Mating à la Spence: Deriving the Market Demand Function for Status Goods

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Abstract
Conspicuous consumption of luxuries plays a central role in the work of Thorstein B. Veblen. More recently, interpersonal effects have emerged as an important factor in consumption theory. However, the inadequate modelling of individuals' interaction often leads to questionable results with regard to the market demand function for status goods. Following Spence, who recommended the application of his Job Market Signaling to conspicuous consumption, in Mating à la Spence potential partners are faced with asymmetric information: Individuals know their own properties, but are incompletely informed about potential partners. However, individuals have the possibility to signal their properties by demonstrative consumption. Mating à la Spence provides a game theoretical derivation of the market demand function for status goods with respect to level and distribution of income in the considered economy: If (1) the price is low, everyone buys the good; if (2) the price is high, only the rich buy the good (a status good in a narrow sense). If (3) the price is located in a very high as well as in a middle range, demand drops. In addition this approach allows conclusions about the potential welfare improving impact of conspicuous consumption.

1 Introduction
In the history of economic thought status seeking and conspicuous consumption play an ancillary role. Among economists, only Thorstein B. Veblen (*1857, †1929) achieved a prominent profile. In his Theory of the Leisure Class he described the conspicuous consumption of the pecuniary upper class [Veblen

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Today, Veblen's continuing popularity derives less from his model of social evolution, but rather from Harvey Leibenstein's contribution *Bandwagon, Snob, and Veblen Effects in the Theory of Consumers' Demand*. There, because of the lack of a better term, Leibenstein named the case of a partly upward sloping demand function the *Veblen Effect* [Leibenstein 1950, 203].

More recently, status seeking and conspicuous consumption as interpersonal effects have emerged as an important factor in consumption theory. Already, Frank [1985], Ireland [1994] as well as Corneo and Jeanne [1997] analyse status seeking and conspicuous consumption in rather classical consumer model frameworks. Following the intuition of Hirsch [1976] that status seeking can be seen as a zero sum game, they claim that the conspicuous consumption of status goods is *social waste* and recommend a *luxury tax* to internalise the negative external effects of consumption. Similar results are found by Jaramillo and Moizéau [2003] as well as Hopkins and Kornienko [2004].

Unlike these approaches, in the models of Bagwell and Bernheim [1996], Cole et al. [1995], Haucap [2001] and Pesendorfer [1995] demonstrative consumption is seen as a useful signaling device in the initiation of social contacts. Furthermore Haucap [2001] demonstrates that demonstrative consumption as a signal in social interaction may be welfare improving. However, the derivation of the market demand function for status goods is not presented there.

The model presented here closes this gap in the tradition of Spence's *Job Market Signaling* [Spence 1973] and provides a game theoretical derivation of the market demand function for status goods. Whether a status good can be a distinctive signal or not depends on its price and the income level and distribution in the considered society.

We will proceed as follows: In section 2 a simple mating model without status signaling is developed, which serves as a benchmark in the following analysis of the welfare. In section 3 individuals have the option to demonstrate their status by conspicuous consumption. Section 4 shows the comparative welfare analysis, before the market demand function for status goods is presented in section 5. Section 6 concludes.


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1 For a concise and worthwhile reading overview of the related literature see the introduction of Hopkins and Kornienko [2004].

2 A World without Status Signaling

Consider a economy with two types of individuals $H$ and $L$. The individuals earn different incomes $w_i$, with $w_H > w_L$. The population shares of the two types are common knowledge and denoted by $q_H$ and $q_L = (1 - q_H)$. Each individual knows her own income, but cannot observe the income of the others directly. Hence, the situation is characterised by asymmetric information. Furthermore, individuals do have the option to enter a partnership or to stay alone. If an individual enters a partnership, she obtains fifty percent of the income of the couple. Thus the payoffs in a partnership result as follows:

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<td>$H$</td>
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<td>$L$</td>
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If the individual stays alone, she can dispose of the full amount of her own income. But in this case psychic costs of being alone $c_A$ emerge. Otherwise, if an existing partnership is dissolved, psychic costs of separating $c_T$ emerge. Both kinds of psychic costs are independent of the type of individual.

In a world without status signaling individual $i$ only consumes the numéraire $y$. The price of the numéraire is normalised to 1. The individual spends her whole income to consume $y$. So, the very simple resource constraint of the individuals is

$$w_i = y_i.$$  \hfill (1)

Only the consumption of the numéraire $y$ enters the utility function, so the utility function for each individual $i$ is given by:\footnote{The assumption of homogenous preferences holds. Furthermore, the standard assumption of a positive but decreasing marginal utility $0 < \alpha < 1$ holds; $\frac{\partial u_i(y_i)}{\partial y_i} > 0$, $\frac{\partial^2 u_i(y_i)}{\partial y_i^2} < 0$.}

$$u_i = f(y_i) = y_i^\alpha.$$  \hfill (2)

Substituting (1) into (2) leads to the utility function

$$u_i = w_i^\alpha.$$  \hfill (3)

2.1 The Desire for Partnership

In the present model, the individual desire for partnership results from the comparison between the expected utility in a partnership and the certain utility of being alone. Index $A$ denotes that individual $i$ stays alone. Index $P$ denotes that individual $i$ enters a partnership. The expected utility from partnership is
\[ E(u_{i,P}) = q_H \left( \frac{w_i + w_H}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_i + w_L}{2} \right)^\alpha. \]  

(4)

If an individual stays alone, her certain utility is

\[ u_{i,A} = w_i^\alpha - c_A. \]

(5)

The individual \( i \) desires a partnership, if

\[ E(u_{i,P}) \geq u_{i,A} \]

(6) The individual \( i \) wants to stay alone, if

\[ E(u_{i,P}) < u_{i,A} \]

(7)

holds.

