

or

$$P^a_* = \frac{[2\gamma\mu - \eta\mu(\beta+1)]\,(\delta + C_a) + \varepsilon[\mu(\alpha - C_c) - \varepsilon C_a]}{4\gamma\mu - \varepsilon^2 - 2\eta\mu(\beta+1)}. \tag{26}$$

A comparison with the static advertising rate is not obvious but can be carried out by simple algebra.

[5] Using the inverse demand function for P^c_t, leads to $\alpha - \gamma Q^c_* + \eta Q^c_* < C_c$, which yields $\eta - \gamma < \frac{C_c - \alpha}{Q^c_*}$. Of course, the right hand side of this inequality is negative, because $Q^c_* > 0$ and $\alpha > C_c$. Therefore, (i) the difference $(\eta - \gamma)$ has to be negative. Because of $\gamma = 1/b$ and $\eta = d/b$, condition (i) holds, as long as $d < 1$, which seems to be appropriate. Furthermore, condition (ii) $|\eta - \gamma| < |\alpha - C_c|/Q^c_*$ has to be fulfilled. This is possible and depends strongly on the magnitude of α. ■

Proposition 4 *(i) The advertising volume, when reading newspapers is habituated is greater than that without any habits. (ii) And also the advertising rate is greater when reading newspapers is habituated. (iii) Furthermore, it always exceeds the usual monopoly rate.*

Proof: (i) Using the first order conditions with respect to advertising demand of both, the dynamic and static model produces the inequality

$$\frac{\delta + \varepsilon Q_*^c - C_a}{2\mu} > \frac{\delta + \varepsilon Q_c^* - C_a}{2\mu}.$$

or

$$Q_*^c > Q_c^*,$$

which has already been proofed in Proposition 3. (ii) Using the inverse demand functions of both, the dynamic and static advertising rates yields the inequality

$$\delta + \varepsilon Q_*^c - \mu Q_*^a > \delta + \varepsilon Q_c^* - \mu Q_a^*,$$

which can be reduced to

$$2(Q_*^c - Q_c^*) > Q_*^c - Q_c^*.$$

As $Q_*^c > Q_c^*$, also Proposition 4 (ii) holds. (iii) Considering the inverse demand function to analyse $P_*^c > P_M^*$ yields the inequality $\delta + \varepsilon Q_*^c - \mu Q_*^a > \delta - \mu Q_M$ or

$$1 > \frac{1}{2},$$

which is always true. ■

The habit effect occurring in the reader market, not surprisingly, leads to a rise in the demand for advertising because of an increasing demand for copies. The newspaper publisher produces a larger circulation in comparison with the situation where habit effects are absent. Hence, the demand for advertising increases only due to the increased demand for copies. Hence, the media firm is able to rise advertising rates in comparison to the static equilibrium without reducing demand for advertising space. More interestingly the advertising rate exceeds the usual monopoly price. Despite of the quantity increment, some kind of "monopolistic exploitation" takes place on the advertising market, in a monopoly-monopoly framework, if habit effects occur.

The effects of habit formation with respect to reader and advertising markets can be summarised as:

$$Q^c_* > Q^*_c, P^c_* \gtreqless P^*_c$$
$$Q^a_* > Q^*_a, P^a_* > P^*_a.$$

Thus, habit formation on reader markets leads to asymmetric effects considering both markets. However, a comparison of the profits from both models is no trivial task. Regarding only advertising markets, the profits are always higher when habit formation on reader markets exists. This result is intuitive, because habituation leads, ceteris paribus, to a higher demand for copies and, therefore, to a higher demand for advertising space. But also the habituated demand for copies should result in higher profits from the reader market.

Proposition 5 *The media monopolist realises larger profits from both markets, if the reader market is characterised by habit formation.*

Proof: To prove $\Pi^a_* > \Pi^*_a$, the profits from the advertising markets out of both models can be compared as:

$$\delta + \varepsilon Q^c_* - C_a)Q^a_* > (\delta + \varepsilon Q^*_c - C_a)Q^*_a.$$

Substituting equation (11) and (23) for Q^*_a and Q^a_* yields the inequality $Q^c_* > Q^*_c$, which has been already proofed in Proposition 4.
The comparison of the profits from the reader market is more complicated. Using the relationship $\Pi^c_* > \Pi^*_c$ leads to the inequality

$$[\alpha + (\eta - \gamma)Q^c_* - C_c]Q^c_* > [\alpha - \gamma Q^*_c - C_c]Q^*_c.$$

Simple algebra yields

$$(\alpha - C_c)(Q^c_* - Q_c*) + \gamma Q^{*2}_c > (\gamma - \eta)Q^{c2}_*,$$

which holds, because (i) $(\alpha - C_c)(Q^c_* - Q_c*) > 0$ and (ii) $\gamma Q^{*2}_c > (\gamma - \eta)Q^{c2}_*$ (see the appendix for a further proof). ■

Hence, the media monopolist realises higher profits in the steady state from both markets, if habit effects exists. Even if copy prices may be lower than in the static case, the increment of quantity leads to an overcompensation of this price effect.

Optimal Consumption Path

Next, the optimal time path of consumption will be considered to analyse the intertemporal behaviour of the media monopolist. The path of copies sold, is determined by the second order difference equation (20). Regarding the reduced equation $AQ^c_{t+1} + BQ^c_t + CQ^c_{t-1} = 0$ yields the characteristic roots:

$$\lambda_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{27}$$

or

$$\lambda_{1,2} = \frac{4\gamma\mu - \varepsilon^2}{4\beta\eta\mu} \pm \frac{1}{2}\sqrt{\left(\frac{4\gamma\mu - \varepsilon^2}{2\beta\eta\mu}\right)^2 - \frac{4}{\beta}}, \tag{28}$$

respectively. Combining the characteristic roots and the particular integral yields the optimal consumption as:

$$Q^{c*}_t = Q^c_* + \psi_1\lambda^t_1 + \psi_2\lambda^t_2. \tag{29}$$

The parameters ψ_1 and ψ_2 can be determined using the initial condition and the terminal state. Since Q^c_0 is given as the initial condition, the terminal condition is not constricted here.[6] Therefore, it is appropriate to assume Q^c_* as the terminal state (see Chiang 1992). But since $\lambda_1 > 1$ and $\lambda_2 < 1$ the optimal path would be unstable for $t \to \infty$, thus ψ_1 hat to be set equal to zero.[7] Hence, we are able to derive the initial and terminal quantity as $Q^c_0 = Q^c_* + \psi_1 + \psi_2$ and $Q^c_\infty = Q^c_*$.

[6]Typical ways to achieve terminal states could be, e.g., the introduction of capacity constraints or the consideration of increasing marginal costs. But also a maximum value of Q_c could lead to a terminal condition.

[7]To realise positive quantities Q^c_* and Q^a_* it is necessary that $-B > A + C$. Using this relation, it follows that $\lambda_1 > 1$ and $\lambda_2 < 1$.

Taking into account that $\psi_1 = 0$ yields $\psi_2 = Q_0^c - Q_*^c$. Thus, the optimal path of copies is

$$Q_t^{c*} = (Q_0^c - Q_*^c)\lambda_2^t + Q_*^c, \tag{30}$$

which can be described by a partial adjustment rule. For $\lambda_2 < 1$ follows that in each period λ_2 times the difference of the current consumption and the steady state is added to the steady state consumption. Assuming $Q_0^c < Q_*^c$, the difference in equation (30) is negative but decreasing with t and Q_t^{c*} is moving towards Q_*^c. Also if $Q_0^c > Q_*^c$, Q_t^{c*} converges to the steady state for $t \to \infty$. The parameter λ_2, therefore, determines the speed of adjustment to the steady state. The smaller the characteristic root, the faster Q_t^{c*} will converge to the equilibrium.

3 Duopolistic advertising markets

Since market delineation of advertising markets has to be figured out in a broader context than market definition of reader markets, we now turn to a more realistic assumption about market structure on advertising markets.[8] Therefore, a reader market monopolist is considered, serving a duopolistic advertising market. A situation that will be denominated as a monopoly-duopoly case in the following. This is not an unusual situation, since most of the German local newspaper markets are of monopolistic or duopolistic structure.

3.1 Static approach

Assuming two monopolistic media firms producing totally differentiated newspapers but competing on the advertising market in a Cournot duopoly, the inverse respective demand functions can be represented by $P_{ci} = \alpha - \gamma Q_{ci}$ and $P_{ai} = \delta + \varepsilon Q_{ci} - \mu(q_{ai} + q_{aj})$. The profit function of firm $i = 1, 2$ is:

[8]Regarding reader markets the consumers are interested in the media and the information provided by the media. The advertising costumer instead is not interested in the media but in the readership or, more exact, in a specific target group. Therefore, especially with respect to daily newspaper or magazines the advertising space in differentiated print media could be near substitutes, even if the newspapers itself are not (see Dewenter 2000).

$$\Pi_i = [\alpha - \gamma Q_{ci} - C_c]Q_{ci} + [\delta + \varepsilon Q_{ci} - \mu(q_{ai} + q_{aj}) - C_a]q_{ai}, \tag{31}$$

where C_c and C_a are, again, marginal costs of both sub-markets. First order conditions with respect to Q_{ci} and q_{ai} yields

$$Q_c = \frac{\alpha - C_c + \varepsilon q_{ai}}{2\gamma} \tag{32}$$

and

$$q_{aj} = \frac{\delta - C_a + \varepsilon Q_c}{\mu} - 2q_{ai}. \tag{33}$$

Where equation (33) is the best response function of firm i considering the advertising market. Note, that because of symmetry of both firms the best response function of firm j is equivalent. Combining the best response functions q_{ai} and q_{aj} yields

$$q_a = \frac{\delta - C_a + \varepsilon Q_{ci}}{3\mu}, \tag{34}$$

which is the optimal advertising volume with respect to the demand for copies. Not surprisingly, neglecting the reader market and setting $Q_{ci} = 0$ yields the standard Cournot output. Inserting equation (34) into (32) produces the optimal quantity of copies

$$Q^*_{ci} = \frac{3\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{6\gamma\mu - \varepsilon^2}. \tag{35}$$

The corresponding optimal advertising space is

$$Q^*_{ai} = \frac{2\gamma(\delta - C_a) + \varepsilon(\alpha - C_c)}{6\gamma\mu - \varepsilon^2}. \tag{36}$$

Again, the optimal quantities are very similar to the static *monopoly-monopoly* case.

