Redistributive Taxation, Inequality, and Intergenerational Mobility

Andrea Schneider
Redistributive taxation, inequality, and intergenerational mobility*

Andrea Schneider
Helmut Schmidt University
Department of Economics and Social Sciences
Holstenhofweg 85
D-22043 Hamburg
Fax +49-40-6541-2069
andrea.schneider@hsu-hh.de

October 2007

Abstract

Education decisions determine a great part of future income. This paper argues that if education is financed by parents’ current income a lump-sum tax reduces inequality if all parents have strict investment incentives. However, if some parents are indifferent there is a possible decrease in the wage gap via a contrary indirect tax effect which drops the returns of schooling. Under strict incentives social mobility is not affected, but it increases if skilled parents have weak incentives and decreases if unskilled parents are indifferent in their investment decision.

JEL classification: D31; D91; I21; J24; J62; H23; H31

Keywords: Intergenerational mobility; Inequality; Redistribution; Lump-sum tax

*I thank Christopher P. Cracknell for his helpful comments.
1 Introduction

Education decisions determine a great part of future income\(^1\) and therefore potential inequality in and across generations. The wage gap, needed to induce investment, implies that it is much easier for rich parents than for poor ones to invest in the education of their children. Thus arises the interest in equal access to schooling and policy interventions which increase social mobility and at the same time reduce inequality. While a higher degree of social mobility benefits intergenerational equity a lower inequality ensures greater intragenerational equity.

This paper is related to two strands of the economic literature. First there are a great number of intergeneration models which analyse steady states (SS) with focus on intra- and intergenerational inequality. This body of literature starts with Gary S. Becker. He shows in a paper with Nigel Tomes that there is a unique unequal equilibrium with no mobility (Becker and Tomes 1979). Wages of the skilled and unskilled are exogenously given and not determined by the measures of both occupation types. Inequality in this model is mainly driven by luck. Some other papers which assume wages endogenously determined find, with the assumption of homogeneous agents, a continuum of SSs which mostly are also characterized by inequality and the absence of social mobility (Banerjee and Newman 1993; Galor and Zeira 1993; Freeman 1996; Mookherjee and Ray 2003).\(^2\) In these models the SS is strongly determined by the conditions at the beginning, i.e. there is great historical dependence. But according to Maoz and Moav (1999) and Mookherjee and Napel (2006) these results are strongly connected to the assumption of homogeneous agents. If children are heterogeneous with respect to their inherited talent it becomes possible that a poor parent invests in his highly talented child and also that a rich parent will not invest in his low-talented child. Here, all steady states are unequal and include mobility. They are generically locally unique and under some conditions global uniqueness is provided. However, in both studies talent is identically and independently distributed (i.i.d.). This is at odds with reality\(^3\) and hence Napel and Schneider (2007) show that if child’s ability depends on its par-

\(^{1}\)One of the earliest studies which show a positive effect of schooling on earning is by Mincer (1958). There is also evidence that the return to schooling has increased over the last decades (Blackburn and Neumark 1993).

\(^{2}\)Galor and Zeira (1993) and Mookherjee and Ray (2003) have equal and unequal SSs.

\(^{3}\)See, e.g., Devlin et al. (1997) for an empirical study.
ent’s ability in a Markovian way the number of steady states with mobility (SSM) is still finite and under some conditions there is a unique SSM. They also argue that the location of the SSMs and thus the wage inequality is not influenced by the strength of the talent connection but that a stronger connection between parent’s and child’s ability reduces social mobility. The model of Napel and Schneider (2007) is the closest to the one described in this paper.

The second strand of related literature analyses interactions of income redistribution and the distribution of education. Ulph (1977) and Hare and Ulph (1979) focus on models where education and income redistribution are optimised simultaneously, i.e. the government sets an income tax as well as an optimal scheme of educational provision (Ulph 1977; Hare and Ulph 1979). It is shown that subsidies on education can reinforce the redistribution effect of an income tax (Hare and Ulph 1979). This positive impact of education subsidies is questioned by Fraja (2002). He finds a strict conflict between efficiency and equity. If the government wants to provide incentives for highly talented children to become educated independent of their parents’ income, poor parents with average-talented children make the greatest contribution to the education budget and, surprisingly, the rich families are subsidised most. An additional effect of such subsidies is discussed in a paper by Dur and Teulings (2001). This paper finds two contrary effects. On the one hand rich families benefit more by the subsidies because they use a greater part of the education system; this increases inequality. On the other hand there is an increase in the fraction of educated people which leads to a drop in the return of human capital via a simple substitution effect. The relative skilled wage decreases and so the wage gap, i.e. the inequality, shrinks. Which effect is more important is ambiguous.

In the present paper the government can - similar to Tuomala (1986) - only use the tax instrument. Parents choose in an intergenerational framework whether their children become educated or not. I analyse the impact of a lump-sum tax on different kinds of SSM in this model. We will see that the second effect of the Dur and Teulings (2001) paper also plays an important role in my model. Generally, a lump-sum tax increases the unskilled wage while it decreases the skilled wage. So, on the one hand it becomes easier for poor parents to invest in the education of their children but on the other

---

4The indirect tax effect also plays a crucial role when Konrad and Spadaro (2006) show that not only low-talented but also highly talented agents may like redistribution.
hand the subjective benefit from investment falls. Because the overall effect of such a lump-sum tax on the global structure of SSMs is very complex I look at the local influence on special types of SSM where one or both occupation types can be indifferent with respect to their investment decision. If, in an SSM, all agents have strict investment incentives the tax does not change its location. So, inequality decreases while social mobility stays constant. In contrast, if some agents are indifferent with respect to their investment decision the SSM decreases. Thus there are two contrary effects on the level of inequality. As a direct tax effect inequality is reduced but as an indirect effect the tax brings down the SSM and thus increases the wage gap. Which tax effect is more important is not clear. While social mobility rises if the skilled have weak incentives, it falls if the unskilled are indifferent in their investment decision. Thus the impact of a lump-sum tax on the degree of inequality and social mobility strongly depends on the type of SSM.

