SCIENCE AND TEACHING: TWO-DIMENSIONAL SIGNALLING IN THE ACADEMIC JOB MARKET

ANDREA SCHNEIDER

Nr./ No. 95
August 2009
Autoren / Authors

Andrea Schneider
Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg
Institut für Finanzwissenschaft / Institute of Public Finance
Holstenhofweg 85
22043 Hamburg
Germany
andrea.schneider@hsu-hh.de

Redaktion / Editors

Helmut Schmidt Universität Hamburg / Helmut Schmidt University Hamburg
Fächergruppe Volkswirtschaftslehre / Department of Economics

Eine elektronische Version des Diskussionspapiers ist auf folgender Internetseite zu finden/
An electronic version of the paper may be downloaded from the homepage:
http://fgvwl.hsu-hh.de/wp-vwl

Koordinator / Coordinator

Kai Hielscher
wp-vwl@hsu-hh.de
SCIENCE AND TEACHING: TWO-DIMENSIONAL SIGNALLING IN THE ACADEMIC JOB MARKET

ANDREA SCHNEIDER

Zusammenfassung / Abstract

Post-docs signal their ability to do science and teaching to get a tenure giving universities the possibility of separating highly talented agents from the low talented ones. However, separating that means signalling effort for the highly talented becomes even more important in a two-dimensional signalling case. This attracts notice to time constraints. Under weak conditions separating equilibria do not exist if time constraints are binding. The existing equilibria are more costly but without additional information compared to the one-dimensional case. Considering this, the efficiency of the current two-dimensional academic job market signalling can be improved by switching to a one-dimensional one.

JEL-Klassifikation / JEL-Classification: I23, D82, J41

Schlagworte / Keywords: Multi-dimensional signalling, Academic job market, Teaching and Research
1 Introduction

Not later than the 19th century the Germans know the concept of the unity of research and teaching. This idea of Wilhelm von Humboldt has mainly influenced especially the German higher education system and is still present today. On the other hand post-docs and professors often rail against the double burden of such a system. These conflicting argumentations in mind economists study the optimal design of the university system (e.g. Del Rey 2001, De Fraja and Valbonesi 2008 or Gautier and Wauthy 2007) as well as their optimal labour contract behaviour (Walckiers 2008). In line with the second part of literature the present paper analyses the possibility of separating highly productive agents from the low-productive ones in a model where post-docs can signal their ability to do science and teaching to get a tenure. Argumenting in line with a job market signalling model it is necessary to mention the work of Michael Spence. Spence as the father of signalling models shows education can be an efficient signal to correct asymmetric information in the job market. It’s due to him that we know about the existence of signalling equilibria (Spence 1973; Spence 1974). In contrast to Spence who mainly deals with the existence of equilibria Cho and Kreps (1987) rank equilibria. They implement an intuitive criterion to eliminate equilibria that are built on implausible out-of-equilibrium beliefs. This stronger equilibrium concept is finally the basic equilibrium concept of the present paper.

Up to here all concepts work in a one-dimensional world. Thus, agents send a one-dimensional signal. Since future professors produce a two-dimensional output consisting of science and teaching a multi-dimensional setup is needed. Unfortunately, papers on multi-dimensional signalling are rare. One of the first is by Rochet and Quinzii (1985). This paper analyses in a formal way the difference between the one- and multi-dimensional signalling set up. Assuming a separable cost structure they give necessary conditions for the existence of a separating equilibrium. In the same kind of model Engers (1987) focuses on pareto-dominant separating equilibria. Armstrong and Rochet (1999) simplify conditions that are necessary to ensure a separating equilibrium by assuming a discrete type distribution. This also is an assumption of my model. A current paper by Kim (2007) is of interest as well because it analyses time binding constraints in a two-dimensional job market signalling.

While Spence’s first paper focuses on the general existence of signalling equilibria the second paper highlights the different market forms.
The aim of this paper is to analyse separating equilibria in a two-dimensional signalling model that describes the academic job market. Post-docs that differ in their ability to do research and teaching can signal both talents to get a tenure. As one result separating equilibria of the two-dimensional case can vanish with time binding constraints. This always happens if teaching (science) productivity of the highly talented is higher than the science (teaching) productivity of the low-talented. Nevertheless, implying the concept of partial separating equilibria it can be shown that under weak conditions there is at least one partial separating equilibrium. More precisely, agents that are highly productive in both outputs send the same signal like the type that is high-talented with respect to the output that is more preferred by the universities. What is important for policy implications is that the signalling effort in the partial separating equilibrium - although it is smaller than in the two-dimensional separating equilibrium - is higher than in the one-dimensional separating equilibrium. This is of interest if signalling only has an effect on costs but not on productivity as it is the case in Spence (1973) and also in the present model. Thus, if time constraints are binding in the academic job market it could be more efficient to let post-docs only signal on the output that is more preferred by the universities. Under time binding constraints universities can only distinguish highly productive from low-productive types in one dimension just like in the one-dimensional case. Having this in mind there is no argument for the two-dimensional signalling process that is currently observed in reality. It just implies additional costs. Only weak conditions concerning the ranking of the productivity parameters are necessary to make a separating equilibrium under time binding constraints impossible in the two-dimensional case. Thus, agents that are highly-talented in both outputs can not be identified by universities.

The remaining paper is structured as follows: Section 2 sets the basic model. The existence of equilibria is analysed in section 3. While section 3.1 focuses on the one-dimensional case that goes in line with Spence (1973) section 3.2 extends the analysis to the two-dimensional case. In this part I also distinguish between a situation where time constraints are not binding resulting in the unique separating equilibrium is most efficient for the universities and a situation where time constraints are binding which may lead to the vanishing of the separating equilibrium. In the second case the existence of partial separating equilibria where some types of agents can be separated while others play the same strategy is analysed. Section 4 concludes.
2 The model

Assume a competitive academic job market with a unit mass of academics. Each university graduate produces science and teaching which requires specific unobservable abilities. Scientists as well as other labourers are not identical but vary in their abilities. There are four types $ij \in \{HH, HL, LH, LL\}$ of future professors. While $i$ denotes research productivity $j$ describes the teaching productivity. Both productivities can be high ($H$) or low ($L$). Future professors can signal both abilities: Science $s_{ij}$ and teaching $t_{ij}$, $i, j \in \{L, H\}$. As in Spence (1973) signals do not have any influence on productivity. Agents use the signals to influence the universities’ beliefs on their abilities. Thus, the pre-tenure research and teaching output functions as a signal for post-tenure productivities. However, there is a time binding constraint $s_{ij} + t_{ij} = \bar{t}$. Signalling effort can not be higher than the available time and therefore is limited. I assume the agents’ $ij$ cost function depending on his type is:

$$c_{ij}(s_{ij}, t_{ij}) = \frac{s_{ij}}{\theta^s_i} + \frac{t_{ij}}{\theta^t_j},$$  

where $\theta^s_i$ and $\theta^t_j$, $i, j \{L, H\}$, are the productivities of science and teaching respectively. Clearly, $\theta^H_i > \theta^L_i, k \in \{s, t\}$ holds. For simplicity I also assume $\theta^L_k \geq 1, k \in \{s, t\}$. Implicitly I assume that the research (teaching) productivity is independent of the ability to teach (research). The fraction of type $ij$ agents in the population is denoted by $\alpha_{ij}$. The distribution of the types is common knowledge.

Universities compete on prospective professors. However, they face asymmetric information and can only form beliefs on the agents’ abilities via signals. The profit of a university is $\pi((\theta_i, \theta_j), w) = \theta^s_i + \theta^t_j - w$, where $w$ is the wage paid to the agent. The competition of the academic job market implies that universities make a profit of zero and therefore wages are given by productivities. Thus, the equilibrium wage offered by the universities is $w^*$

$$w^* \equiv E[\alpha_{ij}(\theta^s_i + \theta^t_j)]$$  

where $E$ is the expectation operator.

---

2In the remaining paper ‘research’, ‘science’ and ‘publishing’ are synonymously used.
3This notation follows Walckiers (2008).
Although pre-tenure publishing and teaching do not influence the productivity universities can condition wage offers on the pre-tenure science and teaching output. The optimal decision of a prospective professor of type $ij$ is

$$\max_{s_{ij}, t_{ij}} U_{ij} = E[w_{ij} - \left( \frac{s_{ij}}{\theta_i^s} + \frac{t_{ij}}{\theta_j^t} \right)].$$

subject to $s_{ij} + t_{ij} \leq \ell$.

In section 3 I will analyse equilibria of this signalling model.

## 3 Signalling in the academic job market

First I focus on signalling equilibria when universities are only interested in science (section 3.1). This analysis goes in line with the signalling model of Spence (1973). Afterwards in section 3.2 I analyse a two-dimensional signalling model where agents signal on science and teaching. In both cases the main question is if there are separating equilibria where signalling can help to solve inefficient results caused by asymmetric information. Therefore, pooling equilibria are only analysed in the margin.