Proposition 6 *The optimal quantities from both markets when the advertising market is duopolistic are (i) less the quantities when the media firm is a monopolist on both markets. (ii) Furthermore, Q^*_{ci} always exceeds the monopoly output.*

Proof: (i) The consideration of $Q^*_{ci} < Q^*_c$ using the optimal quantities yields the inequality

$$0 < 2\gamma\mu\varepsilon(\delta - C_a),$$

which is always true for $\delta - C_a > 0$. (ii) Moreover, using the relation $Q^*_{ai} < Q^*_a$ leads to the inequality

$$\frac{2\gamma(\delta - C_a) + \varepsilon(\alpha - C_c)}{6\gamma\mu - \varepsilon^2} < \frac{2\gamma(\delta - C_a) + \varepsilon(\alpha - C_c)}{4\gamma\mu - \varepsilon^2},$$

which is true, because the denominator of the right hand side is less the denominator of the left hand side and since $\alpha - C_c > 0$ and $\delta - C_a > 0$.

(ii) $Q^*_{ci} > Q^*_M$ is true if

$$\frac{3\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{6\gamma\mu - \varepsilon^2} > \frac{\alpha - C_c}{2\gamma},$$

or

$$2\gamma\varepsilon(\delta - C_a) > -\varepsilon(\alpha - C_c),$$

which holds as long as $\alpha - C_c > 0$ and $\delta - C_a > 0$. ■

Summarising, the optimal quantities from the different static models can be described by $Q^*_M < Q^*_{ci} < Q^*_c$ and $Q^*_M < Q^*_{Cournot} < Q^*_{ai} < Q^*_a$.[9] Not surprisingly, the duopolistic situation on the advertising market leads to a reduction in optimal quantities from both markets. One the one hand, the demand for advertising space can only be satisfied partly by each competitor. Furthermore, the network effect from the copy market is less effective than in a monopoly-monopoly situation. Therefore, also the optimal number of copies declines.

Using the inverse demand function for both markets lead to the optimal prices

[9] See the appendix for the proof of $Q^*_{Cournot} < Q^*_{ai}$.

$$P_{ci}^{*} = \frac{3\gamma\mu(\alpha + C_c) - \varepsilon\left[\gamma(\delta - C_a) + \varepsilon\alpha\right]}{6\gamma\mu - \varepsilon^2} \tag{37}$$

and

$$P_{ai}^{*} = \frac{2\gamma\mu(\delta + C_a) + \varepsilon\left[\mu(\alpha - C_c) - \varepsilon C_a\right]}{6\gamma\mu - \varepsilon^2}. \tag{38}$$

Proposition 7 *The duopolistic advertising rate is (i) probably less than the monopoly-monopoly price for advertising. The copy price of the monopoly-duopoly model, instead (ii) exceeds the copy price of the monopoly-monopoly model but it is (iii) less than the usual monopoly price.*

Proof: see the appendix. ■

Competition on the advertising market leads not unconditionally to a decline in advertising prices. But if ε is relatively large (or $\gamma\mu$ is relatively low), the advertising rate could theoretically exceed the rate from the monopoly-monopoly model. Accordingly, the influence of the demand for copies on the advertising market has to be large and/or the product of the slopes of the inverse demand functions have to be relatively low.

Furthermore, advertising competition leads to an increasing copy price. Because the media monopolist has to share the advertising market with its competitor, enlarging the quantity of copies is no longer as efficient as rising the copy price. Nevertheless, due to the existence of the advertising market, the copy price is still lower than a usual monopoly price.

3.2 Dynamic approach

The next step is to consider the situation where both newspaper firms are faced with habit formation on the reader market. According to the previous sections the inverse demand functions change only slightly due to the parameter η indicating the habit effect. Thus, the optimisation problem of the i'th firm is

$$\max_{Q_t^{ci}, q_t^{ai}} \sum_{t=1}^{\infty} \beta^{t-1} \left[(\alpha + \eta Q_{t-1}^{ci} - \gamma Q_t^{ci} - C_c) Q_t^{ci} + (\delta + \varepsilon Q_t^{ci} - \mu(q_t^{ai} + q_t^{aj}) - C_a) q_t^{ai} \right], \tag{39}$$

where Q_t^{ci} and Q_{t-1}^{ci} are the contemporary and lagged quantities on the reader market and q_t^{ai} (q_t^{aj}) is the contemporary advertising volume of firm i (j). Using the first order conditions with respect to Q_t^{ci} and q_t^{ai} yields the Euler equations

$$\beta^t \eta Q_{t+1}^{ci} + \beta^{t-1}(\alpha + \eta Q_{t-1}^{ci} - 2\gamma Q_t^{ci} - C_c + \varepsilon q_t^{ai}) = 0 \tag{40}$$

and

$$\beta^{t-1}(\delta + \varepsilon Q_t^{ci} - 2\mu q_t^{ai} - \mu q_t^{aj} - C_a) = 0. \tag{41}$$

Rearranging equation (41) leads to the best response functions of both firms as

$$q_t^{ai} = \frac{\delta - C_a + \varepsilon Q_t^{ci}}{\mu} - 2q_t^{aj} \quad \text{and} \quad q_t^{aj} = \frac{\delta - C_a + \varepsilon Q_t^{ci}}{\mu} - 2q_t^{ai}. \tag{42}$$

Combining both equations from (42) yields the optimal quantity on the advertising market

$$q_*^a = \frac{\delta - C_a + \varepsilon Q_t^{ci}}{3\mu}, \tag{43}$$

which still depends on Q_t^{ci}. Thus, inserting equation (43) into the first Euler equation yields the second order difference equation

$$EQ_{t+1}^{ci} + FQ_t^{ci} + GQ_{t-1}^{ci} = -H, \tag{44}$$

where $E = 3\mu\beta\eta$, $F = (\varepsilon^2 - 6\mu\gamma)$, $G = 3\mu\eta$ and $H = 3\mu(\alpha - C_c) + \varepsilon(\delta - C_a)$.

Steady State Equilibrium

The steady state quantities of copies can be derived from equation (44) in analogy to the monopoly-monopoly case as

$$Q_*^{ci} = \frac{3\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{6\mu\gamma - \varepsilon^2 - 3\mu\eta(\beta + 1)}. \tag{45}$$

Obviously, the optimal quantity of copies in the steady state is, again, greater than in the static approach, because only the denominator of equation (45) differs from the denominator of equation (35).

Proposition 8 *The optimal quantity of copies in a monopoly-duopoly situation, when demand on the reader market is denominated by habit formation is (i) larger than in the static approach neglecting habit effects, but (ii) less than in the monopoly-monopoly case.*

Proof: As (i) is obvious, only (ii) has to be proofed. Comparing the optimal quantities Q_*^{ci} and Q_*^{c} yields the inequality

$$\frac{3\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{6\mu\gamma - \varepsilon^2 - 3\mu\eta(\beta + 1)} < \frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\mu\gamma - \varepsilon^2 - 2\mu\eta(\beta + 1)}$$

or

$$[\eta(\beta + 1) - 2\gamma](\delta - C_a) < \varepsilon(\alpha - C_c),$$

which holds, because $2\gamma > \eta(\beta + 1)$ for positive quantities. ■

Hence, the Cournot competition on the advertising market leads to an increased number of copies in comparison to the static case. But on the other hand it leads to a reduction in the quantity of copies in comparison with the monopoly-duopoly model. Therefore, the magnitude of prices on copy markets with habit effects, when advertising markets are duopolistic, is not clear. Using the inverse demand function yields the optimal copy price as

$$P_*^{ci} = \frac{3\mu\left[\gamma - \eta(\beta + 1)\right](\alpha + C_c) + \varepsilon\left[(\gamma - \eta)(\delta - C_a) - \varepsilon\alpha\right]}{6\gamma\mu - \varepsilon^2 - 3\mu\eta(\beta + 1)}. \tag{46}$$

Proposition 9 *The copy price in the monopoly-duopoly case, when habit formation on the reader market is present, is greater than the copy price from the static approach, neglecting habit effects.*

Proof: To prove $P_*^{ci} > P_{ci}^*$ the respective inverse demand function can be used. Simple algebraic manipulation yields

$$\gamma(6\mu\gamma - \varepsilon^2) - 3\mu\eta^2(\beta + 1) > 0,$$

which holds for positive quantities and as long as $\gamma > \eta^2$. As we assumed $\eta < 1$, Proposition 9 is always true. ■

Proposition 9 is an interesting result, since the effect from the competitive situation on the advertising market leads to both, a higher number of copies and a higher price. Thus, the price increment on the copy market is more profitable, than expanding the quantity, since in contrast to the monopoly-monopoly case the effect from a higher demand for newspaper leads to a lower increment in the demand for advertising space. Hence, the "exploitation" of the readers is more profitable than enlarging the market for advertising space.