Substituting (4) and (5) into (6) as well as algebraic transformation lead to

\[ c_A \geq w_i^\alpha - q_H \left( \frac{w_i + w_H}{2} \right)^\alpha - (1 - q_H) \left( \frac{w_i + w_L}{2} \right)^\alpha. \]

(8)

Hence, individuals of type \( H \) desire a partnership, if

\[ c_A \geq (1 - q_H) \left( \frac{w_H}{2} \right)^\alpha - \left( \frac{w_H + w_L}{2} \right)^\alpha \]

holds, and individuals of type \( L \) desire a partnership, if

\[ c_A \geq q_H \left( \frac{w_L}{2} \right)^\alpha - \left( \frac{w_L + w_H}{2} \right)^\alpha \]

(9)

holds. Note that the right hand side of inequation (10) is negative, because \( w_H > w_L \). This means that individuals of type \( L \) desire a partnership even if the psychic costs of being alone are negative, in other words even if they obtain a payoff from being alone. In the present model, individuals of type \( L \) desire a partnership at lower psychic costs of being alone than individuals of type \( H \). This is illustrated by Figure 1. Proposition 1 summarises the resulting conditions.

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\(^4\)The assumption that individuals desire a partnership if \( E(u_{i,P}) = u_{i,A} \), leads to the exact definition of equilibria.
Proposition 1  (a) If psychic costs of being alone $c_A$ are low and $c_A < q_H (w_L^0 - \frac{(w_L + w_H)}{2})$ holds, no individual desires a partnership.

(b) However, if $q_H (w_L^0 - \frac{(w_L + w_H)}{2}) \leq c_A < (1 - q_H)(w_H^0 - \frac{(w_H + w_L)}{2})$ holds, only individuals of type $L$ desire a partnership.

(c) If psychic costs of being alone $c_A$ are high and $c_A \geq (1-q_H)(w_H^0 - \frac{(w_H + w_L)}{2})$ holds, all individuals desire a partnership.

Proof. (a) In this case, individuals of type $L$ represent the critical type, because they desire a partnership at lower psychic costs of being alone $c_A$ than individuals of type $H$. According to (7) individuals of type $L$ desire a partnership if

$$u_{L,A} = w_L^0 - c_A > q_H (\frac{w_L + w_H}{2})^\alpha + (1 - q_H)(\frac{w_L + w_H}{2})^\alpha = E(u_{L,P})$$

(11) holds. Algebraic transformation of (11) leads to Proposition 1(a).

(b) According to (7) individuals of type $H$ want to stay alone, if

$$u_{H,A} = w_H^0 - c_A > q_H (\frac{w_L + w_H}{2})^\alpha + (1 - q_H)(\frac{w_H + w_L}{2})^\alpha = E(u_{H,P})$$

(12) holds. According to (6) individuals of type $L$ desire a partnership, if

$$u_{L,A} = w_L^0 - c_A \leq q_H (\frac{w_L + w_H}{2})^\alpha + (1 - q_H)(\frac{w_L + w_H}{2})^\alpha = E(u_{L,P})$$

(13) holds. Algebraic transformation of (12) and (13) lead to Proposition 1(b).

(c) In this case, individuals of type $H$ represent the critical type, because they do not desire a partnership at lower costs of being alone $c_A$ than individuals of type $L$. According to (6) individuals of type $H$ desire a partnership, if

$$u_{H,A} = w_H^0 - c_A \leq q_H (\frac{w_L + w_H}{2})^\alpha + (1 - q_H)(\frac{w_H + w_L}{2})^\alpha = E(u_{H,P})$$

(14) holds. Algebraic transformation of (14) leads to Proposition 1(c). ■

2.2 Equilibria in a World without Status Signaling

In a world without status signaling, only individuals who desire a partnership mate. If an individual desires a partnership, she is randomly matched with a partner who desires a partnership too. After the individual has entered the partnership she compares the certain utility from the partnership with the expected utility from restarting the mating game after separation $E(u_{i,T})$. Basically, a gain by restarting the mating game is only possible, if individual $i$ was matched with an individual of type $L$ before. In this case individual $i$ has an incentive to dissolve the partnership, if
Figure 1: The Desire for Partnership

\[ E(u_{i,T}) = q_H \left( \frac{w_i + w_H}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_i + w_L}{2} \right)^\alpha - c_T \tag{15} \]

holds. An equilibrium emerges, if no individual can make herself better off by leaving the equilibrium, in other words by dissolving the partnership. The welfare depends on the question which individuals enter a partnership and which not.