Under incomplete information there is need for a definition of a perfect Bayesian equilibrium.

**Definition 1** Perfect Bayesian Equilibrium (PBE): A PBE is a set that consists of a signal $(s_{ij}^*, t_{ij}^*)$ for each type of agent $ij \in \{HH, HL, LH, LL\}$ and a wage offer $w_{ij}(s_{ij}^*, t_{ij}^*)$ used by the universities.

For each signal $(s_{ij}^*, t_{ij}^*)$ the universities make zero profits given the belief $\mu(ij|(s_{ij}, t_{ij}))$ about which types could have sent $(s_{ij}, t_{ij})$.

Each type $ij$ maximises his utility by choosing $(s_{ij}^*, t_{ij}^*)$ given the wage offer $w_{ij}$ of the university.

The university’s belief must be consistent with Bayes’ rule and with the agent’s strategy: $\mu(ij|(s_{ij}, t_{ij})) = \frac{\alpha_{ij}}{\sum_{ij} \alpha_{ij}}$.

Therefore, one can distinguish between a separating equilibrium and a pooling equilibrium. In the first case all types send different signals, i.e. $(s_{ij}^*, t_{ij}^*) \neq (s_{ij'}^*, t_{ij'}^*)$ if $ij \neq i'j'$. In the second case the signal is identical for
all types, i.e. \((s^*_ij, t^*_ij)\), \(\forall i, j \in \{H, L\}\). In contrast to a model set up with two different types of agents that is normally used, in the present model there is also the possibility for an equilibrium in which some but not all agents send the same signal. Such a perfect Bayesian equilibrium will be called a partial separating equilibrium.

### 3.1 One-dimensional signalling

Let us assume for the moment universities are only interested in science and not in teaching. In this case there is no value of teaching and therefore no agent sends a teaching signal. Thus, type \(HH\) and \(HL\) can be interpreted as one type denoted by \(H\). The same applies to \(LH\) and \(LL\). This low productivity type is denoted by \(L\).\(^4\) Then the fraction of the high productivity type is \(\alpha_H \equiv \alpha_{HH} + \alpha_{HL}\) and the fraction of agents with low productivity is \(\alpha_L \equiv \alpha_{LH} + \alpha_{LL}\).

Under complete information the high productivity type would earn a wage of \(\theta^s_H\) while the type with low productivity gets \(\theta^s_L < \theta^s_H\). Since pre-tenure publishing only implies a cost effect but no effect on productivity both types do not publish anything under complete information. Under incomplete information one can distinguish between a pooling and a separating equilibrium. However, the partial separating equilibrium is irrelevant in the case of two different types of agents.

**Proposition 1** Given a two type signalling game where future professors can have high or low productivity of publishing \((\theta^s_H\) or \(\theta^s_L)\) and the universities’ wage offer \(w(s)\) depending on the research signal \(s\) there is the unique separating equilibrium

\[
\begin{align*}
    s^*_H &= \theta^s_L(\theta^s_H - \theta^s_L), \quad s^*_L = 0 \\
    w(s^*_H) &= \theta^s_H, \quad w(s^*_L) = \theta^s_L \\
    \mu(H|s \geq s^*_H) &= 1, \quad \mu(L|s < s^*_H) = 1.
\end{align*}
\]

\(^4\)Clearly, in this two type case the cost and wage structure satisfies the well known Spence-Mirrlees single crossing property condition, i.e. the two-types’ \(w - s_i\)–indifference curves with \(i \in \{H, L\}\) have only one point of intersection.
The detailed proof of proposition 1 can be found in appendix A page 17. The motivation of the results is as follows: In a separating equilibrium there is no incentive for the type with low productivity to invest in publishing because this has just a cost effect but no impact on productivity. Therefore, an agent with high productivity must publish exactly the amount that ensures type \( L \) does not mimic him. However, it is possible that the time constraint is binding, i.e. \( s^*_H > \tilde{l} \). Then the agent with the high productivity can not publish enough to prevent mimicking of the low-productive type.

In addition there is also a unique pooling equilibrium where nobody signals.\(^5\) This is a standard result whenever signals do not have an effect on productivity. In this case nobody has an incentive to invest in signaling playing \( s^* = 0 \). Note, if the time constraint is binding, only the pooling equilibrium persists. However, this paper focuses on efficient separating equilibria.

Of course, all results persists if universities are solely interested in teaching. In this case just replace \( s \) by \( t \) in the previous analysis and redefine \( \alpha_H \equiv \alpha_{HH} + \alpha_{LH} \) and \( \alpha_L \equiv \alpha_{HL} + \alpha_{LL} \) respectively.

3.2 Two-dimensional signalling

A higher load of teaching (and also administrative work) reduces publication output since time to do research can not be used to teach (Mitchell and Rebne 1995). Although teaching can enhance research (Becker and Kennedy 2005) there is no general evidence that good researchers are also good teachers. In contrast economists prefer doing research to teaching (Allgood and Walstad 2005). Since results of the interdependency of science and teaching is unclear I do not make any additional assumptions on the distribution of the four types of agents.\(^6\) Nevertheless note, if both talents are substitutes (complements) \( \alpha_{HL} \) and \( \alpha_{LH} \) are high (small) while \( \alpha_{HH} \) and \( \alpha_{LL} \) are small (high).\(^7\)

\(^5\)For the explicit notation of the pooling PBE and for the proof of its existence see appendix A page 18.

\(^6\)Although there is no clear evidence that research and teaching are complements on the individual perspective level both act complementarily on the university level. For a meta-analysis on this topic see Hattie and Marsh (1996).

\(^7\)Gottlieb and Keith (1997) find in their study that the connection between research and teaching is not just substitutive or complementary but more complex. In detail they show that research can positively affect research but attributes of teaching negatively impact
In subsection 3.1 we have already seen that the time constraint can have an important influence on the existence of PBE. In the one-dimensional case time constraints can lead to a situation where only the pooling equilibrium exists. Now, under two-dimensional signalling I show that the separating equilibrium is even more likely destroyed by time constraints. However, with two dimensions there is the possibility of partial separating equilibria. First, I analyse the separating equilibrium in the two-dimensional case. Then I show that under some conditions (more precisely, if assumption 1 holds) time constraints make a separating equilibrium impossible. Nevertheless, if agents send a two-dimensional signal there is always at least one partial separating equilibrium. In this equilibrium type $HH$ sends the same signal like the type that is highly talented with respect to the output that is more prefered by the universities.

### Time constraint not binding

**Proposition 2** If agents signal science and teaching ability via $(s_{ij}, t_{ij})$, universities offer wages $w(s_{ij}, t_{ij})$ and the time constraint is not binding, i.e. $s_{ij}^* + t_{ij}^* \leq l$, there is a separating equilibrium

$$s_{ij}^* = \begin{cases} \theta^*_L(\theta^*_H - \theta^*_L), & i = H \\ 0, & i = L \end{cases}$$  

and  

$$t_{ij}^* = \begin{cases} \theta^*_L(\theta^*_H - \theta^*_L), & j = H \\ 0, & j = L \end{cases}$$

$$w(s_{ij}^*, t_{ij}^*) = \theta^*_i + \theta^*_j, \ \forall i, j \in \{H, L\}$$

$$\mu(i, j = H | k_{ij} \geq \theta^*_k(\theta^*_H - \theta^*_L)) = 1$$  

and  

$$\mu(i, j = L | k_{ij} < \theta^*_k(\theta^*_H - \theta^*_L)) = 1$$

where $k \in \{s, t\}$.

The detailed proof of proposition 2 is given in appendix B page 19. The basic idea is to derive conditions under which type $ij$ has no incentive to mimic type $i'j'$ for all $i, i', j, j' \in \{H, L\}$. Although these conditions are fulfilled by a continuum of signal combinations $(s_{ij}, t_{ij})$ there is only a unique signal for each type that maximises utility. Caused by additive linearity of costs and productivities the signals in the two-dimensional PBE equal in each of the two components the signals arising in the one-dimensional case. To illustrate the decisions figure 1 shows the incentive compatibility constraints of the research.
different types of agents.\textsuperscript{8}

Figure 1: (a) Incentive compatibility constraint that prevents type $HL$ from mimicking $LL$ and vice versa, (b) Incentive compatibility constraint that prevents type $LH$ from mimicking $LL$ and vice versa, (c) Incentive compatibility constraint that prevents type $HH$ from mimicking $HL$ or $LH$ and vice versa.