Turning to the advertising market, the optimal quantity of advertising and optimal advertising rate can be derived as

$$Q_*^{ai} = \frac{[2\gamma - \eta(\beta + 1)](\delta - C_a) + \varepsilon(\alpha - C_c)}{6\gamma\mu - \varepsilon^2 - 3\mu\eta(\beta + 1)}, \tag{47}$$

and

$$P_*^{ai} = \frac{[2\gamma\mu - \eta\mu(\beta + 1)]\,(\delta + 2C_a) + \varepsilon\,[\mu(\alpha - C_c) - \varepsilon C_a]}{6\gamma\mu - \varepsilon^2 - 3\mu\eta(\beta + 1)}. \tag{48}$$

Proposition 10 *Both, advertising space and advertising rate are higher, when consumption on reader markets is habituated than that from the static model, also if the advertising market is a Cournot duopoly.*

Proof: Comparing the quantities form the advertising market $Q_*^{ai} > Q_{ai}^*$ leads to the inequality

$$3\eta\mu(\alpha - C_c) + \varepsilon(\delta - C_a) > 0,$$

which is always true. To prove $P_*^{ai} > P_{ai}^*$ the respective inverse demand functions can be used. Thus, the inequality

$$\delta + \varepsilon Q_*^{ci} - 2\mu q_*^a > \delta + \varepsilon Q_{ci}^* - 2\mu q_a^*,$$

reduces to

$$3 > 2,$$

when optimal quantities are inserted. ■

Not surprisingly, the habit effects lead to higher prices and quantities, because the increased demand for copies shifts the demand for advertising space. And furthermore, because of the competition on the advertising market, the quantity is higher than in a dynamic monopoly-monopoly case.[10] The magnitude of the price, in contrast, strongly depends of the the elasticities of demand, and other parameters comparing P_*^{ai} and P_*^a.

Optimal Consumption Path

Using the reduced equation $EQ_{i,t+1}^c + FQ_{i,t}^c + GQ_{i,t-1}^c = 0$, and the steady state quantity of copies Q_*^{ci} leads to the optimal consumption path

$$Q_t^{ci*} = (Q_0^{ci} - Q_*^{ci})\lambda_4^t + Q_*^{ci}, \tag{49}$$

where

$$\lambda_4 = \frac{(6\mu\gamma - \varepsilon^2)}{6\mu\eta\beta} - \frac{1}{2}\sqrt{\left(\frac{6\mu\gamma - \varepsilon^2}{3\mu\eta\beta}\right)^2 - \frac{4}{\beta}},$$

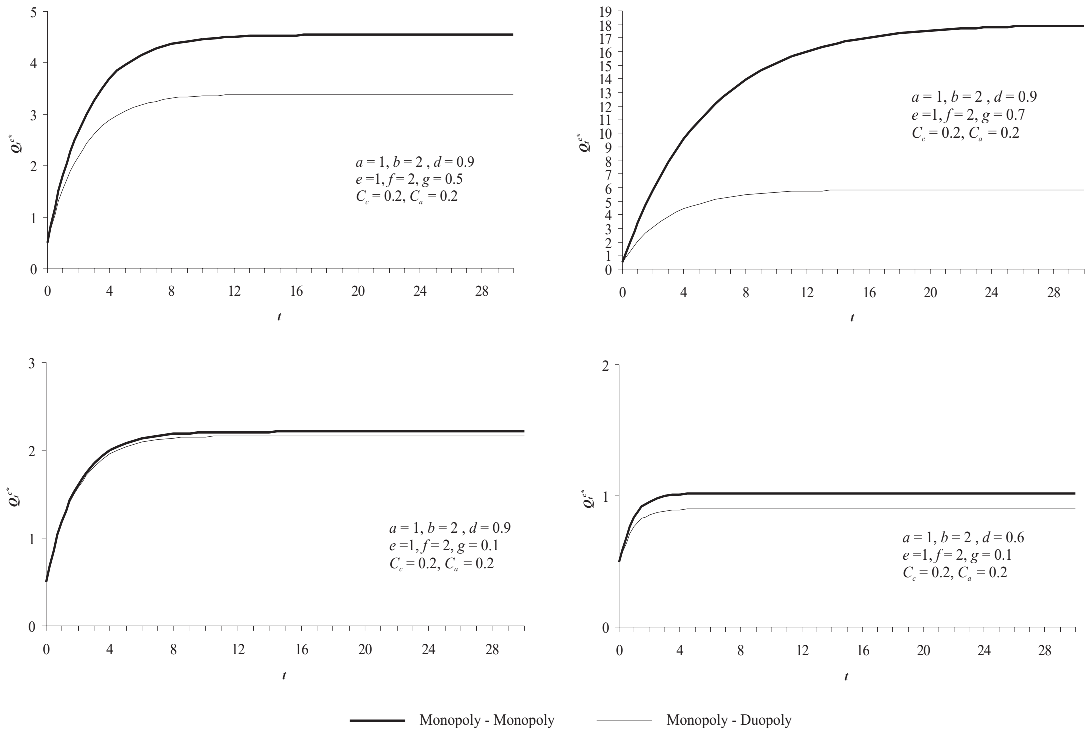
is the smaller of the two characteristic roots of the reduced equation and Q_0^{ci} is the initial value. Again, the optimal consumption path converges towards the steady state value Q_*^{ci} by a partial adjustment rule.

[10]The proof for the quantities can be easily carried out by comparing $Q_*^{ai} > Q_*^a$, which leads to $2\gamma - \eta(\beta + 1) > 0$, which always holds for positive quantities.

Comparing the optimal path with optimal consumption path from the monopoly-monopoly model, it is obvious that the monopoly-monopoly model converges to a higher steady state. However, it is not clear, what kind of consequences from a variation in the different parameters will appear. Especially d, determining the habit effect, and g, measuring the influence of the demand for copies on the demand for advertising space, are of peculiar interest. Therefore, we simulated some models to visualise the respective effects (see Figure 2).

The upper left picture of Figure 2 compares the optimal time paths of both models, assuming a relatively strong habit effect ($d = 0.9$) and an average $g = 0.5$. The steady state consumption from the monopoly-monopoly model is, as expected, higher than that from the monopoly-duopoly (MD) model. Enhancing the influence of the copy market on the demand for advertising space, and therefore rising g to 0.7 (upper right picture), yields a considerable increment of the steady state regarding the monopoly-monopoly model and a less considerable increment regarding the MD model. Hence, the difference in consumption between both models becomes more distinctive. Accordingly, setting $g = 0.1$ (lower left picture) leads, of course, to a reduction of this difference. Thus, a $g = 0$ would lead to identical paths. Varying the strength of the habit effect ($d = 0.9$ to $d = 0.6$), in contrast, leads also to a reduction in the steady state consumption (see lower right picture). But a difference of Q^c_* and Q^{ci}_* will, however, be existent, even if $d \to 0$.

Figure 2: Optimal consumption paths under different parameter regimes



4 Interrelated markets

4.1 Monopolistic advertising markets

Finally, after analysing the influence of habit effects in related media markets with both, monopolistic and duopolistic advertising markets, we now turn to a model of *inter*related markets. Therefore, we relax the assumption that the demand for copies is independent from advertising space by introducing a further parameter (ϕ). Starting from a monopoly-monopoly model the maximum present value of all profits, contemporary and intertemporal is:

$$\max_{Q_t^c, Q_t^a} \sum_{t=1}^{\infty} \beta^{t-1} \left[(\alpha + \eta Q_{t-1}^c - \gamma Q_t^c + \phi Q_t^a - C_c) Q_t^c + (\delta + \varepsilon Q_t^c - \mu Q_t^a - C_a) Q_t^a \right], \tag{50}$$

where ϕ determines the positive influence from the advertising space to the demand for copies. Using the first order conditions with respect to Q_t^c and Q_t^a yields the system of Euler equations:[11]

$$\begin{bmatrix} \eta(L+\beta L^{-1})-2\gamma & \varepsilon+\phi \\ \varepsilon+\phi & -2\mu \end{bmatrix} \begin{bmatrix} Q_t^c & Q_t^a \end{bmatrix} = \begin{bmatrix} -(\alpha-C_c) & -(\delta-C_a) \end{bmatrix}, \quad (51)$$

where L is the standard lag operator, which is defined by $L^n Q_t^c = Q_{t-n}^c$ for $n = \ldots, -2, -1, 0, 1, 2, \ldots$. Solving the system of Euler equations leads to the second order difference equation $AQ_{t+1}^c + IQ_t^c + CQ_{t-1}^c = -J$, where $A = 2\mu\eta\beta$, $I = -[4\gamma\mu - (\varepsilon+\phi)^2]$, $C = 2\mu\eta$, and $J = 2\mu(\alpha - C_c) + (\varepsilon+\phi)(\delta - C_a)$.

Steady state equilibrium

In analogy to the other models the endogenous steady state quantities can be calculated as

$$Q_{**}^c = \frac{2\mu(\alpha-C_c)+(\varepsilon+\phi)(\delta-C_a)}{4\gamma\mu-(\varepsilon+\phi)^2-2\mu\eta(\beta+1)} \quad (52)$$

and

$$Q_{**}^a = \frac{[2\gamma-\eta(\beta+1)]\,(\delta-C_a)+(\varepsilon+\phi)(\alpha-C_c)}{4\gamma\mu-(\varepsilon+\phi)^2-2\mu\eta(\beta+1)}. \quad (53)$$

Conspicuously, both quantities are very similar to Q_*^c and Q_*^a. Merely the parameter ε is extended by ϕ. Thus, substitution of ε by $\varepsilon+\phi$ yields the optimal quantities within an interrelated framework. Whereas, ε and ϕ determining the effects from copy market to the advertising market and vice versa, it is assumed that the interrelationship of the markets lead to an increasing demand for both, copies and advertising space. Because, as long as both parameters are positive, a circulation–advertising spiral will be present.