**Definition 1** The welfare measure \( W \) is defined by the ratio of the absolute value of the welfare and the number of individuals in the considered economy. Thus \( W \) is the welfare per capita. With respect to the different groups of individuals, \( W_i \) represents the relative welfare contribution of the group of individuals of type \( i \in \{L, H\} \) per capita of the whole population

\[ W_i = q_i u_i. \tag{16} \]

The welfare of the whole economy is defined by

\[ W = W_H + W_L. \tag{17} \]

Proposition 2 illustrates equilibria in a world without status signaling as well as the corresponding welfare.

**Proposition 2** (a) If, in a world without status signaling, \( c_A < q_H \left( \frac{w_i + w_H}{2} \right)^\alpha \) holds, an equilibrium emerges in which all individuals stay alone and welfare is \( W_{H,A,L,A} = q_H w_H^o + (1 - q_H) w_L^o - c_A \).

(b) However, if \( q_H \left( \frac{w_i + w_H}{2} \right)^\alpha \leq c_A < (1 - q_H) \left( \frac{w_H + w_L}{2} \right)^\alpha \) holds, an equilibrium emerges in which only individuals of type \( L \) enter a partnership. In this case welfare is \( W_{H,A,L,P} = q_H w_H^o + (1 - q_H) w_L^o - q_H c_A \).
(c) If \( c_A \geq (1-q_H)(w_H^\alpha - (\frac{w_H+w_L}{2})^\alpha) \) and in addition \( c_T \geq q_H(w_H^\alpha - (\frac{w_H-w_L}{2})^\alpha) \) hold, an equilibrium emerges in which all individuals enter a partnership. The welfare is \( W_{H,P,L,P} = q_H w_H^\alpha + (1-q_H)w_L^\alpha \).

(d) \( W_{H,P,L,P} > W_{H,A,L,P} > W_{H,A,L,A} \) holds.

**Proof.** (a) As shown in Proposition 1(a) no individual desires a partnership, if \( c_A < q_H(w_L^\alpha - (\frac{w_L+w_H}{2})^\alpha) \) holds. As a result, all individuals stay alone. According to Definition 1 the welfare per capita is given by

\[
W_{H,A,L,A} = q_H w_H^\alpha + (1-q_H)w_L^\alpha - c_A. \tag{18}
\]

(b) As shown in Proposition 1(b) only individuals of type \( L \) desire a partnership, if \( q_H(w_L^\alpha - (\frac{w_L+w_H}{2})^\alpha) \leq c_A < (1-q_H)(w_H^\alpha - (\frac{w_H+w_L}{2})^\alpha) \) holds. Hence, they are matched only with other individuals of type \( L \). According to Definition 1 the welfare per capita is given by

\[
W_{H,A,L,P} = q_H w_H^\alpha + (1-q_H)w_L^\alpha - q_H c_A. \tag{19}
\]

(c) As shown in Proposition 1(c) all individuals desire a partnership, if \( c_A \geq (1-q_H)(w_H^\alpha - (\frac{w_H+w_L}{2})^\alpha) \) holds. According to (4) individuals expect a specific utility from partnership. If an individual \( i \) is matched with a partner of type \( L \), the expectations of \( i \) are not fulfilled and she tends to dissolve the partnership. In this case, individuals of type \( H \) are the critical type, because they dissolve a partnership at lower separation costs \( c_T \) than individuals of type \( L \).\(^5\) Individuals of type \( H \) have no incentive to dissolve a partnership with an individual of type \( L \), if

\[
u_{H,P} = (\frac{w_H+w_L}{2})^\alpha \geq q_H(\frac{w_H+w_H}{2})^\alpha + (1-q_H)(\frac{w_H+w_L}{2})^\alpha - c_T = E(u_{H,T})
\]

holds. In this case an equilibrium emerges. Algebraic transformation of (20) leads to Proposition 2(c). According to Definition 1 the welfare per capita is given by

\[
W_{H,P,L,P} = q_H w_H^\alpha + (1-q_H)w_L^\alpha. \tag{21}
\]

(d) Proposition 2(d) results directly from comparison between (18), (19) and (21).

In a world without status signaling and low psychic costs of being alone \( c_A < q_H(w_L^\alpha - (\frac{w_L+w_H}{2})^\alpha) \) welfare is given by the average utility minus the costs of aloneness for the whole population \( c_A \).

\(^5\)Individuals of type \( H \) dissolve a partnership at lower separation costs \( c_T \) because of their higher income and the assumption of a positive but decreasing marginal utility. As a result of this, their risk of utility loss with separation and restarting the mating game is lower than the risk of individuals of type \( L \).
If the psychic costs of being alone $c_A$ are higher and $q_H(w_H^A - (\frac{w_H + w_L}{2})^\alpha) \leq c_A < (1 - q_H)(w_H^0 - (\frac{w_H + w_L}{2})^\alpha)$ holds, welfare is given by the average utility minus the costs of being alone for type $H$ individuals $q_Hc_A$. In this case, no psychic costs of being alone emerge among individuals of type $L$.

If the psychic costs of being alone are high and $c_A \geq (1 - q_H)(w_H^0 - (\frac{w_H + w_L}{2})^\alpha)$ holds, welfare per capita is given by the average utility. In this case all individuals enter a partnership and no psychic costs of being alone emerge.