The grey triangle in part (a) of the figure shows all combinations that prevent $HL$ from mimicking $LL$ and vice versa. The cost minimal combination that fulfils these incentive compatibility conditions is $(s_{HL}^*, t_{HL}^*)$. Analogously, part (b) of the figure gives the incentive compatibility constraints that

\textsuperscript{8}The figure refers to the parameter setting $\theta^*_L = 2, \theta^*_H = 3, \theta^*_L = 3$ and $\theta^*_H = 4$.
prevent $LH$ from mimicking $LL$ and vice versa. Here, $(s_{LH}^*, t_{LH}^*)$ is optimal strategy for type $LH$. The grey triangle in part (c) consists of all strategies that prevent $HH$ from mimicking $LH$ and $HL$ and vice versa. The optimal strategy of type $HH$ is then $(s_{HH}^*, t_{HH}^*)$ which is in both components equal to the separating strategy of the high-talented type in the one-dimensional case.

The same argumentation as in the one-dimensional case leads to a pooling PBE where nobody signals, i.e. all agents’ strategy is $(s^* = 0, t^* = 0)$. There is also the possibility for partial separating PBEs in the present case. However, universities are interested in the real type of the agent. So, the most efficient situation is the separating one. I pay more attention to the partial separating PBEs in the next subsection where time constraints play a crucial role.

**Time constraint binding**

Now, I try to answer the question: What happens if time constraints are binding, i.e. if type $HH$ can not play his strategy of the separating equilibrium of proposition 2. More formally, $s_{HH}^* + t_{HH}^* > \bar{l}$ holds. For simplicity I assume that $\theta_k^*(\theta_k^* - \theta_L^*) \leq \bar{l}$, $k \in \{s, t\}$, holds. This guarantees that the equilibria of the one-dimensional case exist. If this is not fulfilled only the pooling equilibrium remains.

As a key mechanism of a separating equilibrium the highly talented agent separates himself by signalling so much that there is no incentive of the low-talented agent to mimic him. This is possible because of the difference in costs. However, if there are not only one but two signals the signalling effort increases and may become too high to be realised in the time given. Before discussing the main result of this section I make an assumption about the ranking of the productivity parameters that is crucial for the remaining analysis.

**Assumption 1** The ranking of the productivity parameters fulfils

$$\theta_H^s \geq \theta_L^t$$

---

9 For the detailed proof see appendix B page 24.

10 In the present model the signalling effort in the two-dimensional case is exactly the sum of the two one-dimensional signalling models where the agent signals on teaching or science. However, this result is driven by the additive structure of productivity and costs.
and

$$\theta^k_H \geq \theta^k_L.$$ 

By definition $$\theta^k_H > \theta^k_L, k \in \{s, t\}$$ always holds. So, for both activities the highly talented agent is more productive than the agent with low productivity. However, nothing is known of the ranking of the productivity parameters comparing both activities. Assumption 1 requires that agents that are highly productive doing one activity are more productive than agents doing the other activity with low talent. Or, the other way round, assumption 1 is violated if the universities’ benefit from one output is so high that producing this output by a low-productive agent is better than producing the other output by a high-productive agent.

**Proposition 3** If agents signal their abilities to do science ($s_{ij}$) and teach ($t_{ij}$), universities offer wages $w(s_{ij}, t_{ij})$, the time constraint is binding, i.e. if in proposition 2 $s^*_{HH} + t^*_{HH} > I$, and assumption 1 holds, there is no separating equilibrium. If assumption 1 does not hold the separating equilibrium from proposition 2 is destroyed but there is again the possibility of separating the four types in equilibrium.

![Figure 2: Incentive compatibility constraints for type HH when assumption 1 holds.](image)
For an illustration of the situation where assumption 1 holds see figure 2.11 The figure describes the incentive compatibility constraints of type HH. All strategies in the light-grey triangle prevent HH from mimicking LH and vice versa. The dark-grey triangle consists of all s-t-combinations that prevent HH from mimicking HL and vice versa. The black triangle therefore gives all strategies that fulfills both conditions. The strategy \((s^*_HH, t^*_HH)\) is the equilibrium strategy. The key idea here is as follows: Because of the pure cost effect of signalling HH realises a cost minimal combination that is tangent to the black triangle at its lower bound. The lower bound of the light-grey triangle has a slope of \(- (\theta^t_H/\theta^s_L)\). The lower bound of the dark-grey triangle has a slope of \(- (\theta^t_L/\theta^s_H)\). Since the slope of HH’s cost function is \(- (\theta^t_H/\theta^s_H)\) and therefore meets the condition \(- (\theta^t_H/\theta^s_L) < - (\theta^t_L/\theta^s_H) < - (\theta^t_L/\theta^s_H)\) strategy \((s^*_HH, t^*_HH)\) becomes the cost minimal strategy that fulfils both incentive compatibility constraints. However, if \((s^*_HH, t^*_HH)\) is the equilibrium strategy of type HH and time constraints are binding there is no strategy that lies south-west of \((s^*_HH, t^*_HH)\) - which is necessary to meet the time constraint - and is located in the black triangle - which is necessary to fulfil the incentive compatibility constraints of type HH. So, if assumption 1 holds there is no separating PBE.

![Figure 3: Incentive compatibility constraints for type HH when assumption 1 is not fulfilled.](image)

11The figure refers to parameter setting \(\theta^s_L = 2, \theta^t_H = \theta^t_L = 3\) and \(\theta^t_H = 4\).
Figure 3 shows a situation in which assumption 1 does not hold. The grey area describes all strategies of $HH$ that fulfil both incentive compatibility constraints. Contrary to figure 2 a decrease in the available time from $\tilde{l}_1$ to $\tilde{l}_2$ shifts the separating PBE from $HH_1$ to $HH_2$. Thus there is still the possibility of separating the different types of agents.

**Proposition 4** If agents signal their abilities to do science $(s_{ij})$ and teach $(t_{ij})$, universities offer a wage $w(s_{ij}, t_{ij})$ equal to the expected productivity there are two partial separating equilibria.

If $\theta^s_H \theta^t_L \geq \theta^s_L \theta^t_H$ holds there is a partial PBE where strategies of the prospective professors are:

$$(s^*_L, t^*_L) = (0, 0), \quad (s^*_L, t^*_H) = (\theta^s_L (\theta^s_H - \theta^t_L), 0) \quad \text{and}$$

$$(s^*_L, t^*_H) = (\theta^s_L C_1^{LH,HH}, 0) \quad \text{with} \quad C_1^{LH,HH} \equiv \frac{\alpha_{HH}}{\alpha_{HL} + \alpha_{HH}} \theta^s_H - (1 - \frac{\alpha_{HL}}{\alpha_{HL} + \alpha_{HH}}) \theta^s_L + \theta^t_H - \theta^t_L.$$  

If $\theta^s_L \theta^t_L \geq \theta^s_L \theta^t_H$ holds there is a partial separating PBE where strategies of the prospective professors are:

$$(s^*_L, t^*_L) = (0, 0), \quad (s^*_L, t^*_L) = (0, \theta^t_L (\theta^s_H - \theta^t_L)) \quad \text{and}$$

$$(s^*_L, t^*_H) = (\theta^s_L C_1^{LH,HH}, 0) \quad \text{with} \quad C_1^{HL,HH} = \theta^s_H - \theta^s_L + \frac{\alpha_{HL}}{\alpha_{HL} + \alpha_{HH}} \theta^t_L - (1 - \frac{\alpha_{HL}}{\alpha_{HL} + \alpha_{HH}}) \theta^t_L.$$  

In cause of clear arrangement proposition 4 only denotes strategies of the prospective professors. The wage setting of the universities is for the separated types equal to the wage setting of proposition 2. The pooled types are paid by average productivities. Thus in the first partial separating PBE it is $w(LH,HH) = \frac{\alpha_{HH}}{\alpha_{HL} + \alpha_{HH}} \theta^s_L + \frac{\alpha_{HH}}{\alpha_{HL} + \alpha_{HH}} \theta^t_L + \theta^t_H$ and in the second partial separating equilibrium it is $w(HL,HH) = \theta^s_H + \frac{\alpha_{HL}}{\alpha_{HL} + \alpha_{HH}} \theta^t_L + \frac{\alpha_{HH}}{\alpha_{HL} + \alpha_{HH}} \theta^t_H$.

---

12The figure refers to the parameter setting $\theta^s_L = 1, \theta^s_H = 2, \theta^t_L = 3$ and $\theta^t_H = 4$.