I set my model in the next section. Section 3 analyses the redistributive effects. Whereas section 3.1 gives some general effects of the tax, section 3.2 studies the local effects on different types of SSM. I look at SSMs where all parents have strict investment incentives in subsection 3.2.1. SSMs with weak incentives of the skilled and unskilled respectively are studied in subsection 3.2.2 and 3.2.3. Section 4 concludes.

2 Model

I consider an intergeneration model with a continuum of families. At each period \( t = 0, 1, 2, \ldots \) a family consists of an adult and a child. Agents are characterised by their observable abilities. There are \( r \) different types of ability which require costs of education \( 0 < x_1 < x_2 < \ldots < x_r \). The fixed measure of each type in the population is denoted by \( \sigma^i \) with \( i \in \{1, 2, \ldots, r\} \) and \( \sum_i \sigma^i = 1 \). A child’s ability depends on its parent’s ability in a Markovian way. The conditional probability that a parent with education costs \( x^i \) has a child with education costs \( x^{i'} \) is given by \( p_{i \rightarrow i'} \) with \( i, i' \in \{1, 2, \ldots, r\} \) and \( \sum_{i'} p_{i \rightarrow i'} = 1 \) for all \( i \). Each adult supplies one unit of labor as a skilled \( s \) or an unskilled \( n \) worker. Only educated agents can take skilled jobs. To become an unskilled worker no investment in education is necessary. The fraction of skilled agents in period \( t \) is denoted by \( \lambda_t \).

The economy produces a single consumption good with a Cobb-Douglas
technology $H$ using both types of labor. It is

$$H(\lambda_t) \equiv \lambda_t^\gamma (1 - \lambda_t)^{(1-\gamma)} \tag{1}$$

with $\gamma \in (0, 1)$. Skilled jobs must pay a premium wage so that there is investment in education. In an SSM $\lambda^*$ this implies $\lambda^* < \gamma$. In equilibrium, pre-tax wages of the skilled $w^s$ and of the unskilled $w^n$ are given by marginal productivity. Equilibrium wages are given by

$$w^s \equiv w^s(\lambda^*) = \gamma \left( \frac{1-\lambda^*}{\lambda^*} \right)^{(1-\gamma)} \tag{2}$$

and

$$w^n \equiv w^n(\lambda^*) = (1-\gamma) \left( \frac{\lambda^*}{1-\lambda^*} \right)^\gamma. \tag{3}$$

There is also a lump-sum tax $\tau \in [0, 1]$ which is constant over time and aims to reduce the inequality, i.e. the gap between skilled and unskilled wages.\(^5\) In equilibrium post-tax wages $w^s_\tau$ and $w^n_\tau$ are given by

$$w^s_\tau \equiv w^s_\tau(\lambda^*) = (1-\tau)w^s(\lambda^*) + \tau \left( \lambda^* w^s(\lambda^*) + (1-\lambda^*)w^n(\lambda^*) \right) \tag{4}$$

and

$$w^n_\tau \equiv w^n_\tau(\lambda^*) = (1-\tau)w^n(\lambda^*) + \tau \left( \lambda^* w^s(\lambda^*) + (1-\lambda^*)w^n(\lambda^*) \right), \tag{5}$$

with $w^s(\lambda^*)$ and $w^n(\lambda^*)$ as defined in equations (2) and (3). While the wage of the skilled $w^s_\tau$ decreases in $\tau$ the wage of the unskilled $w^n_\tau$ increases.\(^6\)

Beside their own consumption $c_{t,\tau}$, parents are interested in the future income of their children $w^k_{t+1,\tau}$ with $k \in \{s, n\}$. The utility function is assumed to be\(^7\)

$$U(c_{t,\tau}, w^k_{t+1,\tau}) = \ln(\underbrace{w^k_{t+1,\tau} - x I_t}_{c_{t,\tau}}) + \delta \ln(w^k_{t+1,\tau}), \tag{6}$$

where $I_t$ equals 1 if a parent invests in his child and 0 otherwise. The parental altruism is scaled by $\delta \in (0, 1)$.

\(^5\)Note, that the pre-tax inequality decreases in $\lambda$.

\(^6\)It is $\frac{\partial w^s_\tau}{\partial \tau} = \frac{1-\lambda^*}{w^s - w^n} < 0$ and $\frac{\partial w^n_\tau}{\partial \tau} = \lambda^*(w^s - w^n) > 0$.

\(^7\)All later results also hold if the utility function is given by $U(c) = \frac{c^{1-\rho}}{1-\rho}$ with $\rho > 1$. 

---

5Note, that the pre-tax inequality decreases in $\lambda$.