### 3 A World with Status Signaling

In a world with status signaling individuals have the option to demonstrate their income by the conspicuous consumption of status goods. Thus status signaling has an important effect on the mating game: If an individual signals, she is matched with another individual, who signals too. If an individual does not signal, despite the option to do so, she expects with a probability of 1 to be matched with an individual of type $L$.

In the considered society only one good is established as a status signal. The status good only works as a status signal, is without intrinsic value and does not influence the individual’s utility directly. Each individual purchases at most one unit of the status good $x$, at the price $p_S \geq 0$, so that $x = \{0, 1\}$ holds. If an individual purchases the status good $x$, beside this, she only consumes the numéraire $y$. However, if the individual does not purchase the status good, she spends her whole income on the consumption of $y$. Only the consumption of the numéraire $y$ causes utility.

#### 3.1 Signaling Costs

In a world with status signaling the budget constraint of the individuals is

$$w_i = p_S + y_i.$$  \hspace{1cm} (22)

The utility function (3) is already known:

$$u_i = f(y_i) = y_i^\alpha.$$  

Now, $y_i$ in (3) can be substituted by (22). This leads the following utility function:

$$u_i = (w_i - p_S)^\alpha.$$  \hspace{1cm} (23)

If an individual signals by means of conspicuous consumption, signaling costs $c_{S,i}$ arise. These signaling costs $c_{S,i}$ reflect the opportunity costs in terms of directly utility enhancing consumption. This results in a utility loss $\Delta u_i$.\footnote{See Corneo and Jeanne [1997, 59].} The signaling costs $c_{S,i}$ are given by
As one can see, the signaling costs $c_{S,i}$ depend on the income $w_i$ and so, depend on the type of consumer. Because of the assumption of a positive but decreasing marginal utility

$$c_{S,H} < c_{S,L}$$

holds. As a result, the ostentatious display of wealth is cheaper for individuals of type $H$ than for individuals of type $L$. With this, the approach presented here provides a utility theoretical explanation for the validity of the single crossing property in the present mating game.

3.2 Signaling Decision

Depending on the signaling decision of the individuals pooling and separating equilibria can emerge.

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7See the illustration in Figure 2. The single crossing property is the condition for the possible emergence of separating equilibria in signaling games. It is also well known as sorting condition, constant sign condition or Spence-Mirrlees condition [Fudenberg/Tirole 1991, 259].
3.2.1 Pooling Equilibrium

In a pooling equilibrium individuals of both types signal. In this case an individual of type $i$ has an incentive to signal, if

$$E(u_{i, \text{pool}}) = q_H \left( \frac{w_i + w_H}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_i + w_L}{2} \right)^\alpha - c_{S,i}$$

$$\geq \left( \frac{w_i + w_L}{2} \right)^\alpha = u_{i,L}. \quad (26)$$

holds. If the individual $i$ is matched with a partner of type $L$, her expectations are not fulfilled. Therefore, she tends to dissolve the partnership, restart the mating game and signal again. However, in this case psychic costs of separation $c_T$ emerge. The individual $i$ only has an incentive to stay inside the partnership, if

$$u_{i, \text{pool}} \geq q_H \left( \frac{w_i + w_H}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_i + w_L}{2} \right)^\alpha - c_{S,i} - c_T = E(u_{i,T}^{\text{pool}})$$

holds. Thereby, individuals of type $H$ are the critical type, because they leave the partnership and so the pooling equilibrium at lower separation costs $c_T$, than individuals of type $L$.\(^8\) Algebraic transformation of (27) leads to the stability condition for pooling equilibria:

$$c_T(p_S) \geq q_H \left( \frac{w_H}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_H + w_L}{2} \right)^\alpha - w_H - (w_H - p_S)^\alpha. \quad (28)$$

The separation costs $c_T$, which are sufficient for the stability for the pooling equilibrium and thus for the stability for partnerships, is dependent on the price of the status good $p_S$. If (26) and (28) hold, an equilibrium emerges in which both the individuals of type $H$ as well as the individuals of type $L$ signal.

3.2.2 Separating Equilibrium

In the separating equilibrium only individuals of type $H$ signal and individuals of type $L$ do not. Individuals of type $H$ have an incentive to signal, if

$$u_H^{\text{sep}} = \left( \frac{w_H + w_H}{2} \right)^\alpha - c_{S,H} \geq \left( \frac{w_H + w_L}{2} \right)^\alpha = u_{H,L} \quad (29)$$

holds. With this, the utility of an individual of type $H$ from a partnership with a partner of type $L$ is not bigger, than her utility from a partnership with a partner of type $H$ minus the type specific signaling costs.

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\(^8\)As before, individuals of type $H$ dissolve a partnership at lower separation costs $c_T$ because of their higher income and the assumption of a positive but decreasing marginal utility. Now, beside their lower risk of utility loss, their signaling costs $c_{S,H}$ are lower than the signaling costs of $L$-type individuals $c_{S,L}$ too.
However, individuals of type \( L \) have no incentive to signal, if
\[
u_{L}^{ep} = \left(\frac{w_{L} + w_{H}}{2}\right)^{\alpha} - c_{S,L} < \left(\frac{w_{L} + w_{H}}{2}\right)^{\alpha} = u_{L,L}
\] holds. With this, the utility of an individual of type \( L \) from a partnership with a partner of type \( L \) is bigger than her utility from a partnership with a partner of type \( H \) minus the type specific signaling costs.