13Proposition 4 only describes two partial separating equilibria. There is also the possibility of other partial separating equilibria, e.g. of $(LL, LH, HH)$. Nevertheless, universities try to identify the highly productive agents. Thus the partial separating PBEs of proposition 4 are the one of interest.
The detailed proof can be found in the appendix C page 26. In the first partial separating PBE universities can distinguish between \( LL, HL \) and \( (LH, HH) \), i.e. they can not separate type \( LH \) from \( HH \). In the second partial separating PBE universities can separate \( LL \) from \( LH \) and \( (HL, HH) \) but not types \( HL \) and \( HH \). The key arrangement of the proof of the first partial separating PBE (and analogously of the second one) is as follows: Type \( LL \) does not signal because of the pure cost effect. Type \( HL \) plays his strategy from the one-dimensional case to prevent \( LL \) from mimicing. Then the incentive compatibility constraints of \( (LH, HH) \) not to mimic \( LL \) or \( HL \) and vice versa are calculated. This results in the equilibrium strategy for \( (LH, HH) \).

Figure 4: Incentive compatibility constraints for \( (LH, HH) \) in the first partial separating PBE (a) when \( \theta_L^* \theta_H^* \geq \theta_H^* \theta_L^* \) holds and (b) if this condition is not fulfilled
To illustrate the necessary condition of the existence of the first partial separating PBE ($LL, HL, (LH, HH)$), i.e. to illustrate the necessity of $\theta_s^L \theta_H^L \geq \theta_H^L \theta_s^L$, look at figure 4. In part (a) of figure 4 it is $\theta_s^L \theta_H^L \geq \theta_H^L \theta_s^L$ and both minimal cost functions of the pooled types, i.e. $c_{LH}$ and $c_{HH}$, are tangent to the black array that consists of all strategies which meet the incentive compatibility constraints at point $(LH, HH)$. This $s-t$-combination is the strategy $LH$ and $HH$ play in the first partial separating equilibrium. In part (b) it is $\theta_s^L \theta_H^L < \theta_H^L \theta_s^L$. Thus, the cost function of $HH$, i.e. $c_{HH}$, runs “too flat”. The minimal cost function of type $HH$ is tangent to the black area where the incentive compatibility constraints are fulfilled at point $HH$. Since the minimal cost function of type $LH$ is tangent to the black array at point $LH$ there is no pooling equilibrium strategy for both types and so no partial separating PBE.

As a first result one can see that both partial separating PBE can only co-exist if $\theta_H^H \theta_L^L = \theta_L^H \theta_H^L$ holds. One example for such a situation is the symmetric case, where low (high) productivity of science equals low (high) productivity of teaching, i.e. $\theta_H^H = \theta_L^L$ and $\theta_L^H = \theta_L^H$. Thus, universities do not have a clear preference for the one or the other output. Assuming that the highly productive agents are the critical one and therefore normalising the productivities of the low-talented to one, i.e. $\theta_L^L = \theta_L^L = 1$, the first partial separating PBE only exists if teaching productivity of the highly talented is higher than his research productivity. Analogously, if the contrary appraisement holds the second partial separating PBE appears. In general an agent that is good in teaching and science pooles with the type that is highly-talented in the output that is more prefered by the universities. This strengthens the argument of Becker (1975) and (1979) that the professors’ research and teaching output positively react on an increase in pecuniary returns.

Secondly, it is clear that without time constraints always at least one of the partial PBEs exists. However, in this case they are less interesting because the separating PBE is more efficient.

---

14 Clearly, an analogous argumentation holds for condition $\theta_H^H \theta_L^L \geq \theta_H^L \theta_s^L$ and the second partial separating PBE.

15 Part (a) of the figure refers to parameter values $\theta_L^L = \theta_L^L = 1, \theta_H^L = 2$ and $\theta_H^L = 3$. More precisely, optimal strategies should be labeled ($s_{LH, HH}^L, t_{LH, HH}^L$). However, caused by clarification I label the strategy with the type.

16 Part (b) of figure 4 refers to $\theta_L^L = 1, \theta_H^L = \theta_L^L = 2$ and $\theta_H^L = 3$. 

Thirdly, proposition 4 shows that if the time constraint is too strong there is even no possibility for a partial separating equilibrium but only for the pooling one. Thus, in both partial separating equilibria the time constraint is relaxed compared to the separating case but not removed. More precisely, the time investment of type $HH$ in the first partial separating equilibrium is $\theta^t_L C_1^{LH,HH} = \theta^t_L \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} (\theta^t_H - \theta^s_L) + \theta^t_L (\theta^t_H - \theta^t_L)$. This is clearly higher than the investment in the one-dimensional case, i.e. $\theta^L_L (\theta^t_H - \theta^t_L)$. So, time constraints can still be binding. They are weakened to the two-dimensional separating equilibrium where time input is $\theta^t_L (\theta^t_H - \theta^t_L) + \theta^t_L (\theta^t_H - \theta^t_L)$ if and only if $\alpha_{HH}/(\alpha_{LH} + \alpha_{HH}) < \theta^s_L/\theta^t_L$. This is always fulfilled if $\theta^s_L \geq \theta^t_L$ and therefore especially in the case where low productivities are normalised to one. By the same argumentation time constraint of the second partial PBE is weaker than in the two-dimensional separating PBE if and only if $\alpha_{HH}/(\alpha_{HL} + \alpha_{HH}) < \theta^t_L/\theta^s_L$ holds. A sufficient condition for this purpose is $\theta^s_L \geq \theta^t_L$.

4 Conclusion

The output of post-docs and professors consists, beside the administrative one that is not mentioned here, of science and teaching. In general universities are interested in both outputs and assign a tenure contract only to those post-docs that are highly talented in both activities. However, since talent is a private information a job market signalling model a la Spence arises. Post-docs signal their ability of science and teaching to get a tenure.

As Spence (1973) has shown in the one-dimensional case signalling can also in the two-dimensional case separate highly talented and low talented agents. So it solves the inefficiency problem of asymmetric information. Unfortunately, the highly productive agents need a signalling effort to separate themselves from the low-productive types and this effort increases in the two-dimensional case. Considering this, time constraints attract notice.

If time constraints are binding and the science (teaching) productivity of the high-talented is higher than the teaching (science) productivity of the type with low talent a separating equilibrium can not exist in the two-dimensional

\[17\] Actually, I can not be sure that this one-dimensional equilibrium implies the stronger time constraint, i.e. that $\theta^t_L (\theta^t_H - \theta^t_L) > \theta^s_L (\theta^t_H - \theta^t_L)$ holds. Nevertheless, this is true if the high-productive agents are of most interest and low productivities are normalised to one.
case. The required assumption is quite weak as it just says that universities should not prefer one output over the other regardless whether the first is created by a high- or low-productive person.

In addition I show that even if the separating equilibrium is destroyed by time constraints there is always at least one partial separating equilibrium where some types can be separated while others pool on the same strategy. More precisely, if the university prefers science to teaching a partial separating equilibrium exists where universities can separate types with high or low research productivity. However, they do not know if an agent with high research productivity is also highly talented in teaching. This is the same result as in the one-dimensional case. Regrettably, the signalling effort that only implies a pure cost effect is higher in the two-dimensional partial separating equilibrium than in the one-dimensional separating one. Corresponding to real life, the two-dimensional signalling system that is currently used in academic admission processes is inefficient if time constraints are binding. In such a situation universities can not identify both talents of the post-doc but only one. The identifiable talent is the one they value more. Then universities can ease requirements on post-docs and can let them - without losing information - just signal on science or teaching.

Appendix

Part A: One-dimensional case

Proof of proposition 1:
Separating equilibrium:
Since pre-tenure publishing implies cost but has no effect on productivity there is no incentive for a \( L \)-type to invest in publishing in a separating equilibrium. Therefore, it is \( s^*_L = 0 \).

In addition any equilibrium must satisfy two incentive compatibility conditions: On the one hand type \( H \) must not have an incentive to mimic the \( L \)-type, i.e.

\[
\begin{align*}
    w(s_H) - c_H(s_H) & \geq w(s_L) - c_H(s_L) \\
    \Leftrightarrow \theta_H^s - \frac{s_H}{\theta_H^s} & \geq \theta_L^s - \frac{s_L}{\theta_H^s}.
\end{align*}
\] (4)
On the other hand the $L$-type must not have an incentive to mimic the $H$-type, i.e.

$$w(s_L) - c_L(s_L) \geq w(s_H) - c_L(s_H)$$

$$\Leftrightarrow \theta^*_L - \frac{s_L}{\theta^*_L} \geq \theta^*_H - \frac{s_H}{\theta^*_H}. \quad (5)$$

Taking into account that the equilibrium strategy of the $L$-type is to publish nothing, i.e. $s^*_L = 0$, inequation (4) results in $s_H \leq \theta^*_H(\theta^*_H - \theta^*_L)$. Analogously, solving inequation (5) by $s_H$ I get $s_H \geq \theta^*_L(\theta^*_H - \theta^*_L)$. Both incentive compatibility conditions together imply that $\theta^*_L(\theta^*_H - \theta^*_L) \leq s_H \leq \theta^*_H(\theta^*_H - \theta^*_L)$ is a necessary condition of a separating equilibrium.