[11] For an application of a similar model of a multivariate dynamic optimisation problem, see Sargent (1987), p. 210.

Proposition 11 *If advertising is informative to readers, the circulation advertising spiral will lead to (i) a higher number of copies and (ii) more advertising space, in contrast to a situation where advertising is neglected by the readership. Moreover, habit effects are leading to higher quantities in both markets than in a static framework (iii),(iv).*

Proof: (i) The proof of $Q^c_{**} > Q^c_*$ yields

$$\frac{2\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{4\gamma\mu - (\varepsilon + \phi)^2 - 2\mu\eta(\beta + 1)} > \frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)},$$

which is true as long as $\phi > 0$, because ϕ increases the nominator and lowers the denominator of Q^c_{**}. (ii) The same is true for the inequality $Q^a_{**} > Q^a_*$. Regarding the comparison of the static and dynamic quantities (iii),(iv), the proofs are also trivial, because setting $\eta = 0$ yields the static quantities, which have lower denominators in both cases (see the appendix for static quantities and prices). Thus, also the static quantities are higher than from related-markets models ($Q^{**}_c > Q^*_c$ and $Q^{**}_a > Q^*_a$). ■

The interdependency of the markets leads, therefore, to a higher number of copies and a higher amount of advertising space, independently from the existence of habit effects. Hence, interrelated markets in combination with habit effects inducing maximum quantities with respect to all of the previous models. The number of copies sold is, therefore, far away from the standard monopoly output.

Inserting optimal quantities into the inverse demand functions lead to the steady state prices of copies and advertising, which can be represented by:

$$P^c_{**} = \frac{2\mu[\gamma(\alpha + C_c) - \eta(\alpha\beta + C_c)] - (\varepsilon + \phi)(\alpha\varepsilon + C_c\phi) + [\varepsilon(\eta - \gamma) + \phi(\gamma - \beta\eta)](\delta - C_a)}{4\mu\gamma - (\varepsilon + \phi)^2 - 2\mu\eta(\beta + 1)} \tag{54}$$

and

$$P^a_{**} = \frac{[2\mu\gamma - \mu\eta(\beta + 1)](\delta + C_a) - (\varepsilon + \phi)(\phi\delta + \varepsilon C_a) + \mu[(\varepsilon - \phi)(\alpha - C_c)]}{4\mu\gamma - (\varepsilon + \phi)^2 - 2\mu\eta(\beta + 1)}. \tag{55}$$

Again, setting $\eta = 0$ yields the static optimal prices (see the appendix for static prices). Assuming $\phi = 0$ leads to the related market model in Section 2. Setting both parameter $\eta = 0$ and $\phi = 0$ yields the standard static prices for related but not interdependent markets. The comparison of prices from the different models is much more complicated with interrelated markets. More exact, it is not clear if $P^c_{**} \gtreqless P^c_*$, $P^c_{**} \gtreqless P^{**}_c$, or if $P^a_{**} \gtreqless P^a_*$, $P^a_{**} \gtreqless P^{**}_a$. And only the relationship $P^a_{**} \gtreqless P^{**}_a$ leads to a simple rule.

Proposition 12 *The advertising rate from interrelated markets, when consumption on reader markets is characterised by habit formation, is larger (less) than the advertising rate from the static approach, if and only if the influence from the copy market to the advertising market is stronger (weaker) than the influence from the advertising market to the copy market.*

Proof: The comparison of the dynamic and static optimal advertising rates $P^a_{**} \gtreqless P^{**}_a$ yields

$$\delta + \varepsilon Q^c_{**} - \mu Q^a_{**} \gtreqless \delta + \varepsilon Q^{**}_c - \mu Q^{**}_a$$

and furthermore the simple inequality

$$\varepsilon \gtreqless \phi.$$

Thus, if $\varepsilon > \phi$ it follows that $P^a_{**} > P^{**}_a$ and vice versa. As $\varepsilon = g/f$ measures the relative influence of the demand for copies on the demand for advertising space and ϕ measures the relative influence of the demand for advertising space on the demand for copies, the relation ε/ϕ determines the price setting behaviour in interrelated markets. ■

Hence, if readers react stronger on extending advertising space than advertising costumers react on enlarging the number of copies sold, then the media monopolist rises advertising rates, if habit effects are realised. If the reaction of advertising customers is stronger, in contrast, the monopolist lowers the advertising rate.

Since the comparison of the other prices leads to more complicated and ambiguous results, we simulated different prices with respect to varying ratios of ε and ϕ. As one can see from Figure (see the appendix), there are always clear price relations to be found by simulation. The results can be summarised as follows:

$$P^c_{**} > P^{**}_c, \quad P^c_{**} > P^c_*, \quad P^{**}_c > P^*_c,$$
$$P^a_{**} \gtreqless P^{**}_a, \quad P^a_{**} < P^a_*, \quad , P^{**}_a < P^*_a.$$

Thus, both, habit formation and interrelated markets lead to higher copy prices. As before, the habit effects have a positive influence on the copy price, in the steady state. The relation $P^c_{**} > P^{**}_c$ is intuitive. The interdependence of the markets has also positive effects on the copy price ($P^c_{**} > P^c_*$ and $P^{**}_c > P^*_c$), because a higher demand for advertising stimulates the demand for copies, as long as $\phi > 0$. The effect on the advertising market are quite different. Apart from the relation $P^a_{**} \gtreqless P^{**}_a$, which depends on ε/ϕ, the price effects on the advertising market as a response on habit formation ($P^a_{**} < P^a_*$) and interdependency ($P^{**}_a < P^*_a$) are negative. As usually habit formation leads to higher prices, the effect from interrelated markets has to be larger than the effect from habituated consumption. Neglecting habituation the advertising rate is lower, when interdependency is observed, because extending the advertising space yields also increasing demand for copies.

Considering the prices in comparison to those from a standard monopoly, the calculated results are ambiguous. Again a simulation can lead to further insights (see Figure 4). Prices from both markets can be greater or less the monopoly rate, depending on the ration of η and ϕ. For the copy market prices are *lower* than the monopoly rate, when ε is relative large. More, exact, if the influence from the copy market to the advertising market is relative strong.[12]

Regarding the advertising market, the same simple rules can be applied like for the comparison of P^a_{**} and P^{**}_a. Therefore, both prices, from the dynamic and the static approach, are *greater* than the monopoly rate, if $\varepsilon/\phi > 1$. Interestingly, choosing a relative high ratio ($\varepsilon/\phi > 1$), could probably lead to a situation, where prices from both markets are larger than the monopoly rates. But this result strongly depends on the magnitude of other parameters.

[12]Overall the results from all simulation are robust against the variations of the parameters. The results did not change qualitatively, at least, if only positive quantities and prices are considered.

Optimal consumption Path

The optimal time path of Q_t^c can be calculated, in analogy to the previous models, as

$$Q_t^{c**} = (Q_0^c - Q_{**}^c)\lambda_6^t + Q_{**}^c \tag{56}$$

with

$$\lambda_6 = \frac{4\mu\gamma - (\varepsilon+\phi)^2}{4\mu\eta\beta} - \frac{1}{2}\sqrt{\left(\frac{4\mu\gamma - (\varepsilon+\phi)^2}{2\mu\eta\beta}\right)^2 - \frac{4}{\beta}},$$

the smaller characteristic root of the reduced equation $AQ_{t+1}^c + IQ_t^c + CQ_{t-1}^c = 0$. The initial value is again Q_0^c and λ_6^t measures the speed of adjustment to the steady state.

4.2 Duopolistic advertising markets

To analyse the monopoly-duopoly model of interrelated markets, the following current value of all profits has to be considered:

$$\max_{Q_t^c, Q_t^a} \sum_{t=1}^{\infty} \beta^{t-1} \left[(\alpha + \eta Q_{t-1}^c - \gamma Q_t^c + \phi q_t^{ai} - C_c)Q_t^c + (\delta + \varepsilon Q_t^c - \mu(q_t^{ai} + q_t^{aj}) - C_a)q_t^{ai}\right], \tag{57}$$

Using the first order conditions yields the system of Euler equations:

$$\begin{bmatrix} \eta(L+\beta L^{-1}) - 2\gamma & \varepsilon+\phi \\ \varepsilon+\phi & -2\mu \end{bmatrix} \begin{bmatrix} Q_t^{ci} & q_t^{ai} \end{bmatrix} = \begin{bmatrix} -(\alpha - C_c) & -(\delta - C_a) + \mu q_t^{aj} \end{bmatrix}. \tag{58}$$

Some algebraic manipulation of the second Euler equation leads to the best response functions $q_t^{ai} = \frac{\delta - C_a + (\varepsilon+\phi)Q_t^{ci}}{\mu} - 2q_t^{aj}$ and $q_t^{aj} = \frac{\delta - C_a + (\varepsilon+\phi)Q_t^{ci}}{\mu} - 2q_t^{ai}$. Combining the best response functions yields the modified system

$$\begin{bmatrix} \eta(L+\beta L^{-1}) - 2\gamma & \varepsilon+\phi \\ \varepsilon+\phi & -3\mu \end{bmatrix} \begin{bmatrix} Q_t^{ci} & q_t^{a} \end{bmatrix} = \begin{bmatrix} -(\alpha - C_c) & -(\delta - C_a) \end{bmatrix}. \tag{59}$$

Solving the system of Euler equations for the endogenous quantities and few algebra produces the optimal quantities and prices. The steady state quantities are

$$Q_{**}^{ci} = \frac{3\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{6\gamma\mu - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)} \tag{60}$$

and

$$Q_{**}^{ai} = \frac{[2\gamma - \eta(\beta + 1)]\,(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{6\gamma\mu - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)}. \tag{61}$$

Of course, as $\phi > 0$, the comparison of the quantities is analogous to the related monopoly-duopoly model. Hence, the quantity of copies, when habit effects occur is greater than in the static case ($Q_{**}^{ci} > Q_{ci}^{**}$). Furthermore, it is also greater than in the monopoly-duopoly model of related markets ($Q_{**}^{ci} > Q_{*}^{ci}$). The same is true for the static quantities ($Q_{ci}^{**} > Q_{ci}^{*}$). Again both interdependency and habit effects lead to higher optimal quantities. Comparing the optimal quantity from the monopoly-duopoly model with the quantity from the monopoly-monopoly model, leads to $Q_{**}^{ci} < Q_{**}^{c}$ (see the appendix for a proof). The competition on the advertising market leads, again, to a reduction of the quantity.