If (29) and (30) hold, an equilibrium emerges in which only the individuals of type \( H \) signal.

### 3.3 Sequence of Moves

The game unfolds as follows:

1. According to the probability distribution \( \{ q_{H}; 1 - q_{H} \} \) the types of individuals are randomly assigned. Individuals know their own type, but the type is not directly observable by the others. However, the good which is established as a status good, its price \( p_{S} \) as well as the probability distribution \( \{ q_{H}; 1 - q_{H} \} \) are common knowledge.

2. Individuals compare their utility from being alone with their expected utility from partnership and decide to initiate a partnership or not.

3. If the individuals decide to stay alone, the mating game stops. However, if the individuals decide to initiate a partnership, in a further step they compare the certain utility from a partnership with a partner of type \( L \) with the expected utility from partnership in the pooling equilibrium or certain utility from partnership in the separating equilibrium with regard to the type specific signaling costs and decide to buy the status good or not \( x = \{ 0, 1 \} \).

4. According to the matching technology in the pooling equilibrium each individual is randomly matched with a partner; in the separating equilibrium each individual of type \( H \) is matched with a partner of type \( H \) and each individual of type \( L \) is matched with a partner of type \( L \).

5. Each individual compares the utility obtained with the expected utility from dissolving the partnership and restarting the mating game. Then, the individual decides to stay in the partnership or not.

6. Final Payoffs accrue.

Whether pooling equilibrium oder separating equilibrium emerge depends on the price of the status good. Proposition 3 encapsulates this.

**Proposition 3** (a) If in a world with status signaling
\[
0 \leq p_{S} \leq w_{L} - \sqrt{w_{L}^{\alpha} + q_{H} w_{L}^{\alpha} - (w_{L}^{\alpha} + w_{H}^{\alpha})^{\alpha}}, \text{ the stability condition } c_{T}(p_{S}) \geq q_{H} (w_{H}^{\alpha} - (w_{L}^{\alpha} + w_{H}^{\alpha})^{\alpha}) - (w_{H}^{\alpha} - (w_{H} - p_{S})^{\alpha}), \text{ and the budget condition } \\
\alpha \leq \ln \left( \frac{1 + x}{2} \right) / \ln \left( \frac{w_{H}^{\alpha} + w_{L}^{\alpha}}{2w_{L}} \right) \text{ hold, a pooling equilibrium emerges in which all individuals purchase the status good. In this case welfare is}
\]
\[ W^{pool} = q_H w^*_H + (1 - q_H) w^*_L - [q_H c_{S,H} + (1 - q_H) c_{S,L}] . \]

(b) However, if
\[ w_L - \sqrt{2 w^*_L - \left( \frac{w_L + w_H}{2} \right)^\alpha} \leq p_S \leq \left( \frac{w_H + w_L}{2} \right)^\alpha, \]
and the budget condition 
\[ \alpha \leq \ln(2)/\ln(\frac{w_H + w_L}{w_H + w_L}) \]
hold, a separating equilibrium emerges in which only individuals of type \( H \) purchase the status good. In this case welfare is
\[ W^{sep} = q_H w^*_H + (1 - q_H) w^*_L - q_H c_{S,H} . \]

(c) No signaling equilibria emerge, if
\[ w_L - \sqrt{w^*_L - q_H (w^*_L - \left( \frac{w_L + w_H}{2} \right)^\alpha)} \leq p_S \leq w_L - \sqrt{2 w^*_L - \left( \frac{w_L + w_H}{2} \right)^\alpha} \]
or
\[ p_S > \left( \frac{w_H - w_L}{2} \right) \]
holds.

**Proof.** (a) In this case individuals of type \( L \) are the critical type, because they leave the pooling equilibrium at a lower price of the status good \( p_S \) than individuals of type \( H \). According to (26), individuals of type \( L \) have an incentive to signal, if
\[ E(u^*_L) = q_H \left( \frac{w_H + w_L}{2} \right)^\alpha + (1 - q_H) \left( \frac{w_H + w_L}{2} \right)^\alpha - c_{S,L} \]
holds. Substituting (24) into (31) and algebraic transformation lead to Proposition 3(a). Inequality (28) is the stability condition for the equilibrium. The budget condition \( \alpha \leq \ln(1 + q)/\ln(\frac{w_H + w_L}{2 w_L}) \) prevents that the willingness to pay of individuals of type \( L \) may exceed their budget in the case of low income \( w_L < w_H \left( \frac{q}{1 - q} \right) \), so that (22) holds.\(^9\) According to Definition 1 welfare is given by
\[ W^{pool} = q_H w^*_H + (1 - q_H) w^*_L - [q_H c_{S,H} + (1 - q_H) c_{S,L}] . \]

(b) A separating equilibrium emerges, if individuals of type \( H \) signal and individuals of type \( L \) do not. According to (29) individuals of type \( H \) have an incentive to signal, if
\[ u^{sep}_H = \left( \frac{w_H + w_L}{2} \right)^\alpha - c_{S,H} \geq \left( \frac{w_H + w_L}{2} \right)^\alpha = u_{H,L} \]
holds. According to (30) individuals of type \( L \) do not have an incentive to signal, if
\[ u^{sep}_L = \left( \frac{w_L + w_H}{2} \right)^\alpha - c_{S,L} < \left( \frac{w_L + w_H}{2} \right)^\alpha = u_{L,L} \]
holds. Substituting (24) into (33) and (34) as well as algebraic transformation lead to Proposition 3(b). The budget condition \( \alpha \leq \ln(2)/\ln(\frac{w_H + w_L}{2 w_L}) \) prevents that the willingness to pay of individuals of type \( L \) may exceed their budget.