6It is $\frac{\partial w^s_\tau}{\partial \tau} = \frac{1-\lambda^*}{w^s - w^n} < 0$ and $\frac{\partial w^n_\tau}{\partial \tau} = \lambda^*(w^s - w^n) > 0$.

7All later results also hold if the utility function is given by $U(c) = \frac{c^{1-\rho}}{1-\rho}$ with $\rho > 1$. 

5
The subjective benefit from investment given the tax rate $\tau$ is
\[
B_\tau(\lambda_{t+1}) \equiv \delta \left( \ln w_{t+1,\tau}^s - \ln w_{t+1,\tau}^n \right);
\] (7)
the subjective costs from investment are
\[
C_\tau^k(\lambda_t, x) \equiv \ln w_{t,\tau}^k - \ln (w_{t,\tau}^k - x),
\] (8)
where $k \in \{s, n\}$ denotes parent’s occupation and $x$ are the education costs of his child. An agent will invest (or not invest) in the education of his child if the subjective benefit is higher (or smaller) than the subjective costs. When benefit equals costs agents invest with arbitrary probability. The indifference curves of the skilled and unskilled are defined by
\[
I^s_\tau = \{ (\lambda, x) : B_\tau(\lambda) = C^s_\tau(\lambda, x) \}\]
and analogously
\[
I^n_\tau = \{ (\lambda, x) : B_\tau(\lambda) = C^n_\tau(\lambda, x) \}\.
\] (10)
The indifference curves of the skilled and unskilled before (solid lines) and after tax (dashed lines) are illustrated in Figure 1.\(^8\)

Figure 1: Indifference curves of the skilled and unskilled before and after tax

Implementing a lump-sum tax influences subjective benefit and costs of investment in education and therefore maybe the investment incentives. Lemma 1 concludes the influence of a lump-sum tax $\tau$ on the subjective benefit and costs of the skilled and unskilled.

\(^8\)The subscript $\tau$ refers to the after tax case. Index $s$ and $n$ respectively refers to the skilled and unskilled agents.
**Lemma 1** Given a fixed proportion of educated people $\lambda_t$ subjective benefit and subjective costs of the unskilled decrease in $\tau$; subjective costs of the skilled increase in $\tau$.

The lemma is proved directly by calculating the first order derivatives.

Considering Lemma 1 and the special forms of the production and utility functions the net benefit of investment of the skilled $B_{s}(\lambda) - C^{s}_{\tau}(\lambda)$ is smaller after tax; the net benefit of investment of the unskilled $B_{\tau}(\lambda) - C^{\tau}_{\tau}(\lambda)$ is higher after tax for all $\lambda < \hat{\lambda}$ and smaller for all $\lambda > \hat{\lambda}$. The fraction of educated people, where $\tau$ does not influence the net benefit of the unskilled, i.e. $B(\lambda) - C^{n}(\lambda) = B_{\tau}(\lambda) - C^{\tau}_{\tau}(\lambda)$, is denoted by $\hat{\lambda}$.

In each period $t$ the aggregate parental occupation and cost distribution is given by

$$\pi(t) \equiv (\pi_{s1}(t), \pi_{s2}(t), ..., \pi_{s'}(t), \pi_{n1}(t), \pi_{n2}(t), ..., \pi_{n'}(t)). \quad (11)$$

For more details of the case of two different types of ability see also Napel and Schneider (2007). The sum of the first $r$ components of the distribution gives the proportion of educated people $\lambda_t$. The occupation and cost distribution in generation $t + 1$ is determined by the investment decisions of the agents in $t$. This is a heterogeneous Markov chain where the transition matrix is determined by $\lambda_t$.

In my analysis I focus on the influence of a lump-sum tax on equilibria with stationary skill ratios, i.e. $\lambda_t = \lambda_{t+1}$ for all $t$. In this case the transition matrix is stationary and the Markov chain becomes homogeneous. Because a situation without mobility is at odds with reality I restrict the analysis to SSMs, i.e. equilibria with stationary skill ratios in which the measure of unskilled investors is positive and equals the measure of skilled non-investors. There are different SSM types where a type is characterised by the investment decision of the skilled and unskilled concerning all possible costs of education.

---

9The fraction $\hat{\lambda}$ depends on the tax rate $\tau$. The higher $\tau$ the smaller $\hat{\lambda}$. So, the interval where the tax rate increases the net benefit of the unskilled decreases in $\tau$. 

---
3 Influence of redistribution on SSM

3.1 General influence of redistribution

As a result of Mookherjee and Napel (2006) and Napel and Schneider (2007) for a model without redistribution we know that there are never more than $2(r - 1)$ SSMs and that under some conditions a unique SSM occurs.\(^{10}\) However, given the $r$ different cost types and all conditional probabilities $p_{i \rightarrow i'}$, $i, i' \in \{1, 2, ..., r\}$ a lump-sum tax can change this SSM structure extremely. So, while the global influence on the number and kind of SSM is not clear I examine the influence on local points precisely.

For every proportion of educated agents $\lambda$ there exist marginal unskilled investors and marginal skilled non-investors. I will refer to the respective cost types by $x^l$ and $x^h$ where $l, h \in \{1, 2, ..., r\}$ and $x^l < x^h$. If - for a fixed fraction of skilled agents - unskilled (strictly or weakly) invest in cost type $x^l$ they also strictly invest in $x^v$, $v < l$ and do not invest otherwise. Analogously, if skilled do (strictly or weakly) not invest in cost type $x^h$ they also do not invest in $x^w$, $w > h$, and invest for all other cost types. Because of the local view I can reduce the types of SSM to:

Type I: SSM with strict incentives. Skilled do not invest in cost type $x^h$ and unskilled invest in cost type $x^l$.