Regarding the advertising market, also the optimal advertising space increases in comparison with the static model ($Q_{**}^{ai} > Q_{ai}^{**}$). The same holds for $Q_{**}^{ai} > Q_{*}^{ai}$ (and $Q_{ai}^{**} > Q_{ai}^{*}$). Due to the interdependency of the markets, the optimal advertising space is larger than considering related markets. Furthermore, competition on the advertising markets leads to increased advertising space in the steady state.[13]

Optimal prices can be represented as

$$P_{**}^{ci} = \frac{3\mu[\gamma(\alpha + C_c) - \eta(\alpha\beta + C_c)] - (\varepsilon + \phi)(\alpha\varepsilon + C_c\phi) + [\varepsilon(\eta - \gamma) - \phi(\gamma + \beta\eta)](\delta - C_a)}{6\mu\gamma - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)} \tag{62}$$

[13]All of the proofs which are not trivial can be found in the appendix.

and

$$P^{ai}_{**} = \frac{[2\mu\gamma - \mu\eta(\beta+1)](\delta+2C_a) - (\varepsilon+\phi)(\phi\delta+\varepsilon C_a) + \mu[(\varepsilon-2\phi)(\alpha-C_c)]}{6\mu\gamma - (\varepsilon+\phi)^2 - 3\mu\eta(\beta+1)}, \qquad (63)$$

which are very similar to the prices from the monopoly-monopoly model. In contrast to the related markets model, simple and unambiguous relations of the prices are no longer valid. As in the monopoly-monopoly model of interrelated markets the interdependency of the reader and advertising markets leads to ambiguous results.

First of all, habit formation does not unconditionally lead to unambiguous higher or lower prices. Therefore, P^{ci}_{**} can be larger, less or equal to P^{**}_{ci}, if $\varepsilon \lesseqgtr 1/2[\phi(\beta-1) + \sqrt{\phi^2(\beta+1)^2 + 12\gamma\mu(1-\beta)}]$, which is, of course, positive. Unfortunately this expression is too complex for further results. Thus, we simulated copy and advertising rates, using the same parameters as for the monopoly-monopoly model (see appendix B.3). Regarding this simulation and some variations in the respective parameters, P^{ci}_{**} is always larger than P^{**}_{ci}, for positive prices and quantities. Hence, one would expect that habit formation also in a monopoly-duopoly situation will rise the optimal number of copies.

Similar is true for the comparison of P^{ai}_{**} and P^{**}_{ai}. Here it is only the relation of ε and ϕ that is determining the price ratio (see the appendix A.6.4). Hence, only if the relative influence of the demand for copies on the advertising market is more than twice as strong as the relative influence of the demand for advertising on the reader market, the advertising rate is higher when habit formation exists. Economically this results can be interpreted as follows: if and only if advertising customers react more than twice as strong on a variation in the number of copies sold, than the readers react on varying the advertising volumes (relative to price variations), the habit formation leads to a higher advertising rate than neglecting the habit effects. So in contrast to the monopoly-duopoly model of *related* markets, habit formation do not necessarily lead to higher advertising volumes. In a situation where the reader market volume is relatively not very important to the advertising customers, habit effects could probably lead to decreasing advertising rates. Because lower rates would lead to higher advertising volumes and, therefore, to a more efficient enlargement of copies. But this situation seems to be unlikely.

A further interesting relation that between P^{ci}_{**} and P^{ci}_{*} (and P^{ai}_{**}, P^{ai}_{*} respectively) determining the effects of the interdependency of the markets. Again, algebraic modifications does not yield an unambiguous result, but simulation of prices can lead to further insights. Following Figure 5 (in the appendix) a clear result exists: regarding the reader market the interdependency has a positive effect on prices. The copy price from an interrelated approach is always higher than that from a related model, because the positive effect from advertising allows to rise the copy price. Considering the advertising market the opposite is true. Here P^{ai}_{**} is always less than P^{ai}_{*}, independently of the ratio ε/ϕ.[14] If markets are interdependent, the media firm has to lower the advertising rate to stimulate a larger advertising volume and, thus, an increasing demand for copies. But also a comparison of the monopoly-duopoly and the monopoly-monopoly model is of further interest:

Proposition 13 *Both, copy price and advertising rate are probably higher in a monopoly-duopoly situation, when copy consumption is habituated, than in a monopoly-monopoly model. Furthermore, both prices probably exceed respective monopoly rates.*

Proof: See the appendix. ■

If the relative influence of the demand for copies on the advertising market (ε) is enough stronger than the relative influence of the demand for advertising space on the reader market (ϕ), the copy price from an interrelated monopoly-duopoly model could probably exceed the prices from the monopoly-monopoly situation. The interdependency could, therefore, lead to higher prices although one market is characterised by a competitive situation. As P^{ci}_{**} is also higher assuming the

[14]Indeed, all of the simulation results are very robust against variations of different parameters, as long as quantities and prices are assumed to be positive.

usual magnitudes of the parameters (see Figure 6), situations where $P^{ai}_{**} > P^{a}_{**}$ are only realised for special parameter regimes (see appendix A.6.2).

Figure 4 includes not only the simulation of the prices from the monopoly-monopoly model and the usual monopolistic prices, but also the comparison of the monopoly-duopoly prices and the monopoly rates. As in the MM model only a small area exists where $P^{a}_{**} > P_M$, the MD model generates a different outcome. Not only the copy price, but also the advertising rates are higher than usual monopoly rates in a much broader area.

Comparison of profits

Due to the introduction of the interdependency not only the prices, but also the comparison of the profits becomes more complicated. For this reason a simulation of profits has been carried out. As one can see from Figure 7 (see the appendix), the relations of profits seems to be clear.[15] Profits from the monopoly-monopoly model are as expected usually larger than from the monopoly-duopoly model. The same is true for the comparison of dynamic and static models. Habit formation leads, ceteris paribus, to increasing profits. Interestingly, the profits from a static monopoly-monopoly model could probably exceed the profit from a dynamic monopoly-duopoly model, when habituation is low and ε rises.

Moreover, all profits except for `G*is` are larger than the profits from two isolated monopoly markets (`GM`). In the monopoly-duopoly model of related markets profits are not unconditionally larger than usual monopoly profits. But the stronger the habit effects and the stronger the (inter-)relationship between the markets, the larger the gap between media market profits and the profits from two isolated markets.

Optimal Consumption Path

For the sake of completeness the optimal path of Q^c_t is calculated. Again using the smaller of the characteristic roots

[15] Various profits are confronted with each other, where * (**) stands for the related (inter-related) model and s (d) denominating the static (dynamic) approach. The index i stands for the monopoly-duopoly models. Simulated profits are calculated in dependency of ε/ϕ (varying ε and fixed $\phi = 0.2$). The upper four pictures are calculated with a low habit effect of $d = 0.1$, the lower four pictures assume a stronger effect ($d = 0.4$).

$$\lambda_8 = \frac{6\mu\gamma - (\varepsilon+\phi)^2}{6\mu\eta\beta} - \frac{1}{2}\sqrt{\left(\frac{6\mu\gamma - (\varepsilon+\phi)^2}{3\mu\eta\beta}\right)^2 - \frac{4}{\beta}},$$

yields the optimal path

$$Q_t^{ci**} = (Q_0^c - Q_{**}^{ci})\lambda_8^t + Q_{**}^{ci}. \tag{64}$$

It is noticeable that λ_8 is very similar to λ_6 and λ_4. A comparison of the optimal paths is not necessary, because we do not expect further insights. Furthermore, the main focus of this paper is to analyse the optimal quantities, prices and profits in the steady state.

5 Conclusions

The consumption of mass media, like newspaper or magazines, but also of other media like the internet, is frequently characterised by habit formation. Though, the literature on economic aspects of mass media is considerable, effects of habit formation in consumption have not been analysed yet. Therefore, the aim of this paper was to determine the pricing behaviour of a newspaper publisher serving reader and advertising markets when consuming newspaper is characterised by habituation. Furthermore, the interaction of the interdependency of the markets on the one hand and the habit effects on the other hand was of special interest. To receive preferably accurate results, we started from a simple static monopoly-monopoly model of related but not interdependent markets, neglecting habituation. The partly strong assumptions were relaxed piecewise and we ended with a monopoly-duopoly model with interrelated markets and habit effects. Both, monopolistic and duopolistic situations have been considered with respect to the advertising sector, where the reader market was assumed to be monopolistic in all respects.

The first important result from the analysis is that, considering the reader and the advertising market, media firms produce quantities which are larger than the standard monopoly output. This is already true for the simplest model and is in line with the results from different authors. A reduction in the optimal number of copies is only possible, when demands are interdependent and if readers have negative attitudes against advertising. Otherwise a circulation-advertising spiral exists. Price effects, in contrast, are ambiguous. While copy prices are lower in related markets and could eventually lie under marginal costs, the advertising rate could even rise.