Since the universities never pay a wage higher than $\theta^*_H$ only the lower bound of the interval, i.e. $s^*_H = \theta^*_L(\theta^*_H - \theta^*_L)$, maximises utility of type $H$.

However, if $s^*_H > \bar{l}$ holds the separating equilibrium vanishes. Having the optimal decisions of the agents in mind universities belief that they focus on an agent of type $H$ whenever $s \geq s^*_H$ and that they focus on an agent of type $L$ whenever $s < s^*_H$. $\square$

**Pooling equilibrium:**

The pooling equilibrium in the one-dimensional case is

$$s^* = s_H = s_L = 0$$

$$w(s^*) = \alpha_H \theta^*_H + \alpha_L \theta^*_L$$

$$\mu(H|s \leq \alpha_H(\theta^*_L(\theta^*_H - \theta^*_L))) = \alpha_H, \mu(L|s \leq \alpha_H(\theta^*_L(\theta^*_H - \theta^*_L))) = \alpha_L$$

$$\mu(L|s > \alpha_H(\theta^*_L(\theta^*_H - \theta^*_L))) = 1$$

if the time constraint is not binding, i.e. $s^*_i \leq \bar{l}, i \in \{H, L\}$. The argumentation is as follows:

In every pooling equilibrium agents send identical signals, i.e. $s^*_H = s^*_L = s^*$. Since universities cannot distinguish between both types they set a unique wage that equals average valuation of the universities, i.e.

$$w(s) = \alpha_H \theta^*_H + \alpha_L \theta^*_L. \quad (6)$$

In a pooling equilibrium both types must not get lower utility than without signaling getting $\theta^*_L$, i.e.

$$\theta^*_L \leq w(s) - c_H(s)$$

$$\Leftrightarrow \theta^*_L \leq \alpha_H \theta^*_H + \alpha_L \theta^*_L - \frac{s}{\theta^*_H}. \quad (7)$$
and

\[ \theta_L^s \leq w(s) - c_L(s) \]
\[ \Leftrightarrow \theta_L^s \leq \alpha_H \theta_H^s + \alpha_L \theta_L^s - \frac{s}{\theta_L^s}. \]  

(8)

With \( \theta_H^s > \theta_L^s \) only condition (8) becomes critical. It implies that in every pooling equilibrium

\[ \theta_L^s \leq \alpha_H \theta_H^s + \alpha_L \theta_L^s - \frac{s}{\theta_L^s} \]
\[ \Leftrightarrow s \leq \theta_L^s (\alpha_H \theta_H^s - (1 - \alpha_L) \theta_L^s) \]
\[ \Leftrightarrow s \leq \alpha_H \theta_L^s (\theta_H^s - \theta_L^s) \]  

(9)

must hold. However, since publishing only implies a cost effect both types prefer the signal \( s_H^* = s_L^* = s^* = 0 \). This is a pooling perfect Bayesian equilibrium and in addition always satisfies the time constraint \( s^* \leq \bar{t} \). □

Part B: Two-dimensional case without time constraints

Proof of proposition 2:

In a separating PBE universities pay an agent \( ij \) a wage equal to his productivity. Thus, \( w(s_{ij}^*, t_{ij}^*) = \theta_{ij}^s + \theta_{ij}^t \) holds.

This directly gives \( (s_{LL}^*, t_{LL}^*) = (0, 0) \) as equilibrium signal of type \( LL \). In a next step, signals of types \( HL \) and \( LH \) must meet the incentive compatibility constraints so that both types have no incentive to mimic \( LL \) and vice versa. This automatically prevents \( HH \) from mimicing \( LL \).

Type \( HL \) does not mimic \( LL \) if

\[ w(s_{LL}^*, t_{LL}^*) - c_{HL}(s_{LL}^*, t_{LL}^*) \leq w(s_{HL}, t_{HL}) - c_{HL}(s_{HL}, t_{HL}) \]
\[ \Leftrightarrow \theta_L^s + \theta_L^t \leq \theta_H^s + \theta_L^t - s_{HL} \quad \theta_H^s - \theta_L^s \]
\[ \Leftrightarrow \frac{1}{\theta_H^s} s_{HL} + \frac{1}{\theta_L^s} t_{HL} \leq \theta_H^s - \theta_L^s \]
holds. Analogously, \( LL \) does not mimic \( HL \) whenever
\[
 w(s_{LL}^{*}, t_{LL}^{*}) - c_{LL}(s_{LL}^{*}, t_{LL}^{*}) \geq w(s_{HL}^{*}, t_{HL}^{*}) - c_{LL}(s_{HL}^{*}, t_{HL}^{*}) \\
 \Leftrightarrow \theta_{L}^{s} + \theta_{L}^{t} \geq \theta_{H}^{s} + \theta_{L}^{t} - \frac{s_{HL}}{\theta_{L}^{s}} - \frac{t_{HL}}{\theta_{L}^{t}} \\
 \Leftrightarrow \frac{1}{\theta_{L}^{s}} s_{HL} + \frac{1}{\theta_{L}^{t}} t_{HL} \geq \theta_{H}^{s} - \theta_{L}^{s}
\]
holds. Therefore the incentive compatibility constraint that prevents \( HL \) from mimicking \( LL \) and vice versa is
\[
\frac{1}{\theta_{H}^{s}} s_{HL} + \frac{1}{\theta_{L}^{t}} t_{HL} \leq \theta_{H}^{s} - \theta_{L}^{s} \leq \frac{1}{\theta_{L}^{s}} s_{HL} + \frac{1}{\theta_{L}^{t}} t_{HL}.
\]
A signal that maximises utility of type \( HL \) must lie on the lower bound which one can rewrite as
\[
s_{HL} = \frac{\theta_{H}^{s} (\theta_{H}^{s} - \theta_{L}^{s})}{\theta_{H}^{s}} - \frac{\theta_{L}^{t} t_{HL}}{\theta_{L}^{s}}.
\]
Type \( HL \) will now choose the signal that fulfils this condition and minimises costs. Since costs are (taking the last equation into account)
\[
c_{s_{HL}, t_{HL}} = \frac{s_{HL}}{\theta_{H}^{s}} + \frac{t_{HL}}{\theta_{L}^{t}}
\]
\[
= \frac{\theta_{H}^{s} (\theta_{H}^{s} - \theta_{L}^{s})}{\theta_{H}^{s}} - \frac{\theta_{L}^{t} t_{HL}}{\theta_{L}^{s}} + \frac{1}{\theta_{L}^{t}} t_{HL}
\]
\[
= \frac{\theta_{H}^{s} (\theta_{H}^{s} - \theta_{L}^{s})}{\theta_{H}^{s}} + (1 - \frac{\theta_{L}^{s}}{\theta_{H}^{s}}) \frac{1}{\theta_{L}^{t}} t_{HL}
\]
the minimal cost combination is \( t_{HL}^{*} = 0 \) and therefore \( s_{HL}^{*} = \theta_{L}^{s} (\theta_{H}^{s} - \theta_{L}^{s}) \). Type \( HL \)'s strategy in the separating PBE is \( (s_{HL}^{*}, t_{HL}^{*}) \).

In the same way type \( LH \) does not mimic type \( LL \) if
\[
 w(s_{LL}^{*}, t_{LL}^{*}) - c_{LH}(s_{LL}^{*}, t_{LL}^{*}) \leq w(s_{LL}^{*}, t_{LL}^{*}) - c_{LH}(s_{LL}^{*}, t_{LL}^{*}) \\
 \Leftrightarrow \theta_{L}^{s} + \theta_{L}^{t} \leq \theta_{L}^{s} + \theta_{L}^{t} - \frac{s_{LH}}{\theta_{L}^{s}} - \frac{t_{LH}}{\theta_{L}^{t}} \\
 \Leftrightarrow \frac{1}{\theta_{L}^{s}} s_{LH} + \frac{1}{\theta_{L}^{t}} t_{LH} \leq \theta_{L}^{s} - \theta_{L}^{t}
\]
holds.