Type II: SSM with weak incentives of the skilled. Skilled are indifferent facing cost type $x^h$ and unskilled invest in cost type $x^l$.

Type III: SSM with weak incentives of the unskilled. Unskilled are indifferent facing cost type $x^l$ and skilled do not invest in cost type $x^h$.

Type IV: SSM with weak incentives. Skilled and unskilled are indifferent facing their relevant cost types.

Figure 2 illustrates the four types of SSM with the corresponding upward and

---

\(^{10}\)Mookherjee and Napel (2006) show on page 15 that with $r$ discrete cost types up to $2(r - 1)$ SSMs can exist. While their model examines i.i.d. talents Napel and Schneider (2007) find the same results for a model in which parent’s talent determines child’s talent (see the conclusion of their paper). According to the fact that a lump-sum tax only shifts upward and downward flow to the left the results of both papers concerning the number of SSMs also hold for the present model.
downward flows.\textsuperscript{11} Upward flow $u(\lambda^*)$ and downward flow $d(\lambda^*)$ determine social mobility. In an SSM $\lambda^*$ both flows are equal and given by

$$u(\lambda^*) = (1 - \lambda^*)(\sum_{w=1}^{r} \sum_{v=1}^{l-1} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v} + \alpha \sum_{w=1}^{r} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v} + \alpha \sum_{w=1}^{r} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v})$$

(12)

and

$$d(\lambda^*) = \lambda^*(\sum_{w=1}^{r} \sum_{v=h-1}^{r} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v} + \beta \sum_{w=1}^{r} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v} + \beta \sum_{w=1}^{r} \frac{\pi_{w}}{\pi_{v}} + \ldots + \frac{\pi_{w}}{\pi_{v}} p_{w-v})$$

(13)

with appropriate $\alpha$ and $\beta$ in $[0, 1]$ and $l, h$ refering to the respective marginal cost types. According to equations (12) and (13) the upflow is characterised by up to $r$ up jumps and the corresponding down jumps whereas the downflow is a strictly increasing function with up to $r$ up jumps.

![Figure 2: Upward and downward mobility flows for the four types of SSM](image)

The following analysis targets the question how the lump-sum tax $\tau$ influences the inequality and social mobility in the different types of SSM. I only consider cases where the respective SSM type is not destroyed by the tax.\textsuperscript{12}

\textsuperscript{11}The dashed lines represent the upward and downward flow after tax. An SSM type IV is always destroyed by the tax. So, in Figure 2 (d) only the pre-tax SSM can be illustrated.

\textsuperscript{12}For appropriate small tax rates this assumption always holds. Thus we get always at least the marginal effect of introducing a lump-sum tax.
3.2 Influence on special types of SSM

3.2.1 SSM with strict investment incentives

I start with an SSM in which the skilled strictly invest in children with cost types up to \( h - 1 \), the unskilled strictly invest in cost types up to \( l \), and in which there is no investment otherwise.

**Proposition 1** Given an SSM type I the SSM does not change, i.e. \( \lambda^I = \lambda_I^\tau \).

*Proof:* In this case an SSM is given by \( \lambda^I = \sum_{v=1}^I \rho_v + \lambda^I \sum_{w=l+1}^{h-1} \rho_w \). So, \( \lambda^I \) is given by \( \lambda^I = \sum_{v=1}^I \rho_v / (1 - \sum_{w=l+1}^{h-1} \rho_w) \) and is therefore independent of \( \tau \). \( \square \)

Wage inequality in SSM is fixed by the fraction of skilled people \( \lambda^I \) and the lump-sum tax rate \( \tau \). According to Proposition 1 the proportion of skilled people does not change with the tax \( \tau \). So the inequality changes as follows:

**Proposition 2** If the SSM is of type I the degree of inequality after taxation, i.e. the after-tax wage gap, equals \((1 - \tau)\) times the wage gap before taxation.

The proof follows directly by calculating the wage gap after taxation. So, a one percent increase in the tax rate implies a \( \frac{\tau}{1 - \tau} \) percent decrease in the wage gap.

**Proposition 3** If the SSM is of type I social mobility is not influenced by the tax.

*Proof:* Upward and downward flow are only determined by \( \lambda^I \) and the transition probabilities \( p_{i \to i'}, i, i' \in \{1, \ldots, r\} \). According to Proposition 1 neither of these change. \( \square \)

A lump-sum tax in a situation where all agents have strict incentives does not help making more poor children become educated. Just the inequality, i.e. the relative poverty, is reduced.
3.2.2 SSM with weak investment incentives of the skilled

In this type of SSM the skilled are indifferent with respect to their marginal cost type $x^h$. They invest with probability $\beta$ in a child with education costs $x^h$. Let $\lambda^\text{II}$ be the SSM before taxation. An SSM type II is characterised by equal benefit and costs for the skilled, i.e. $B(\lambda^\text{II}) = C^s(\lambda^\text{II}, x^h)$. After tax the equality at $\lambda^\text{II}$ is destroyed because $B(\lambda^\text{II}) > B^\tau(\lambda^\text{II})$ but $C^s(\lambda^\text{II}, x^h) < C^s(\lambda^\text{II}, x^h)$.