Introducing habit effects to the various models produces, as expected, the same positive impact on the optimal quantities. Independently of the framework, habituation always leads to increasing circulation and advertising space. But also prices tend to increase due to habituation and lie possibly over usual monopoly rates. Consequently, also the profits realised by a media firm which is faced with habituation are usually larger than from comparable market structures.

The most interesting outcomes, but unfortunately also the most ambiguous results, arise from the interdependent markets models. Not surprisingly, these approaches produce the highest quantities possible, driven by the circulation-advertising spiral on the one hand and the habituation effects on the other hand. With respect to both the monopoly-monopoly and the monopoly-duopoly model, copy price tend to rise due to habit effects and as a result of interdependency. The more readers value the advertising, the higher the optimal copy price. The advertising rates, in contrast, usually decreases with an increasing valuation of advertising. But regarding the monopoly-duopoly model advertising rates are higher with interrelated markets.

Furthermore, prices from both markets could possibly lie above monopoly rates. Interestingly, such a situation is more probable in a monopoly-duopoly framework than in the monopoly-monopoly case; a situation which is typical for German regional newspaper markets. The reason for this seemingly contra-intuitively result is that on a duopolistic advertising market advertising space is split over the two competitors. Lowering the advertising rate does not unconditionally stimulate a larger demand for copies. Some kind of "monopolistic exploitation" is, therefore, possible on both markets. Which is a very important result from a competition policy perspective. But this result is very interesting in the light of seemingly collusive behaviour. Though Dewenter and Kraft (2001) found empirical evidence for monopolistic prices on duopolistic German newspaper advertising markets, it is questionable if those prices are an expression of collusion or simply the result of habit effects.

Finally, also the profits from both dynamic approaches, in general, exceed the profits generated from two isolated monopolies and also those from the static models. Moreover, with an increasing influence of the copy market size on the demand for advertising space, absolute and relative to the readers valuation of advertising, we found increasing magnitudes of profits. We assume that such a situation where advertising customers value the reader markets stronger than readers valuate advertising is more realistic than vice versa.

Further research should focus on empirical tests of the models to evaluate the economic contents and to check for validity. Not only the existence of habit effects, which has already been analysed, but moreover the implications of habituation and interdependency with respect to markets structures and prices should be of special interest.

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A Mathematical Appendix

A.1 Proof of $\gamma Q_c^{*2} > (\gamma - \eta) Q_*^{c2}$

Condition (ii) of Proposition 5 holds if

$$\frac{\gamma}{\gamma - \eta} > \frac{Q_*^{c2}}{Q_c^{*2}}.$$

Inserting optimal quantities Q_*^c and Q_c^* yields

$$\frac{\gamma}{\gamma - \eta} > \frac{\left(\frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\gamma\mu - \varepsilon^2}\right)^2}{\left(\frac{2\mu(\alpha - C_c) + \varepsilon(\delta - C_a)}{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)}\right)^2},$$

or

$$\frac{\gamma}{\gamma - \eta} > \left(\frac{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)}{4\gamma\mu - \varepsilon^2}\right)^2.$$

Because $\gamma > \eta$, the left hand side is larger than unity. The right hand side, in contrast, is less than one because $2\mu\eta(\beta + 1) > 0$. Therefore, assuming positive quantities ($Q_c^* > 0$ and $Q_*^c > 0$), the profit of a media monopolist considering habit effects is always larger than that of a monopolist neglecting habituation.

A.2 Proof of $Q^*_{Cournot} < Q^*_{ai}$

The inequality $Q^*_{Cournot} < Q^*_{ai}$ holds if

$$\frac{2\gamma(\delta - C_a) + \varepsilon(\alpha - C_c)}{6\gamma\mu - \varepsilon^2} > \frac{\delta - C_a}{3\mu},$$

or

$$3\mu\varepsilon(\alpha - C_c) > -\varepsilon^2(\delta - C_a).$$

Therefore, the Cournot output on advertising markets is larger than the usual Cournot output and thus also larger than the monopoly output.

A.3 Proof of Proposition 7

Proposition 7 (i) can be proofed using the inverse demand functions for P_{ai}^* and P_a^* which lead to

$$\delta + \varepsilon Q_{ci}^* - \mu Q_{ai}^* < \delta + \varepsilon Q_c^* - \mu Q_a^*.$$

Inserting quantities and simple algebraic manipulation yields:

$$\varepsilon^2 < 2\gamma\mu,$$

which is ambiguous.

Also Proposition 7 (ii) and (iii) can be proofed using the respective inverse demand equations. Thus, proof of (ii) is

$$\alpha - \gamma Q_{ci}^* > \alpha - \gamma Q_c^*$$

or

$$-Q_{ci}^* > -Q_c^*,$$

which holds because $Q_{ci}^* < Q_c^*$. And proof of (iii) is

$$\alpha - \gamma Q_{ci}^* < \alpha - \gamma Q_M^*$$

or

$$-Q_{ci}^* < -Q_M^*,$$

which is true, since $Q_{ci}^* > Q_M^*$. Hence, the prices from both markets can be ranked as $P_M^* > P_{ci}^* > P_c^*$ and $P_{ai}^* < P_a^*$. The relationship of P_{ai}^* and P_M^* remains unclear.

A.4 Quantities and prices from interrelated markets

A.4.1 Monopoly-Monopoly model

Equilibrium quantities

$$Q_c^{**} = \frac{2\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{4\gamma\mu - (\varepsilon + \phi)^2}$$

$$Q_a^{**} = \frac{2\gamma(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{4\gamma\mu - (\varepsilon + \phi)^2}$$

Equilibrium prices

$$P_c^{**} = \frac{2\mu\gamma(\alpha + C_c) - (\varepsilon + \phi)(\alpha\varepsilon + C_c\phi) + \gamma(\varepsilon + \phi)(\delta - C_a)}{4\mu\gamma - (\varepsilon + \phi)^2}$$

$$P_a^{**} = \frac{2\mu\gamma(\delta + C_a) - (\varepsilon + \phi)(\phi\delta + \varepsilon C_a) + \mu[(\varepsilon - \phi)(\alpha - C_c)]}{4\mu\gamma - (\varepsilon + \phi)^2}$$

A.4.2 Monopoly-Duopoly model

Equilibrium quantities

$$Q_{ci}^{**} = \frac{3\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{6\gamma\mu - (\varepsilon + \phi)^2}$$

$$Q_{ai}^{**} = \frac{2\gamma(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{6\gamma\mu - (\varepsilon + \phi)^2}$$

Equilibrium prices

$$P_{ci}^{**} = \frac{3\mu\gamma(\alpha + C_c) - (\varepsilon + \phi)(\alpha\varepsilon + C_c\phi) + \gamma(\varepsilon + \phi)(\delta - C_a)}{6\mu\gamma - (\varepsilon + \phi)^2}$$

$$P_{ai}^{**} = \frac{2\gamma\mu(\delta + 2C_a) - (\varepsilon + \phi)(\phi\delta + \varepsilon C_a) + \mu[(\varepsilon - 2\phi)(\alpha - C_c)]}{6\mu\gamma - (\varepsilon + \phi)^2}$$

A.5 Comparison of optimal quantities from the interrelated monopoly-duopoly model

A.5.1 Proof of: $Q^{ci}_{**} > Q^{c}_{**}$

Inserting the quantities yield the inequality

$$\frac{3\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{6\gamma\mu - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)} > \frac{2\mu(\alpha - C_c) + (\varepsilon + \phi)(\delta - C_a)}{4\gamma\mu - (\varepsilon + \phi)^2 - 2\mu\eta(\beta + 1)}$$

which reduces to

$$\mu[(\varepsilon + \phi)[\eta(\beta + 1) - 2\gamma](\delta - C_a) - (\varepsilon + \phi)^2(\alpha - C_c)] < 0,$$

which is, of course, negative for positive quantities.

A.5.2 Proof of: $Q^{ai}_{**} > Q^{**}_{ai}$

$$\frac{[2\gamma - \eta(\beta + 1)]\,(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{6\gamma\mu - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)} > \frac{2\gamma(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{6\gamma\mu - (\varepsilon + \phi)^2},$$

if

$$\eta(\beta + 1)(\varepsilon + \phi)[(\varepsilon + \phi)(\delta - C_a) + 3\mu(\alpha - C_c)] > 0,$$

which is true, since all parameters are positive and $\delta > C_a$ and $\alpha > C_c$.

A.5.3 Proof of: $Q^{ai}_{**} > Q^{a}_{**}$

To analyse

$$\frac{[2\gamma - \eta(\beta + 1)]\,(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{6\gamma\mu - (\varepsilon + \phi)^2 - 3\mu\eta(\beta + 1)} > \frac{[2\gamma - \eta(\beta + 1)]\,(\delta - C_a) + (\varepsilon + \phi)(\alpha - C_c)}{4\gamma\mu - (\varepsilon + \phi)^2 - 2\mu\eta(\beta + 1)},$$

only the denominators have to be compared, which yields the inequality

$$2\gamma - \eta(\beta + 1) > 0.$$

A.6 Comparison of optimal prices from the interrelated monopoly-duopoly model

A.6.1 Proof of $P^{ci}_{**} > P^{c}_{**}$

Using the inverse demand functions yields $\alpha + \phi Q^{ai}_{**} - \gamma Q^{ci}_{**} > \alpha + \phi Q^{a}_{**} - \gamma Q^{c}_{**}$ or

$$\frac{\phi\gamma - \phi\beta - \varepsilon\gamma - \varepsilon\eta}{(\varepsilon + \phi)(\gamma - \eta)} \gtreqless 0,$$

which is ambiguous. Solving for ε yields

$$P^{ci}_{**} \gtreqless P^{c}_{**}, \quad \text{if} \quad \varepsilon \lesseqgtr \phi\frac{\gamma - \eta\beta}{\gamma - \eta}.$$

Since $\gamma > \eta$ and $\beta \in [0, 1]$, ϕ is multiplied by a factor which is larger than one. Thus, the lower β, the higher is the right hand side of the inequality.