Type LL does not mimic type LH if

\[
\begin{align*}
w(s_{LL}^*, t_{LL}^*) - c_{LL}(s_{LL}^*, t_{LL}^*) & \geq w(s_{LH}, t_{LH}) - c_{LL}(s_{LH}, t_{LH}) \\
\Leftrightarrow \theta_s^* + \theta_t^* & \geq \theta_s^* + \theta_t^* - \frac{s_{LH}}{\theta_s^* - \theta_t^*} - \frac{t_{LH}}{\theta_t^*} \\
\Leftrightarrow \frac{1}{\theta_s^*} s_{LL}^* + \frac{1}{\theta_t^*} t_{LL}^* & \geq \theta_s^* - \theta_t^*
\end{align*}
\]

is fulfilled. Taking both conditions together type LH has no incentive to mimic type LL and vice versa if

\[
\frac{1}{\theta_s^*} s_{LH}^* + \frac{1}{\theta_t^*} t_{LH}^* \leq \theta_s^* - \theta_t^* \leq \frac{1}{\theta_s^*} s_{LL}^* + \frac{1}{\theta_t^*} t_{LL}^*
\]

holds. Again LH chooses a signal on the lower bound given by the second part of the condition. Thus it is

\[
t_{LH} = \theta_t^* (\theta_s^* - \theta_t^*) - \frac{\theta_t^*}{\theta_s^*} s_{LH}^*.
\]

This in mind costs of type HL are given by

\[
c_{HL}(s_{LH}, t_{LH}) = \frac{s_{LH}}{\theta_s^*} + \frac{t_{LH}}{\theta_t^*} = \frac{\theta_t^* (\theta_s^* - \theta_t^*)}{\theta_s^*} + \frac{1}{\theta_s^*} s_{LH}^* - \frac{\theta_t^*}{\theta_s^*} s_{LH}^* = (1 - \frac{\theta_t^*}{\theta_s^*}) \frac{1}{\theta_s^*} s_{LH}^* + \frac{\theta_t^* (\theta_s^* - \theta_t^*)}{\theta_t^*}
\]

To minimise costs and therefore maximise utility given the wage \(\theta_s^* + \theta_t^*\) type LH plays \(s_{LH}^* = 0\) and \(t_{LH}^* = \theta_t^* (\theta_s^* - \theta_t^*)\) in equilibrium.

With \((s_{HL}^*, t_{HL}^*)\) and \((s_{LH}^*, t_{LH}^*)\) type HL has no incentive to mimic type LH and vice versa cause

\[
w(s_{HL}^*, t_{HL}^*) - c_{HL}(s_{HL}^*, t_{HL}^*) \leq w(s_{HL}^*, t_{HL}^*) - c_{HL}(s_{HL}^*, t_{HL}^*)
\]

\[
\Leftrightarrow \theta_s^* + \theta_t^* - \frac{\theta_t^* (\theta_s^* - \theta_t^*)}{\theta_t^*} \leq \theta_s^* + \theta_t^* - \frac{\theta_t^* (\theta_s^* - \theta_t^*)}{\theta_s^*}
\]

\[
\Leftrightarrow 0 \leq (\theta_s^* - \theta_t^*)^2
\]

21
and
\[
\begin{align*}
    w(s_{HL}^*, t_{HL}^*) - c_{LH}(s_{HL}^*, t_{HL}^*) & \leq w(s_{LH}^*, t_{LH}^*) - c_{LH}(s_{LH}^*, t_{LH}^*) \\
    \Leftrightarrow \theta_H^s + \theta_L^t - \frac{\theta_H^s(\theta_H^s - \theta_L^s)}{\theta_L^s} & \leq \theta_L^t + \theta_H^s - \frac{\theta_L^t(\theta_H^t - \theta_L^t)}{\theta_H^t} \\
    \Leftrightarrow 0 & \leq (\theta_H^t - \theta_L^t)^2
\end{align*}
\]
are always fulfilled.

In a last step one has to make sure that $HH$ does neither mimic $HL$ nor $LH$ and vice versa. Type $HH$ does not mimic $HL$ whenever
\[
\begin{align*}
    w(s_{HH}^*, t_{HH}^*) - c_{HL}(s_{HH}^*, t_{HH}^*) & \leq w(s_{HL}^*, t_{HL}^*) - c_{HL}(s_{HL}^*, t_{HL}^*) \\
    \Leftrightarrow \theta_H^s + \theta_L^t - \frac{s_{HH}^*}{\theta_H^s} - \frac{t_{HH}^*}{\theta_L^t} & \leq \theta_H^s + \theta_L^t - \frac{s_{HL}^*(\theta_H^s - \theta_L^s)}{\theta_H^s} \\
    \Leftrightarrow s_{HH} + \frac{\theta_H^s}{\theta_L^t}t_{HH} & \leq \theta_H^s(\theta_H^t - \theta_L^t) + \theta_L^t(\theta_H^t - \theta_L^t)
\end{align*}
\]
holds.

Analogously, type $HL$ has no incentive to mimic $HH$ if
\[
\begin{align*}
    w(s_{HH}^*, t_{HH}^*) - c_{HL}(s_{HH}^*, t_{HH}^*) & \leq w(s_{HL}^*, t_{HL}^*) - c_{HL}(s_{HL}^*, t_{HL}^*) \\
    \Leftrightarrow \theta_H^s + \theta_L^t - \frac{s_{HH}^*}{\theta_H^s} - \frac{t_{HH}^*}{\theta_L^t} & \leq \theta_H^s + \theta_L^t - \frac{s_{HL}^*(\theta_H^s - \theta_L^s)}{\theta_H^s} \\
    \Leftrightarrow s_{HH} + \frac{\theta_H^s}{\theta_L^t}t_{HH} & \geq \theta_H^s(\theta_H^t - \theta_L^t) + \theta_L^t(\theta_H^t - \theta_L^t)
\end{align*}
\]
is fulfilled. Both conditions together are the incentive compatibility condition that prevents $HH$ from mimicing $HL$ and vice versa. Because of the pure cost effect of signalling the lower bound of the second condition, i.e.
\[
\begin{align*}
    s_{HH} + \frac{\theta_H^s}{\theta_L^t}t_{HH} & = \theta_H^s(\theta_H^t - \theta_L^t) + \theta_L^t(\theta_H^s - \theta_L^s) \\
    \Leftrightarrow s_{HH} & = \theta_H^s(\theta_H^t - \theta_L^t) + \theta_L^t(\theta_H^s - \theta_L^s) - \frac{\theta_H^s}{\theta_L^t}t_{HH}
\end{align*}
\]
is a necessary condition for a separating PBE. However additionally, type
$HH$ does not have an incentive to mimic type $LH$ and vice versa. Therefore,

$$w(s_{LH}^*, t_{LH}^*) - c_{HH}(s_{LH}^*, t_{LH}^*) \leq w(s_{HH}, t_{HH}) - c_{HH}(s_{HH}, t_{HH})$$

$\Leftrightarrow \theta^*_L + \theta^*_H - \frac{\theta^*_L(\theta^*_H - \theta^*_L)}{\theta^*_H} \leq \theta^*_H + \theta^*_L - \frac{s_{HH} - t_{HH}}{\theta^*_H}$

$\Leftrightarrow \frac{\theta^*_H}{\theta^*_L} s_{HH} + t_{HH} \leq \theta^*_H(\theta^*_H - \theta^*_L) + \theta^*_L(\theta^*_H - \theta^*_L)$

and

$$w(s_{HH}, t_{HH}) - c_{LH}(s_{HH}, t_{HH}) \leq w(s_{LH}^*, t_{LH}^*) - c_{LH}(s_{LH}^*, t_{LH}^*)$$

$\Leftrightarrow \theta^*_H + \theta^*_L - \frac{s_{HH} - t_{HH}}{\theta^*_L} \leq \theta^*_H + \theta^*_L - \frac{\theta^*_L(\theta^*_H - \theta^*_L)}{\theta^*_H}$

$\Leftrightarrow \frac{\theta^*_H}{\theta^*_L} s_{HH} + t_{HH} \geq \theta^*_H(\theta^*_H - \theta^*_L) + \theta^*_L(\theta^*_H - \theta^*_L)$

must hold. Both conditions together are the incentive compatibility constraint that prevent $HH$ from mimicking $LH$ and vice versa. Cause of the pure cost effect of signalling the lower bound of the second condition, i.e.

$$\frac{\theta^*_H}{\theta^*_L} s_{HH} + t_{HH} = \theta^*_H(\theta^*_H - \theta^*_L) + \theta^*_L(\theta^*_H - \theta^*_L)$$