**Proposition 4** Given an SSM type II the SSM decreases by the tax, i.e. $\lambda^\text{II} < \lambda^\tau$. The new SSM is defined by\(^{13}\)

$$\lambda^\text{II} = \{ \lambda : B^\tau(\lambda) - C^s(\lambda, x^h) = 0 \}. \quad (14)$$

**Proof:** Given the SSM $\lambda^\text{II}$ before tax it is $B^\tau(\lambda^\text{II}) - C^s(\lambda^\text{II}, x^h) < 0$ after tax. A drop in the skill ratio leads to the equality of benefit and costs after taxation because the incentives of the skilled are strictly decreasing in $\lambda$. \(\square\)

Note, in this case there are two different mechanisms which influence wage inequality in opposite directions. On the one hand - holding the skilled fraction constant - the tax lowers the wage gap. This I will call the direct tax effect. On the other hand the skilled fraction decreases by the tax, so that there is an increase in the wage gap. This effect I will call the indirect tax effect. As stated in Proposition 5 the resulting overall effect is generally not clear.

**Proposition 5** If the SSM is of type II a lump-sum tax $\tau$ reduces inequality if

$$\frac{w^s(\lambda^\text{II}) - w^\tau_s(\lambda^\text{II})}{w^s(\lambda^\text{II}) - w^\tau_s(\lambda^\text{II})} < \frac{1}{1 - \tau} \quad (15)$$

holds, with “>” the wage gap increases, and if “=” holds the tax has no influence on the wage gap.

**Proof:** The right hand side of equation (15) is the proportion of the pre-tax and post-tax wage gap given a fixed SSM $\lambda^\text{II}$. The left side of the equation

\(^{13}\)The explicit reduction can be calculated for a concrete SSM and tax rate but not in a general way because of the given utility and production functions.
is the proportion of the post-tax wage gap with SSM $\lambda^H$ and the smaller post-tax wage gap with SSM $\lambda^H_\tau$ (see Prop. 4). If the left hand side of the equation is smaller (higher) than the right hand side the direct tax effect is stronger (weaker) than the indirect tax effect. If both sides of equation (15) are equal the indirect tax effect balances the direct tax effect. □

So, if skilled agents have weak investment incentives in the SSM there can be the paradox situation that a tax rate that should lower the inequality increases the wage gap.\(^\dagger\)

**Proposition 6** If the SSM is of type II social mobility increases by

\[
\Delta u^H \equiv u(\lambda^H_\tau) - u(\lambda^H) = \sum_{w=l+1}^{h} \sum_{v=1}^{l} (\Delta^H \sigma^w p_{w \rightarrow v} + \sum_{v=1}^{l} (\beta_\lambda^H - \beta_{\tau} \lambda^H_\tau) \sigma^h p_{h \rightarrow v} \quad (16)
\]

where $\beta_{\tau} < \beta$ refers to the market clearing probability that a skilled agent invests in a cost type $x^h$ after tax and $\Delta^H \equiv \lambda^H - \lambda^H_\tau$.

**Proof:** I refer to the smallest $\lambda$ for which unskilled agents have a weak incentive to invest in a cost type $x^l$ by $\underline{\lambda}$. For increasing $\lambda$ there is a $\lambda$ for which the unskilled either additionally want to invest in the cost type $x^{l+1}$, i.e. the upflow jumps to a higher level, or again becomes indifferent for the cost type $x^l$, i.e. the upflow jumps to a lower level. In the interval $(\Delta, \underline{\lambda})$ the upflow decreases linearly by $-\sigma^l$. So, the decrease in $\lambda^H$ by the tax implies an increase in the upflow and therefore, in equilibrium, an increase in the downflow. Focusing on a situation where the SSM type II persists, the fraction of indifferent skilled agents who finally invest shrinks, i.e $\beta_{\tau} < \beta$. According to equation (12) the upward flow is given by

\[
u(\lambda^H) = \sum_{w=l+1}^{h-1} \sum_{v=1}^{l} (1 - \lambda^H) \sigma^w p_{w \rightarrow v} + \sum_{v=1}^{l} \left( (1 - \lambda^H) + (1 - \beta) \lambda^H \right) \sigma^h p_{h \rightarrow v} + \sum_{w=h+1}^{r} \sum_{v=1}^{l} \sigma^w p_{w \rightarrow v}.
\]

\(^\dagger\)This typically happens only for high tax rates.
Thus the difference between the upflow before and after taxation is given by equation (16).

Considering an SSM $\lambda^\text{II}$ where the skilled have weak investment incentives, the tax reduces the net benefit of the skilled so that for given $\lambda^\text{II}$ no skilled would invest in the cost type $x^h$. Thus the downflow would be higher than the upflow and the SSM would be destroyed. Therefore the fraction of educated people decreases until the skilled are again indifferent for the cost type $x^h$. In this new SSM $\lambda^\text{II}_\tau < \lambda^\text{II}$ the upflow is increased and so must also the downflow. The market clearing probability that a skilled agent invests in a child with costs $x^h$ drops to $\beta_\tau < \beta$. All in all, in this situation the fraction of poor children who become educated increases but the aggregated number of educated agents shrinks by the tax.