A.6.2 Proof of $P^{ai}_{**} > P^{a}_{**}$

Comparing P^{ai}_{**} and P^{a}_{**} yields the inequality $\delta + \varepsilon Q^{ci}_{**} - 2\mu Q^{ai}_{**} > \delta + \varepsilon Q^{c}_{**} - \mu Q^{a}_{**}$ or

$$\frac{\varepsilon}{2\mu} \gtreqless \frac{2\gamma - \eta(\beta + 1)}{\varepsilon + \phi}$$

which is, again, ambiguous. Solving for ϕ leads to

$$P^{ai}_{**} \gtreqless P^{a}_{**}, \quad \text{if} \quad \phi \gtreqless \frac{4\gamma\mu - \varepsilon^2 - 2\mu\eta(\beta + 1)}{\varepsilon},$$

which is positive for positive quantities.

A.6.3 Proof of $P^{ci}_{**} > P^{**}_{ci}$

Comparing the prices using the inverse demand functions
$\alpha + \phi Q^{ai}_{**} - \gamma Q^{ci}_{**} > \alpha + \phi Q^{**}_{ai} - \gamma Q^{**}_{ci}$ yields

$$P^{ci}_{**} \gtreqless P^{**}_{ci}, \quad \text{if} \quad \varepsilon \lesseqgtr \frac{1}{2}\left[\phi(\beta - 1) + \sqrt{\phi^2(\beta + 1)^2 + 12\gamma\mu(1 - \beta)}\right].$$

A.6.4 Proof of $P^{ai}_{**} > P^{**}_{ai}$

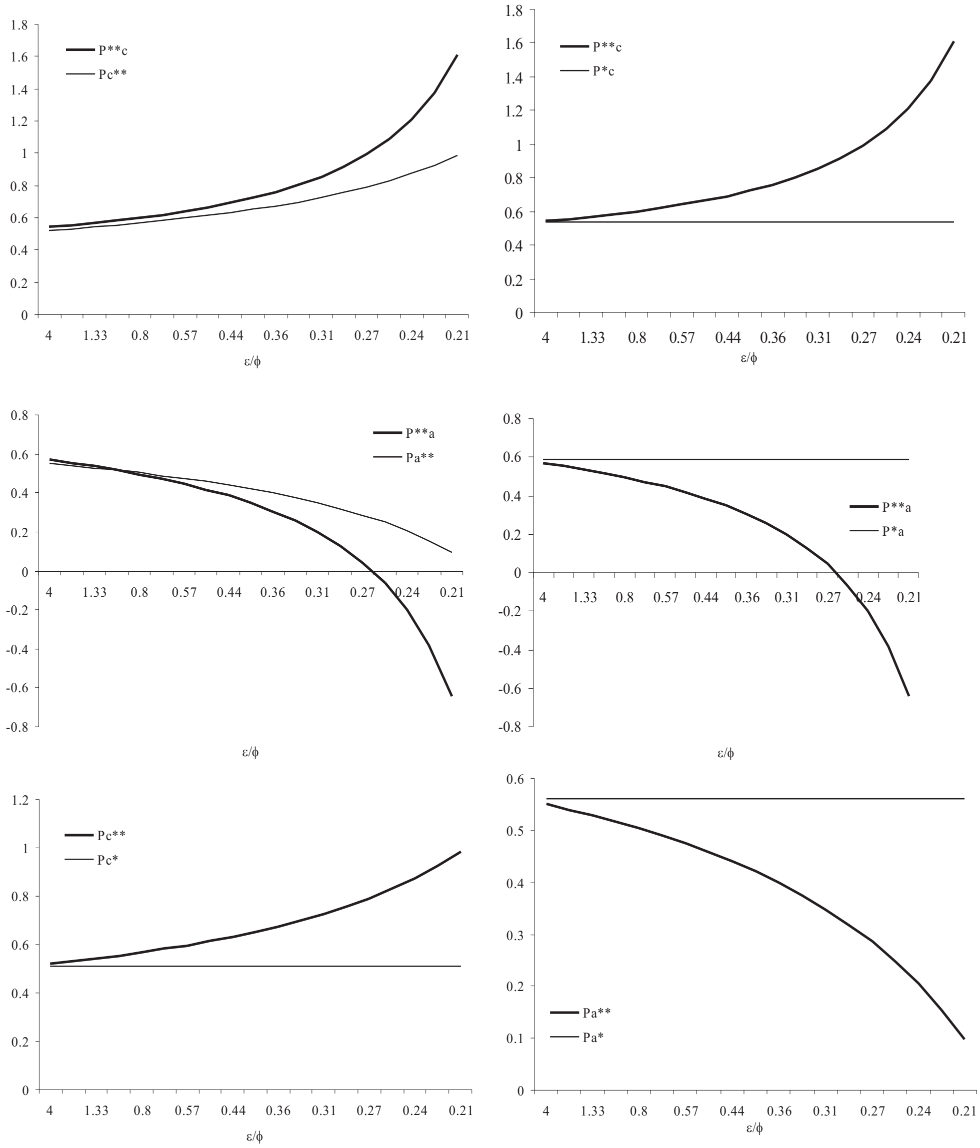
The result from comparing the prices is $\delta + \varepsilon Q^{ci}_{**} - 2\mu Q^{ai}_{**} > \delta + \varepsilon Q^{**}_{ci} - 2\mu Q^{**}_{ai}$ or

$$P^{ai}_{**} \gtreqless P^{**}_{ai}, \quad \text{if} \quad \varepsilon \gtreqless 2\phi.$$

B Figures

B.1 Simulation of related and interrelated prices (MM)

Figure 3: Simulated prices

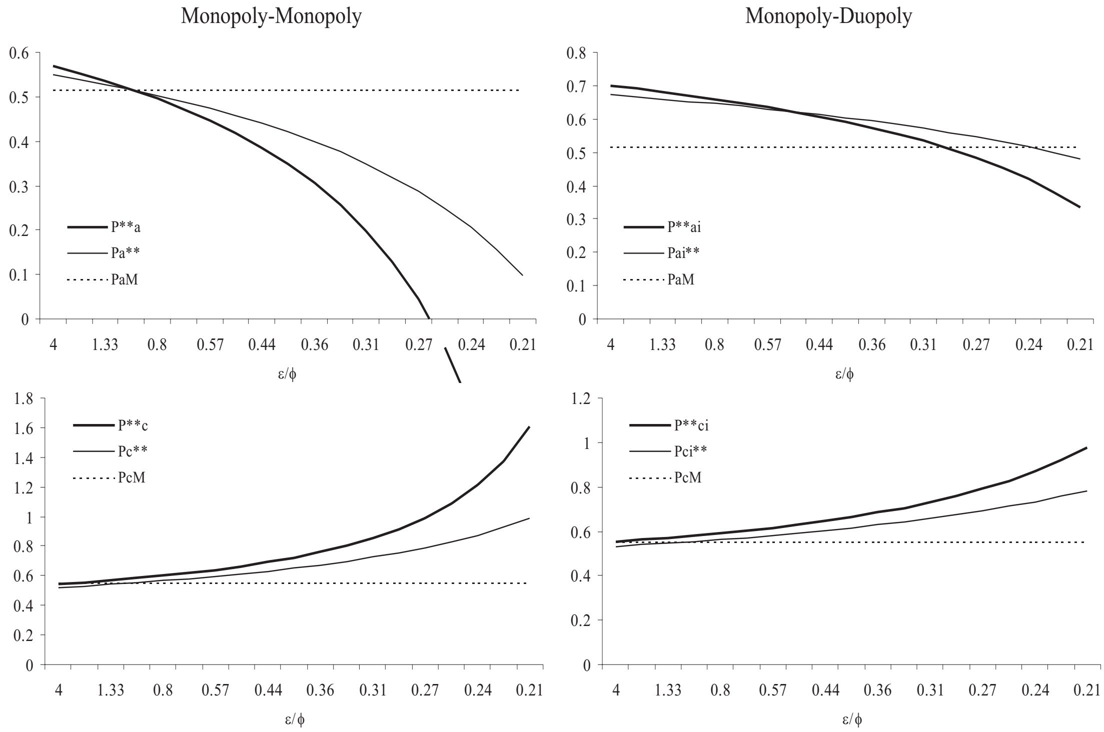


$\alpha = 0.90$, $\beta = 0.8$, $\gamma = 0.90$, $\delta = 0.83$, $\varepsilon = 0.2$, $\eta = 0.36$, $\mu = 0.83$, $C_c = 0.2$, $C_a = 0.2$

Pc* = Copy price, related markets, static
P*c = Copy price, realted markets, dynamic
Pc** = Copy price, interrelated markets, static
P**c = Copy price, interrelated markets, dynamic