$\Leftrightarrow s_{HH} = \theta^*_L(\theta^*_H - \theta^*_L) + \frac{(\theta^*_H - \theta^*_L)\theta^*_L}{\theta^*_H} - \frac{\theta^*_L}{\theta^*_H} t_{HH}$ \hspace{1cm} (11)

is a necessary condition for a PBE. In a separating PBE type $HH$ neither mimics $HL$ nor $LH$. Thus, conditions (10) and (11) must hold. Both linear functions describe the lower bound of the area that fulfils both incentive compatibility constraints. Because of the pure cost effect of signalling the optimal strategy is element of this lower bound. To make sure that the optimal strategy is unique the slope of this lower bound must be unequal to the slope of the cost function of $HH$.\footnote{If this condition is not fulfilled the minimal cost combination would be tangent to the area that fulfills the incentive compatibility constraints on a whole section represented by a part of the linear function (10) or (11) and not to a unique point.} The cost function of type $HH$ is $c_{HH}(s_{HH}, t_{HH}) = (s_{HH}/\theta^*_H) + (t_{HH}/\theta^*_H)$. So, the slope of this function in a $s$-$t$-area is $-(\theta^*_H/\theta^*_H)$. As the slope of equation (10) in such an area is $-(\theta^*_L/\theta^*_L)$ and the slope of equation (11) is $-(\theta^*_L/\theta^*_H)$ there is a unique optimal strategy.
of $HH$ that is given by the point of intersection of the linear combinations (10) and (11). Calculating this point of intersection leads to

\[
\theta^*_H(\theta^*_H - \theta^*_L) + \theta^*_L(\theta^*_H - \theta^*_L) - \frac{\theta^*_H t_{HH}}{\theta^*_L} = \theta^*_L(\theta^*_H - \theta^*_L) + \frac{(\theta^*_H - \theta^*_L)\theta^*_L t^*_H}{\theta^*_L} - \frac{\theta^*_H t_{HH}}{\theta^*_L},
\]

\[
\Leftrightarrow \theta^*_H\theta^*_H(\theta^*_H - \theta^*_L) - \theta^*_L\theta^*_L(\theta^*_H - \theta^*_L) = \frac{\theta^*_H\theta^*_H - \theta^*_L\theta^*_L}{\theta^*_L} t_{HH},
\]

\[
\Leftrightarrow (\theta^*_H\theta^*_H - \theta^*_L\theta^*_L)(\theta^*_H - \theta^*_L) = \frac{\theta^*_H\theta^*_H - \theta^*_L\theta^*_L}{\theta^*_L} t_{HH},
\]

\[
\Leftrightarrow t^*_{HH} = \theta^*_L(\theta^*_H - \theta^*_L).
\]

Inserting this in equation (10) gives the first part of the equilibrium signal $s^* = \theta^*_L(\theta^*_H - \theta^*_L)$.

\[\square\]

**The pooling PBE in the two-dimensional case:**

The pooling PBE in the two dimensiona case is

\[
(s^*, t^*) \equiv (s^*_{ij}, t^*_{ij}) = (0, 0) \quad \forall i, j \in \{H, L\},
\]

\[
w(s^*, t^*) = \sum_{ij} \alpha_{ij}(\theta^*_i + \theta^*_j),
\]

\[
\mu(ij | (s, t)) = \left(I \cdot (\alpha_{HH} + \alpha_{HL})\theta^*_L(\theta^*_H - \theta^*_L), J \cdot (\alpha_{HH} + \alpha_{LH})\theta^*_L(\theta^*_H - \theta^*_L)\right) = \alpha_{ij},
\]

where $I = 1$ if $i = H$ and 0 otherwise and $J = 1$ if $j = H$ and 0 otherwise.

The proof of this result is as follows:

Universities’ wage setting seeing the pooled signal $(s^*, t^*)$ is

\[
w(s^*, t^*) = \sum_{ij} \alpha_{ij}(\theta^*_i + \theta^*_j).
\]

Since each agent $ij$ can always get the lowest wage $\theta^*_i + \theta^*_j$ utility with
the pooled signal must be higher than this reward, i.e.

\[ \theta^s_L + \theta^t_L \leq w(s, t) - c_{ij}(s, t) \]
\[ \Leftrightarrow \theta^s_L + \theta^t_L \leq \sum_{ij} \alpha_{ij} (\theta^s_i + \theta^t_j) - \frac{s}{\theta^s_i} - \frac{t}{\theta^t_j} \]
\[ \Leftrightarrow s + \frac{t}{\theta^s_j} \leq (\alpha_{HH} + \alpha_{HL} - 1)\theta^s_L + (\alpha_{HH} + \alpha_{HL})\theta^t_H + (\alpha_{LL} + \alpha_{HL} - 1)\theta^t_L + (\alpha_{HH} + \alpha_{LH})\theta^t_H \]
\[ \Leftrightarrow \frac{s}{\theta^s_i} + \frac{t}{\theta^t_j} \leq -(\alpha_{HH} + \alpha_{HL})\theta^s_L + (\alpha_{HH} + \alpha_{HL})\theta^t_H - (\alpha_{HH} + \alpha_{LH})\theta^t_L + (\alpha_{HH} + \alpha_{LH})\theta^t_H \]
\[ \Leftrightarrow \frac{s}{\theta^s_i} + \frac{t}{\theta^t_j} \leq (\alpha_{HH} + \alpha_{HL})(\theta^s_H - \theta^s_L) + (\alpha_{HH} + \alpha_{LH})(\theta^t_H - \theta^t_L) \]
\[ \Leftrightarrow t \leq (\alpha_{HH} + \alpha_{HL})s. \]

This is just a linear equation in \( s \). Clearly type \( LL \) is the restricting type. Thus every linear combination of \( (s, t) \) for which

\[ \frac{s}{\theta^s_L} + \frac{t}{\theta^t_L} \leq (\alpha_{HH} + \alpha_{HL})(\theta^s_H - \theta^s_L) + (\alpha_{HH} + \alpha_{LH})(\theta^t_H - \theta^t_L) \]

holds meets the incentive compatibility constraints. However, the pure cost effect of signalling makes \( (s^*, t^*) = (0, 0) \) the unique pooling PBE.

The incentive conditions that prevent agents from breaking out of the pooling PBE are illustrated in figure 5. It refers to the symmetric case with \( \theta^s_H = \theta^t_H = 2, \theta^s_L = \theta^t_L = 1 \) and \( \alpha_{ij} = \frac{1}{4} \forall i, j \in \{H, L\} \). The grey area describes all combinations that prevent \( LL \) - and therefore also the other types - from breaking out of the pooling PBE. However, only \( (s^*, t^*) = (0, 0) \) implies minimum costs and therefore is PBE.
Figure 5: Incentive compatibility constraint of type $LL$ in a pooling PBE of two dimensions

**Part C: Two-dimensional case with time constraints**

**Proof of proposition 4:**
The sequence of the proof of the partial separating PBE $(LL, HL, (LH, HH))$ is as follows:
First of all I find the optimal strategy for $HL$ that prevents him from mimicking $LL$. Secondly I give the incentive compatibility constraint that prevents $LL$ from mimicing $(LH, HH)$ and vice versa. Thirdly, I give the incentive compatibility constraint that prevents $HL$ from mimicing $(LH, HH)$ and vice versa. Step two and three together result in an optimal strategy for $(LH, HH)$.

In a PBE where $LL$ is separated he has no incentive to signal. Thus, $(s_{LL}^*, t_{LL}^*) = (0, 0)$. Then refering to the first step $HL$ signals $(s_{HL}^*, t_{HL}^*) = (\theta_L^*(\theta_H - \theta_L^*), 0)$ to prevent $LL$ from mimicing him. This strategy directly results from the separating PBE.
To make sure that in a second step $LL$ does not mimic $(LH, HH)$

\[
\begin{align*}
& w_{LL} - c_{LL}(s_{LL}^*, t_{LL}^*) \geq w_{(LH, HH)} - c_{LL}(s_{(LH, HH)}, t_{(LH, HH)}) \\
\iff & \theta_s^L + \theta_t^L \geq \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^L + \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H + \theta_t^L \\
& \theta_s^L \frac{s_{(LH, HH)}}{t_{(LH, HH)}} - \theta_t^L \frac{t_{(LH, HH)}}{t_{(LH, HH)}} \geq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L. \\
\iff & \theta_s^L \frac{s_{(LH, HH)}}{\theta_s^L} + \theta_t^L \frac{t_{(LH, HH)}}{\theta_t^L} \geq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L. \\
\end{align*}
\]

must hold.

Analogously, $LH$ and therefore $(LH, HH)$ does not mimic $LL$ if

\[
\begin{align*}
& w_{LL} - c_{LL}(s_{LL}^*, t_{LL}^*) \leq w_{(LH, HH)} - c_{LL}(s_{(LH, HH)}, t_{(LH, HH)}) \\
\iff & \theta_s^L + \theta_t^L \leq \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^L + \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H + \theta_t^L \\
& \theta_s^L \frac{s_{(LH, HH)}}{t_{(LH, HH)}} - \theta_t^L \frac{t_{(LH, HH)}}{t_{(LH, HH)}} \leq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L. \\
\iff & \theta_s^H \frac{s_{(LH, HH)}}{\theta_s^H} + \theta_t^L \frac{t_{(LH, HH)}}{\theta_t^L} \leq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L.
\end{align*}
\]

is fulfilled. Since signals can not be negative a necessary condition for the existence of the partial separating PBE is $C1_{(LH, HH)} > 0$. I will come to this later on.