### 3.2.3 SSM with weak investment incentives of the unskilled

In this situation the skilled do not invest with strict incentives in their marginal cost type $x^h$ and the unskilled are indifferent with respect to their marginal cost type $x^l$. They invest with probability $\alpha$. Let $\lambda^\text{III}$ be the SSM before tax and $\lambda^\text{III}_\tau$ the SSM after tax. The SSM type III implies equal benefit and costs of the unskilled in respect to their marginal cost type, i.e. $B(\lambda^\text{III}) = C^u(\lambda^\text{III}, x^l)$. Analogously to the SSM type II, the fraction of educated agents changes as follows:

**Proposition 7** Given an SSM type III the SSM decreases by the tax, i.e. $\lambda^\text{III}_\tau < \lambda^\text{III}$.

*Proof:* The investment decision of the unskilled with respect to a cost type $x^l$ changes twice. For small $\lambda$ the wage of the unskilled is so low that they do not invest in their children although the benefit of investment would be very high. An increasing $\lambda$ cause that the wage of the unskilled rises. Now unskilled parents invest because of the high benefit of investment. If $\lambda$ increases more the benefit of investment, which is determined by the gap between skilled and unskilled wages, shrinks. So, there is a point where investment is no longer beneficial for the unskilled. Taxation now raises the wage of the unskilled, so that they can benefit from investment in a cost type $x^l$ at a smaller $\lambda$. As a second effect taxation reduces the wage gap at every $\lambda$,
so that the benefit of investment diminishes at a smaller fraction of educated agents. Concluding, both values of \( \lambda \) at which the unskilled are indifferent with respect to a cost type \( x^l \) decrease by the tax. So, the SSM decreases. \( \square \)

The inequality in the SSM type III changes with the tax analogously to the SSM type II (see Prop. 5):

**Proposition 8** If the SSM is of type III a lump-sum tax \( \tau \) decreases the inequality if

\[
\frac{w^s_\tau(\lambda^{III}) - w^n_\tau(\lambda^{III})}{w^s_\tau(\lambda^{III}) - w^n_\tau(\lambda^{III})} < \frac{1}{1 - \tau}
\]  

(17)

holds, with “\( > \)” the wage gap increases, and if “\( = \)” holds the tax has no influence on the wage gap.

The proof goes analogously to the proof of Proposition 5. But in contrast to an SSM type II taxation influences the social mobility in a negative way.

**Proposition 9** If the SSM is of type III social mobility decreases by

\[
\Delta d^{III} \equiv d(\lambda^{III}) - d(\lambda^{III}_\tau) = \sum_{w=\bar{l}}^{\bar{r}} \sum_{v=h}^{r} \Delta \lambda^{III} \sigma^w p_{w\rightarrow v} + \sum_{v=h}^{r} \left( \alpha(1 - \lambda^{III}) - \alpha(1 - \lambda^{III}_\tau) \right) \sigma^l p_{l\rightarrow v}
\]  

(18)

where \( \alpha_\tau < \alpha \) refers to the market clearing probability that an unskilled agent invests in cost type \( x^l \) after tax and \( \Delta \lambda^{III} \equiv \lambda^{III} - \lambda^{III}_\tau \).

*Proof:* Let \( \Delta^{h} \) denote the proportion of educated agents where the skilled are indifferent with respect to the cost type \( x^h \). For increasing \( \lambda \) the downflow increases until the skilled become also indifferent for the lower cost type \( x^{h-1} \) at \( \bar{\lambda}^h \), i.e. the downflow jumps to a higher level, or \( \lambda = \gamma \). In the interval \( (\Delta^{h}, \bar{\lambda}^h) \) the downflow increases linear by \( \sigma^h \). According to Proposition 7 the SSM and therefore the downflow decreases. So, in equilibrium, the upflow must also decrease. In a situation where the SSM type III persists, the fraction of unskilled agents who invest in a cost type \( x^l \) must shrink by the
taxation, i.e. \( \alpha_r < \alpha \). Considering equation (13) and the fact that nobody invests in cost types \( x^h, \ldots, x^r \) the downflow in the SSM is given by

\[
d(\lambda^{\text{III}}) = \sum_{w=1}^{l-1} \sum_{v=h}^{r} \sigma^w p_{w \rightarrow v} + \sum_{v=h}^{r} \left( \lambda^{\text{III}} + \alpha(1 - \lambda^{\text{III}}) \right) \sigma^l p_{l \rightarrow v} + \sum_{w=l+1}^{h-1} \sum_{v=h}^{r} \lambda^{\text{III}} \sigma^w p_{w \rightarrow v}.
\]

Thus the difference in the downflow before and after taxation is given by equation (18).

If the unskilled invest in the cost type \( x^l \) at a whole interval \([\lambda^l, \lambda^l]\) and not only at one point, an SSM type III could appear at both endpoints of this interval.\(^{15}\) If \( \lambda^{\text{III}} = \lambda^l \) no unskilled would invest in the cost type \( x^l \) after tax because of a decrease in the net benefit. So, the upflow would decrease and be smaller than the downflow. The SSM would be destroyed. For decreasing \( \lambda \), the downflow decreases and the upflow increases. That way the difference between downflow and upflow decreases until it diminishes at the new SSM \( \lambda^{\text{III}}_l < \lambda^{\text{III}} \). If \( \lambda^{\text{III}} = \lambda^l \) all unskilled would invest in the cost type \( x^l \). For fixed \( \lambda^{\text{III}} \) the upflow would increase and the pre-tax SSM would diminish. The new SSM \( \lambda^{\text{III}}_l \) occurs at a lower fraction of educated agents with a smaller arbitrary probability that a unskilled agent invest in the cost type \( x^l \). In both cases a redistribution lowers social mobility. So, the tax decreases the probability that a poor child becomes educated.