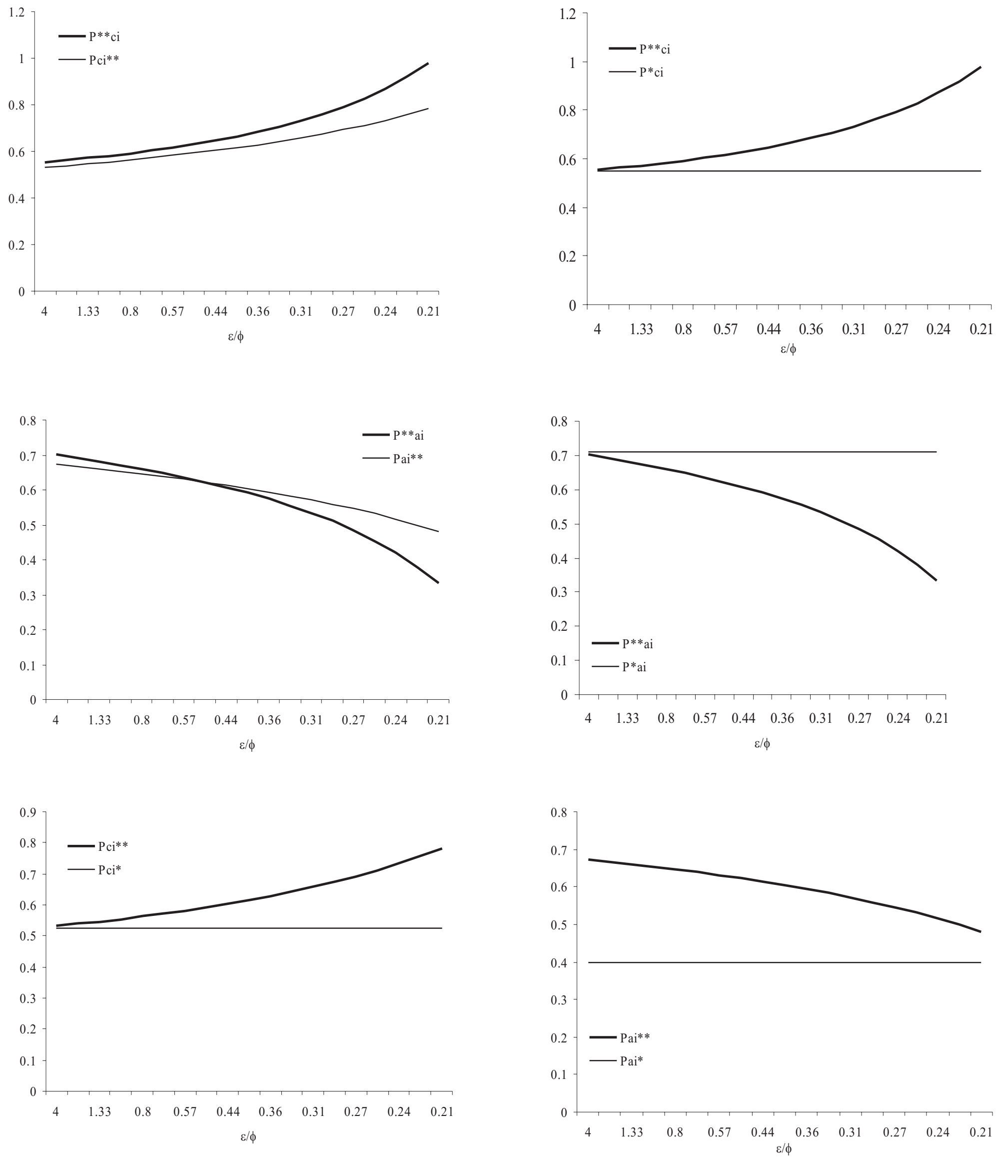
B.2 Comparison of copy, adverting and monopoly prices

Figure 4: Simulated prices from interrelated markets



B.3 Simulation of related and interrelated prices (MD)

Figure 5: Simulated prices

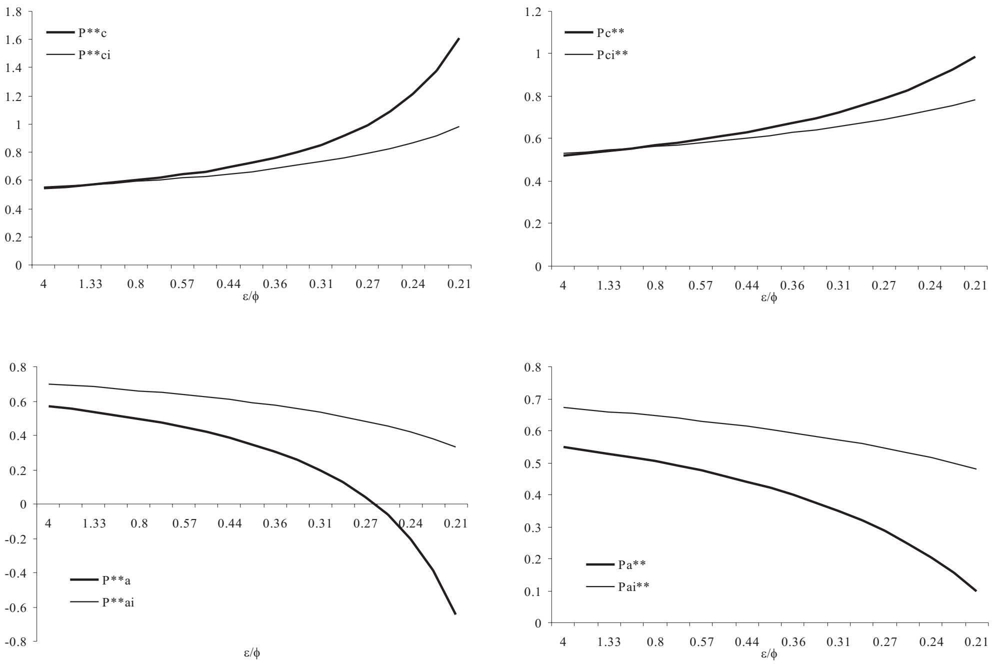


$\alpha = 0.90$, $\beta = 0.8$, $\gamma = 0.90$, $\delta = 0.83$, $\varepsilon = 0.2$, $\eta = 0.36$, $\mu = 0.83$, $C_c = 0.2$, $C_a = 0.2$

Pci* = Copy price, related markets, static
P*ci = Copy price, realted markets, dynamic
Pci** = Copy price, interrelated markets, static
P**ci = Copy price, interrelated markets, dynamic

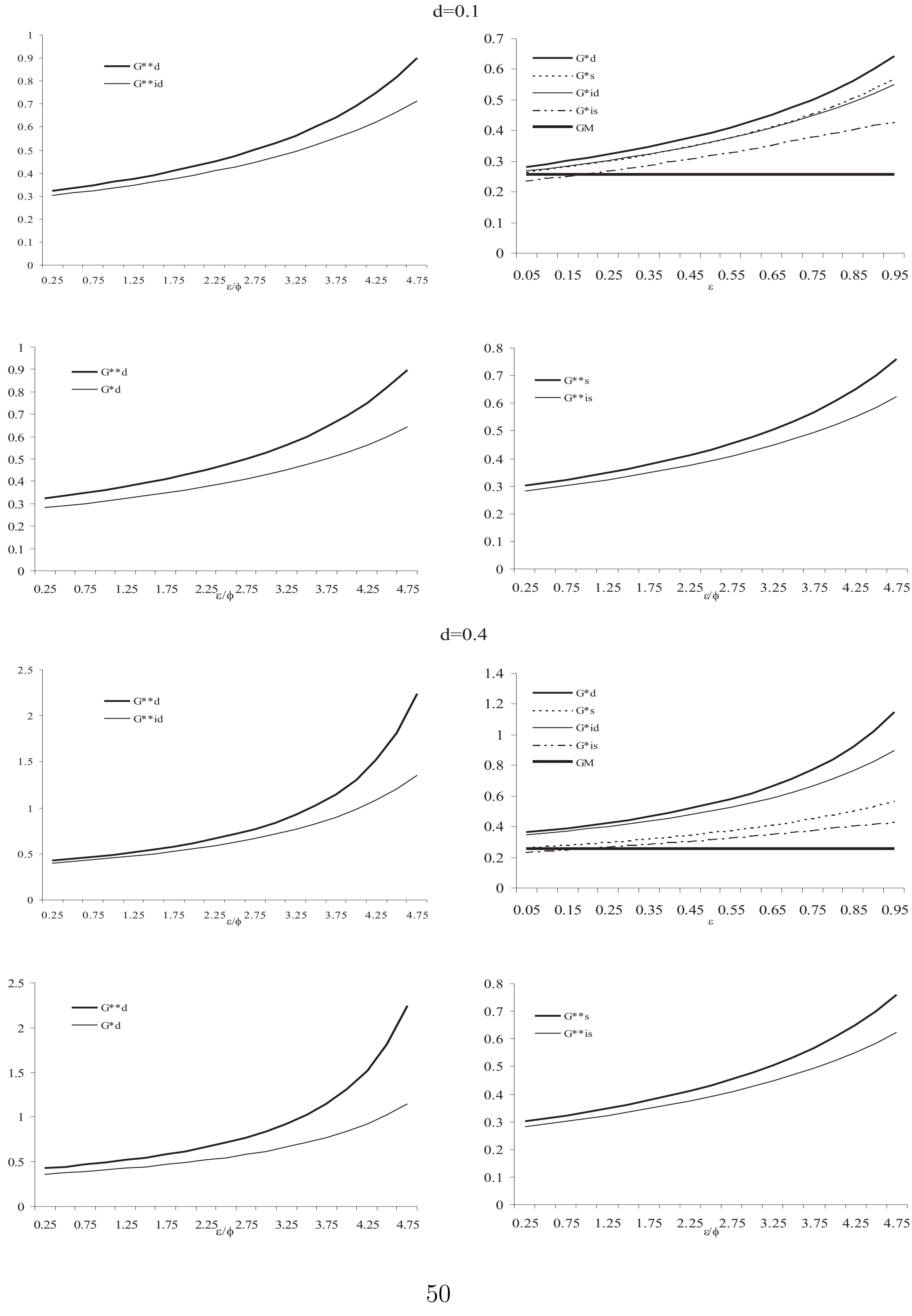
B.4 Simulation of interrelated prices (MM-MD)

Figure 6: Simulated prices



B.5 Simulation of profits

Figure 7: Simulated profits



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