To prevent $HL$ from mimicking $(LH, HH)$ (third step) the following condition must hold:

\[
\begin{align*}
& w_{HL} - c_{HL}(s_{HL}^*, t_{HL}^*) \geq w_{(LH, HH)} - c_{HL}(s_{(LH, HH)}, t_{(LH, HH)}) \\
\iff & \theta_s^H + \theta_t^L - \frac{\theta_s^H}{\theta_s^H} (\theta_s^H - \theta_t^L) \geq \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H + \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H + \theta_t^L \\
& \theta_s^H \frac{s_{(LH, HH)}}{t_{(LH, HH)}} - \theta_t^L \frac{t_{(LH, HH)}}{t_{(LH, HH)}} \geq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L. \\
\iff & \theta_s^H \frac{s_{(LH, HH)}}{\theta_s^H} + \theta_t^L \frac{t_{(LH, HH)}}{\theta_t^L} \geq \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}} \theta_s^H - (1 - \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}) \theta_s^L + \theta_t^H - \theta_t^L.
\end{align*}
\]
\[-(1 - \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}})\theta^s_H + (1 + \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}})\theta^s_L + \theta^t_H - \theta^t_L - \frac{(\theta^s_L)^2}{\theta^s_H}\]

\(\equiv c_{2(LH,HH)}\)

Analogously, to prevent HH and therefore \((LH,HH)\) from mimicking HL

\[w_{HL} - c_{HH}(s^*_L, t^*_L) \leq w_{(LH,HH)} - c_{HH}(s_{(LH,HH)}, t_{(LH,HH)})\]

\[\Leftrightarrow \theta^s_H + \theta^t_L - \frac{\theta^s_H(\theta^s_H - \theta^s_L)}{\theta^s_H} \leq \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}}\theta^s_L + \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}}\theta^s_H + \theta^t_L - \frac{s_{(LH,HH)}}{\theta^s_H} - \frac{t_{(LH,HH)}}{\theta^t_H} \]

\[\Leftrightarrow \frac{s_{LH,HH}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} \leq \]

\[-(1 - \frac{\alpha_{HH}}{\alpha_{LH} + \alpha_{HH}})\theta^s_H + (1 + \frac{\alpha_{LH}}{\alpha_{LH} + \alpha_{HH}})\theta^s_L + \theta^t_H - \theta^t_L - \frac{(\theta^s_L)^2}{\theta^s_H}\]

\(\equiv c_{2(LH,HH)}\)

must hold. A necessary condition for the existence of the partial separating PBE is again that \(C^2_{2(LH,HH)} > 0\) is fulfilled. This condition is even stronger than \(C^2_{1(LH,HH)}\) from above because

\[C^2_{2(LH,HH)} - C^1_{(LH,HH)} = -\theta^s_H + 2\theta^s_L - \frac{(\theta^s_L)^2}{\theta^s_H}\]

\[= -\frac{(\theta^s_H)^2 + 2\theta^s_H\theta^s_L - (\theta^s_L)^2}{\theta^s_H}\]

\[= -\frac{(\theta^s_H - \theta^s_L)^2}{\theta^s_H} < 0\]

holds. Although \(C^2_{2(LH,HH)} < C^1_{(LH,HH)}\) is fulfilled one can not directly see if equation (12) or equation (13) is the stronger condition because of the different LHS. If you compare both conditions you find that the relationship depends on the exact parameter values. However, I show that the optimal - cost minimal - behavior for type LH and HH is the same regardless whether equation (12) or equation (13) is the stronger condition.

Thus assume that equation (12) is stronger than equation (13) then
\[ t_{(LH,HH)} = \theta^t_L C1_{(LH,HH)} - \frac{\partial L}{\partial L} s_{(LH,HH)} \] holds. This in mind costs of \( LH \) are

\[ \frac{s_{(LH,HH)}}{\theta^s_L} + \frac{t_{(LH,HH)}}{\theta^t_H} = \frac{1}{\theta^s_L} \left( 1 - \frac{\theta^t_L}{\theta^t_H} \right) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C1_{(LH,HH)}. \]

Since costs increase in \( s_{(LH,HH)} \) the optimal strategy of \( LH \) is \( s_{(LH,HH)} = 0 \).

Analogously, costs of \( HH \) are

\[ \frac{s_{(LH,HH)}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} = \left( 1 - \frac{\theta^t_L}{\theta^t_H} \right) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C1_{(LH,HH)}. \]

If \( \frac{1}{\theta_H} - \frac{\theta^t_L}{\theta^t_H} \leq 0 \) holds the optimal strategy is to maximise \( s_{(LH,HH)} \). However, then the partial separating PBE is destroyed. Type \( LH \) and \( HH \) do not play the same strategy. Therefore \( \frac{1}{\theta_H} - \frac{\theta^t_L}{\theta^t_H} \geq 0 \) must hold to ensure the described PBE. If equation (12) is the stronger condition one \( \theta^t_L \theta^t_H \geq \theta^t_H \theta^t_L \) becomes a necessary condition of the partial separating PBE.

Now assume that instead of equation (12) equation (13) is the stronger condition then \( t_{(LH,HH)} = \theta^t_L C2_{(LH,HH)} - \frac{\partial L}{\partial H} s_{(LH,HH)} \) holds and costs of type \( LH \) are

\[ \frac{s_{(LH,HH)}}{\theta^s_L} + \frac{t_{(LH,HH)}}{\theta^t_H} = \left( \frac{1}{\theta^s_L} - \frac{\theta^t_L}{\theta^t_H} \right) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)} \]

\[ = \left( \frac{\theta^t_H \theta^t_L - \theta^t_L \theta^t_H}{\theta^t_L \theta^t_H} \right) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)}. \]

Again it is optimal for type \( LH \) to play \( s_{(LH,HH)} = 0 \). Analogously, costs of type \( HH \) are

\[ \frac{s_{(LH,HH)}}{\theta^s_H} + \frac{t_{(LH,HH)}}{\theta^t_H} = \frac{1}{\theta^s_H} \left( 1 - \frac{\theta^t_L}{\theta^t_H} \right) s_{(LH,HH)} + \frac{\theta^t_L}{\theta^t_H} C2_{(LH,HH)}. \]

As costs increase in \( s_{(LH,HH)} \) type \( HH \) sets \( s_{(LH,HH)} = 0 \).

Summarising, under both assumption \( s^*_H(\LH,HH) = 0 \) is an optimal strategy for both pooling types. This reduces condition (12) to \( t_{(LH,HH)} = \theta^t_L C1_{(LH,HH)} \).
and condition (13) to \( t_{(LH,HH)} = \theta^t_L C2_{(LH,HH)} \). With \( C1_{(LH,HH)} > C2_{(LH,HH)} \) from the above condition (12) becomes the crucial condition for the existence of the partial separating PBE. The equilibrium strategy of \((LH, HH)\) is \( (s^*_{(LH,HH)}, t^*_{(LH,HH)}) = (0, \theta^t_L C1_{(LH,HH)}) \). A necessary condition for the existence of the equilibrium is \( \theta^*_L \theta^t_H > \theta^*_H \theta^t_L \).

Finally, the proof of the second partial PBE, i.e. of \((LL, LH, (HL, HH))\) is analogous and is therefore not specified here.

\[ \square \]

References


DISKUSSIONSPAPIERE DER FÄCHERGRUPPE VOLKSWIRTSCHAFTSLEHRE
DISCUSSION PAPERS IN ECONOMICS

Die komplette Liste der Diskussionspapiere ist auf der Internetseite veröffentlicht / for full list of papers see:
http://fgwl.hsu-hh.de/wp-vwl

2009
95 Schneider, Andrea. Science and teaching: Two-dimensional signalling in the academic job market, August 2009.
93 Hackmann, Johannes. Ungereimtheiten der traditionell in Deutschland vorherrschenden Rechtfertigungsansätze für das Ehegattensplitting, Mai 2009.
92 Schneider, Andrea; Klaus W. Zimmermann. Mehr zu den politischen Segnungen von Föderalismus, April 2009.
91 Beckmann, Klaus; Schneider, Andrea. The interaction of publications and appointments - New evidence on academic economists in Germany, März 2009.
90 Beckmann, Klaus; Schneider, Andrea. MeinProf.de und die Qualität der Lehre, Februar 2009.
89 Berlemann, Michael; Hielscher, Kai. Measuring Effective Monetary Policy Conservatism, February 2009.
88 Horgos, Daniel. The Elasticity of Substitution and the Sector Bias of International Outsourcing: Solving the Puzzle, February 2009.
87 Rundshagen, Bianca; Zimmermann, Klaus W.. Buchanankaoperation und Internationale Öffentliche Güter, Januar 2009.

2008
82 Beckmann, Klaus; Engelmann, Dennis. Steuerwettbewerb und Finanzverfassung, Juli 2008.
76 Beckmann, Klaus; Gattke, Susan. Status preferences and optimal corrective taxes: a note, February 2008.

2007
74 Dewenter, Ralf. Netzneutralität, Dezember 2007