I will not analyse the SSM type IV because such an SSM only exists under the special condition \( B(\lambda^{IV}) = C^u(\lambda^{IV}, x^l) = C^s(\lambda^{IV}, x^h) \) and this situation is definitely destroyed by the tax. The tax always influences the incentives of the skilled more than these of the unskilled.

\(^{15}\) It is \( \lambda^l < \hat{\lambda} < \lambda^l \) with \( \hat{\lambda} \) as defined on page 7. Therefore \( \lambda^l \) is located in the interval where the tax raises the net benefit of the unskilled whereas \( \lambda^l \) is located in the interval where the tax lowers the net benefit of the unskilled.
4 Concluding remarks

This paper shows that a lump-sum tax which should increase opportunities for poor parents to invest in their children can only definitely reduce inequality if all agents have strict investment incentives. However, if some parents are indifferent with respect to an investment in the education of their children then, as a contradictory effect of the tax, the fraction of skilled people drops. This yields decreased wages for the unskilled\footnote{In contrast, Hendel, Shapiro, and Willen (2005) claim that an increasing number of unskilled workers drives up their wages. They argue that with a high fraction of uneducated agents the average unskilled worker is more likely to be of high ability. Thus, firms are willing to pay more for unskilled workers.} and increased wages for the skilled and so drives up inequality in outcomes. The impact of a lump-sum tax on social mobility depends even more on the type of SSM. In a situation where all parents have strict incentives, the probability that a poor child becomes educated is only determined by the fixed measures of the different types of talent in the population and so is not influenced by the tax. However, if skilled agents are indifferent with respect to their investment decision social mobility increases as result of a reduced proportion of educated people and therefore an increased measure of unskilled investors. In the case where unskilled parents have weak investment incentives, the measure of skilled non-investors and thus social mobility decreases in the SSM.

In general, the model illustrates two contrary impacts of a lump-sum tax on the investment incentives: On the one hand subjective costs of the poor are narrowed but on the other hand also the subjective benefit of schooling is reduced. The overall effect of such a policy on inter- and intragenerational inequality strongly depends on the investment incentives before taxation.

In my model, children’s wages depend only on the education decision of the parents but not on the inherited talent of the child. So, this model does not consider a possible positive effect in aggregated productivity or growth if the most talented agents become educated (see e.g. Hassler and Rodríguez Mora (1998) for such a model). Additionally, a lump-sum tax - although it can not reduce inequality and increase social mobility in all cases- may increase the aggregate utility in a Bentham way. This could be a starting point for further research.
References


Bisher erschienen:
Diskussionspapiere der Fächergruppe Volkswirtschaftslehre

- Schneider, Andrea, Redistributive taxation, inequality, and intergenerational mobility, No. 68, (November 2007).
- Kruse, Jörn, Exklusive Sportfernsehrechte und Schutzlisten, Nr. 67 (Oktober 2007).
- Dewenter, Ralf, Crossmediale Fusionen und Meinungsvielfalt: Eine ökonomische Analyse, Nr. 65 (Oktober 2007).
- Dewenter, Ralf, Justus Haucap & Ulrich Heimeshoff, Regulatorische Risiken in Telekommunikationsmärkten aus institutionenökonomischer Perspektive, Nr. 64 (September 2007).
- Thomas, Tobias, Mating à la Spence: Deriving the Market Demand Function for Status Goods, No. 63 (September 2007).
- Carlberg, Michael, Monetary and Fiscal Policies in the Euro Area, No. 61 (August 2007).
- Zimmermann, Klaus W. & Tobias Thomas, Internalisierung externer Kosten durch Steuern und Verhandlungen: Eine Nachlese, Nr. 60 (Juni 2007), erscheint in Wirtschaftswissenschaftliches Studium (WiSt).
- Dluhosch, Barbara & Klaus W. Zimmermann, Zur Anatomie der Staatsquote, Nr. 58 (Januar 2007).
- Göbel, Markus, Andrea Schneider & Tobias Thomas, Consumer behavior and the aspiration for conformity and consistency, No. 57 (Januar 2007).
- Haucap, Justus & Ralf Dewenter, First-Mover Vorteile im Schweizer Mobilfunk, Nr. 56 (Dezember 2006).
- Kruse, Jörn, Mobilterminierung im Wettbewerb, Nr. 55 (Dezember 2006).
- Dluhosch, Barbara & Klaus W. Zimmermann, Some Second Thoughts on Wagner’s Law, No. 54, (December 2006).
- Napel, Stefan & Andrea Schneider, Intergenerational talent transmission, inequality, and social mobility, No. 52 (October 2006).
Papenfuss, Ulf & Tobias Thomas, Eine Lanze für den Sachverständigenrat?, Nr. 51 (Oktober 2006), erscheint in: Perspektiven der Wirtschaftspolitik.

Kruse, Jörn, Das Monopol für demokratische Legitimation: Zur konstitutionellen Reform unserer staatlichen und politischen Strukturen, Nr. 50 (Juli 2006).

Hackmann, Johannes, Eine reinvermögenszugangstheoretisch konsequente Unternehmensbesteuerung, Nr. 49 (Juni 2006).