\(^9\)In parametric constellations \( w_L > w_H \left( \frac{q}{1 - q} \right) \) the condition is already fulfilled by the standard utility assumption \( 0 < \alpha < 1 \).
constraint in the case of low income \( w_L < w_H / 3 \), so that (22) holds. According to Definition 1 welfare is given by

\[
W^{sep} = q_H w_H^0 + (1 - q_H) w_L^0 - q_H c_{S,H}.
\]

(c) Proposition 3(c) results directly from (a) and (b).

\section{Welfare Comparison}

Now, no less than five different cases are to be compared with respect to the welfare: (a) a world without status signaling, where all individuals stay alone, (b) a world without status signaling and only individuals of type \( L \) entering a partnership, (c) a world without status signaling and all individuals entering a partnership, (d) a world with status signaling and a pooling equilibrium and (e) a world with status signaling and a separating equilibrium.

The following table summarises the welfare of the five cases.

<table>
<thead>
<tr>
<th>Case</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( W_{L,A;H,A} = q_H w_H^0 + (1 - q_H) w_L^0 - c_A )</td>
</tr>
<tr>
<td>(b)</td>
<td>( W_{L,P;H,A} = q_H w_H^0 + (1 - q_H) w_L^0 - q_H c_A )</td>
</tr>
<tr>
<td>(c)</td>
<td>( W_{L,P;H,P} = q_H w_H^0 + (1 - q_H) w_L^0 )</td>
</tr>
<tr>
<td>(d)</td>
<td>( W^{pool} = q_H w_H^0 + (1 - q_H) w_L^0 - [q_H c_{S,H} + (1 - q_H) c_{S,L}] )</td>
</tr>
<tr>
<td>(e)</td>
<td>( W^{sep} = q_H w_H^0 + (1 - q_H) w_L^0 - q_H c_{S,H} )</td>
</tr>
</tbody>
</table>

\begin{proof}
(a) Status signaling is welfare improving, if \( c_A < q_H (w_H^0 - (\frac{w_L + w_H}{2})^\alpha) \) holds and in addition \( c_A > q_H c_{S,H} + (1 - q_H) c_{S,L} \) in the pooling equilibrium or \( c_A > q_H c_{S,H} \) in the separating equilibrium holds.

(b) Status signaling is also welfare improving, if \( q_H (w_L^0 - (\frac{w_L + w_H}{2})^\alpha) \leq c_A < (1 - q_H) (w_H^0 - (\frac{w_H + w_L}{2})^\alpha) \) holds and in addition \( q_H c_A > q_H c_{S,H} + (1 - q_H) c_{S,L} \) in the pooling equilibrium or \( q_H c_A > q_H c_{S,H} \) in the separating equilibrium holds.
\end{proof}

Proof. (a) As shown in Proposition 1(a), no individual desires a partnership, if \( c_A < q_H (w_H^0 - (\frac{w_L + w_H}{2})^\alpha) \) hold. In this case psychic costs of being alone \( c_A \) emerge in the whole population. From the table directly follows that \( W^{pool} > W_{L,A;H,A} \), if

\[
c_A > q_H c_{S,H} + (1 - q_H) c_{S,L}
\]

holds and that \( W^{sep} > W_{L,A;H,A} \), if

\[
c_A > q_H c_{S,H}
\]

holds.

(b) As shown in Proposition 1(b), only individuals of type \( L \) desire a partnership, if \( q_H (w_L^0 - (\frac{w_L + w_H}{2})^\alpha) \leq c_A < (1 - q_H) (w_H^0 - (\frac{w_H + w_L}{2})^\alpha) \) holds. In this case

\footnote{In parametric constellations \( w_L > w_H / 3 \) the condition is already fulfilled by the standard utility assumption \( 0 < \alpha < 1 \).}
psychic costs of being alone $c_A$ only emerge in the type $H$ part of the whole population. From the table directly follows, that $W_{pool}^H > W_{L,P,H}^{L,H}$, if

$$q_H c_A > q_H c_S, H + (1 - q_H) c_{S,L}$$

holds and that $W_{sep}^H > W_{L,P,H}^{L,H}$, if

$$q_H c_A > q_H c_S, H$$

holds. \[\blacksquare\]

In summary it can be ascertained that the lower the costs of being alone $c_A$ are, the more likely status signaling is to be welfare improving. Consequentially, in no way can status signaling be seen as social waste.