Carlberg, Michael, Interactions between Monetary and Fiscal Policies in the Euro Area, No. 48 (March 2006).

Bayer, Stefan & Jacques Méry, Sustainability Gaps in Municipal Solid Waste Management: The Case of Landfills, No. 47 (February 2006).

Schäfer, Wolf, Schattenwirtschaft, Äquivalenzprinzip und Wirtschaftspolitik, Nr. 46 (Januar 2006).


Kruse, Jörn, Zugang zu Premium Content, Nr. 44 (Dezember 2005).

Dewenter, Ralf & Jörn Kruse, Calling Party Pays or Receiving Party Pays? The Diffusion of Mobile Telephony with Endogenous Regulation, No. 43 (November 2005).

Schulze, Sven, An Index of Generosity for the German UI-System. No. 42 (Oktober 2005).


Carlberg, Michael, International Monetary Policy Coordination, No. 39 (March 2005).


Hackmann, Johannes, Die Bestimmung der optimalen Bevölkerungsgröße als (wirtschafts-)ethisches Problem, Nr. 37 (März 2005).


• Josten, Stefan Dietrich & Klaus W. Zimmermann, Unanimous Constitutional Consent and the Immigration Problem, No. 31 (Dezember 2004), erscheint in: Public Choice.

• Bleich, Torsten, Importzoll, Beschäftigung und Leistungsbilanz: ein mikrofundierter Ansatz, Nr. 30 (September 2004).

• Dewenter, Ralf, Justus Haucap, Ricardo Luther & Peter Rötzel, Hedonic Prices in the German Market for Mobile Phones, No. 29 (August 2004), erscheint in: Telecommunications Policy, 2007.

• Carlberg, Michael, Monetary and Fiscal Policy Interactions in the Euro Area, No. 28 (März 2004).


• Kruse, Jörn, Ökonomische Konsequenzen des Spitzensports im öffentlich-rechtlichen und im privaten Fernsehen, Nr. 26 (Januar 2004).


• Haucap, Justus & Tobias Just, Der Preis ist heiß. Aber warum? Zum Einfluss des Ökonomiestudiums auf die Einschätzung der Fairness des Preissystems, Nr. 24 (November 2003), erschienen in Wirtschaftswissenschaftliches Studium (WiSt) 33 (9), 2004, 520-524.

• Dewenter, Ralf & Justus Haucap, Mobile Termination with Asymmetric Networks, No. 23 (October 2003), erschienen unter dem Titel “The Effects of Regulating Mobile Termination Rates for Asymmetric Networks” erschienen in: European Journal of Law and Economics 20, 2005, 185-197.


• Quitzau, Jörn, Erfolgsfaktor Zufall im Profifußball: Quantifizierung mit Hilfe informations-effizienter Wettmärkte, Nr. 20 (September 2003).


• Zimmermann, Klaus W. & Tobias Just, On the Relative Efficiency of Democratic Institutions, No. 16 (July 2003).

• Meyer, Dirk, Die Energieeinsparverordnung (EnEV) - eine ordnungspolitische Analyse, Nr. 14 (Juli 2003).

• Zimmermann, Klaus W. & Tobias Thomas, Patek Philippe, or the Art to Tax Luxuries, No. 13 (June 2003).

• Dewenter, Ralf, Estimating the Valuation of Advertising, No. 12 (June 2003).

• Otto, Alkis, Foreign Direct Investment, Production, and Welfare, No. 11 (June 2003).

• Dewenter, Ralf, The Economics of Media Markets, No. 10 (June 2003).

• Josten, Stefan Dietrich, Dynamic Fiscal Policies, Unemployment, and Economic Growth, No. 9 (June 2003).


• Bräuninger, Michael, A Note on Health Insurance and Growth, No. 6 (June 2003).

• Dewenter, Ralf, Media Markets with Habit Formation, No. 5 (June 2003).

• Haucap, Justus, The Economics of Mobile Telephone Regulation, No. 4 (June 2003).

• Josten, Stefan Dietrich & Achim Truger, Inequality, Politics, and Economic Growth. Three Critical Questions on Politico-Economic Models of Growth and Distribution, No. 3 (June 2003).

• Dewenter, Ralf, Rational Addiction to News?, No. 2 (June 2003).

• Kruse, Jörn, Regulierung der Terminierungsentgelte der deutschen Mobilfunknetze?, Nr. 1 (Juni 2003).

**Frühere Diskussionsbeiträge zur Wirtschaftspolitik**


Frühere Diskussionsbeiträge aus dem Institut für Theoretische Volkswirtschaftslehre

- Bräuninger, Michael, Social Capital and Regional Mobility, Nr. 4/2002.
- Heppke, Kirsten, On the Existence of the Credit Channel in Poland, Nr. 8/1999.
- Bräuninger, Michael, Unemployment and International Lending and Borrowing in an Overlapping Generations Model, Nr. 8/1999.
- Henning, Andreas & Wolfgang Greiner, Organknappheit im Transplantationswesen - Lösungsansätze aus ökonomischer Sicht, Nr. 7/1999.

Frühere Diskussionsbeiträge zur Finanzwissenschaft


• Zimmermann, Klaus W. & Tobias Just, The Euro and Political Credibility in Germany, 2000, erschienen in: *Challenge* 44, 2001, S. 102-120