5 Market Demand Function for Status Goods

The number of the status goods purchased is given by the part of the population, which buys just one unit of the status good. Thus in a world with status signaling price ranges of the status good have to be considered, in which pooling or separating equilibria emerge. Proposition 5 shows the market demand function for the status good.\[11\]

**Proposition 5** The market demand function for the status good $D(p_S)$ is given by:

$$D(p_S) = \begin{cases} 
1, & \text{if } 0 \leq p_S \leq w_L - \sqrt{w_L^2 + q_H(w_L^2 - (\frac{w_H+w_L}{2})^\alpha)} \\
0, & \text{if } w_L - \sqrt{w_L^2 + q_H(w_L^2 - (\frac{w_H+w_L}{2})^\alpha)} < p_S \\
& < w_L - \sqrt{2w_L^2 - (\frac{w_H+w_L}{2})^\alpha} \\
q_H, & \text{if } w_L - \sqrt{2w_L^2 - (\frac{w_H+w_L}{2})^\alpha} \leq p_S \leq (\frac{w_H-w_L}{2}) \\
0, & \text{if } p_S > (\frac{w_H-w_L}{2}) 
\end{cases}$$

**Proof.** The market demand function for the status good $D(p_S)$ follows directly from Proposition 3 and its proof. \[\blacksquare\]

There exist four ranges of the market demand function for the status good $D(p_S)$ which are determined by the upper and lower price limits of the pooling and separating Equilibrium. Inside the different ranges the demand for the status good is inelastic. The location of those ranges depend on the level of income $w_H$.

---

\[11\] Figure 3 illustrates the market demand function for the status good for $w_H = 100000$, $w_L = 50000$, $q_H = 0.3$ and $\alpha = 0.90$.  

14
Figure 3: Market Demand for Status Goods

and $w_L$ and its distribution in the considered economy $q_H$ and $q_L = (1 - q_H)$ as well as on the exponent of the utility function $\alpha$. The ranges of the market demand function react to changes of the parameters as follows:

<table>
<thead>
<tr>
<th>Reaction of the price ranges</th>
<th>(a) Upper limit of the pooling case</th>
<th>(b) Lower limit of the separating case</th>
<th>(c) Upper limit of the separating case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_H$ $\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
</tr>
<tr>
<td>$w_L$ $\uparrow$</td>
<td>$\uparrow$</td>
<td>$\uparrow$</td>
<td>$\downarrow$</td>
</tr>
<tr>
<td>$q_H$ $\uparrow$</td>
<td>$\uparrow$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$q_L$ $\uparrow$</td>
<td>$\downarrow$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\alpha$ $\uparrow$</td>
<td>$\uparrow$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

The first order derivatives of the $p_S$-limits with respect to the different parameter can be found in the Appendix.

6 Conclusion

In the current mating game, conspicuous consumption of status goods only serves as a signal at the initiation of partnership in a world of asymmetric information. If an individual purchases the status good, signaling costs as utility loss emerge, because of the lower amount of other goods consumed. This definition of signaling costs allows a utility theoretical explanation for the validity of the single crossing property in the present mating game.
With respect to the level and distribution of income in the considered economy we compute a critical price for the status good to act as a distinctive signal (a status good in a narrow sense). Taking this special importance of the price of status goods into account, the model allows the derivation of the market demand function for status goods.

The derived market demand function clearly differs from the standard case: If (1) the price is low, a pooling equilibrium emerges and everyone in the considered economy buys the good. In this case the good is no status good in a narrow sense, because it does not act as a distinctive signal. If (2) the price is high, a separating equilibrium emerges and only the rich part of the population purchases the good. In this case the good is a status good in a narrow sense. If (3) the price is located in a very high as well as in a middle range, demand drops. The location of the price ranges in which pooling or separating equilibria emerge depend on the level and distribution of income in the considered economy.

Interpreting demonstrative consumption as a useful signaling device in the initiation of social contacts in a world of asymmetric information this approach allows conclusions about the potential welfare improving impact of conspicuous consumption. Consequently, it has to be stated that in no way status signaling can be seen as social waste. Taking these results into account, recommendations by numerous economists to prevent the welfare losses of conspicuous consumption by introducing a luxury tax are highly questionable.

The present model is for sure quite basic. It only considers two groups of individuals and only one status good. Nevertheless, the basic mechanisms of status driven demand were presented on a market level. With regard to future research the model could be developed into a more sophisticated model with \( n \) types of individuals and \((n - 1)\) status goods.

Another challenge is the empirical verification of the market demand function for status goods. A starting point could be the contribution of Basmann, Molina and Slottje [1983/1988], Phillips and Slottje [1983] as well as Creedy and Slottje [1991]. The authors show empirically Veblen Effects as the positive price dependency of demand for commodities. However, whether this special price dependency refers to interpersonal consumption effects stays ambiguous there.

References


Appendix

Note, that all parameter $w_H, w_L, q_H, \alpha > 0$.

(a) The upper limit of the pooling case is given by

$$p_S = w_L - \sqrt{w_L^\alpha + q_H(w_L^\alpha - \frac{w_L + w_H}{2})}.$$

Note, that in the pooling equilibrium the budget condition $\alpha < \ln(\frac{1-q_H}{q_H})/\ln(\frac{w_H+w_L}{2w_L})$ holds, so that $w_L^\alpha + q_H(w_L^\alpha - \frac{w_L + w_H}{2}) > 0$.

The first order derivatives of $p_S$ with respect to $w_H$, $w_L$, $q_H$ and $\alpha$ are given as follows:

$$\frac{\partial p_S}{\partial w_H} = \frac{1}{2} q_H(\frac{w_L + w_H}{2})^{\alpha-1}.$$

(41)

$$\frac{\partial p_S}{\partial w_L} = 1 - (w_L^{\alpha-1} + q_H(w_L^{\alpha-1} - \frac{1}{2}(w_L + w_H)\alpha^{-1})),$$

(42)
If \( w_L \) is low, \( \frac{\partial p_S}{\partial w_L} > 0 \) holds. If \( w_L \) is high, \( \frac{\partial p_S}{\partial w_L} < 0 \) holds. One can construct parametric constellations for each case.

\[
\frac{\partial p_S}{\partial q_H} = \frac{1}{a} \left( \frac{w_L + w_H}{2} \right)^a - w_L^a . \tag{43}
\]

\[
(w_L^a + q_H (w_L^a - \left( \frac{w_L + w_H}{2} \right)^a))^\left( \frac{1}{a} - 1 \right) > 0 .
\]

\[
\frac{\partial p_S}{\partial \alpha} \begin{aligned}
&= \left( \frac{w_L^a + q_H (w_L^a - \left( \frac{w_L + w_H}{2} \right)^a)}{\alpha^2 (w_L^a + q_H (w_L^a - \left( \frac{w_L + w_H}{2} \right)^a))} \right) \\
&\quad \cdot \alpha (w_H^a + q_H \ln \left( \frac{w_L + w_H}{2} \right)) + \left( (1 + q) w_L^a \ln (w_L) \right) \\
&\quad - \frac{\alpha (w_L^a + q_H (w_L^a - \left( \frac{w_L + w_H}{2} \right)^a))}{\alpha^2 (w_L^a + q_H (w_L^a - \left( \frac{w_L + w_H}{2} \right)^a))} .
\end{aligned} \tag{44}
\]

(b) The lower limit of the separating case is given by

\[
p_S = w_L - \sqrt{2 w_L^a - \left( \frac{w_L + w_H}{2} \right)^a} .
\]

Note, that in the separating equilibrium the budget condition \( \alpha < \ln(2) / \ln(\frac{w_L + w_H}{2w_L}) \) holds, so that \( 2 w_L^a - \left( \frac{w_L + w_H}{2} \right)^a > 0 \).

The first order derivatives of \( p_S \) with respect to \( w_H, w_L, q_H \) and \( \alpha \) are given as follows:

\[
\frac{\partial p_S}{\partial w_H} = \frac{1}{2} \left( \frac{w_L + w_H}{2} \right)^{a-1} . \tag{45}
\]

\[
(2 w_L^a - \left( \frac{w_L + w_H}{2} \right)^a)^\left( \frac{1}{a} - 1 \right) > 0 , \quad \text{and}
\]

\[
\frac{\partial p_S}{\partial w_L} = 1 - (2 w_L^{a-1} - \frac{1}{2} \left( \frac{w_L + w_H}{2} \right)^{a-1}) . \tag{46}
\]

\[
(2 w_L^a - \left( \frac{w_L + w_H}{2} \right)^a)^\left( \frac{a}{a-1} \right) \leq 0 .
\]

If \( w_L \) is low, \( \frac{\partial p_S}{\partial w_L} > 0 \) holds. If \( w_L \) is high, \( \frac{\partial p_S}{\partial w_L} < 0 \) holds. One can construct parametric constellations for each case.
\[ \frac{\partial p_S}{\partial q_H} = 0. \tag{47} \]

\[ \frac{\partial p_S}{\partial \alpha} = \left( \frac{2w_L^\alpha - (\frac{w_L + w_H}{2})^\alpha}{\alpha^2(2w_L^\alpha - (\frac{w_L + w_H}{2})^\alpha)} \right) \ln \left( \frac{w_L + w_H}{2} \right) + \frac{\alpha((\frac{w_L + w_H}{2})^\alpha \ln(\frac{w_L + w_H}{2}) - 2w_L^\alpha \ln(w_L))}{\alpha^2(2w_L^\alpha - (\frac{w_L + w_H}{2})^\alpha)} \ln \left( \frac{w_L + w_H}{2} \right) \right) \ln \left( \frac{w_L + w_H}{2} \right) > 0. \tag{48} \]

(c) The upper limit of the separating case is given by

\[ p_S = \left( \frac{w_H - w_L}{2} \right). \]

The first order derivatives of \( p_S \) with respect to \( w_H, w_L, q_H \) and \( \alpha \) are given as follows:

\[ \frac{\partial p_S}{\partial w_H} = \frac{1}{2} > 0, \quad \frac{\partial p_S}{\partial w_L} = -\frac{1}{2} < 0, \quad \frac{\partial p_S}{\partial q_H} = 0 \quad \text{and} \quad \frac{\partial p_S}{\partial \alpha} = 0. \tag{49} \]
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